# Channel Characterization using Jitter Measurements

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***Abstract*—This paper proposes a technique for characterization of the frequency-dependent losses in a wireline communications link. By using the measured jitter at the output of a receiver front-end as its only input, this method is able to estimate both the pulse response and frequency response of the link, including the effects of the transmitter output, the channel itself, and the receiver front-end. Simulated and measured results verify the accuracy of this technique, which can be used to efficiently adapt the settings of critical circuit blocks in the link, such as equalizer tap weights.**

## I. Introduction

Communicating high-speed data robustly over copper interconnects presents enormous research challenges. These interconnects are harsh communications channels, which introduce frequency-dependent loss and noise. At either end of such links are tunable transmit and receive circuits that generally include equalization, permitting them to accommodate variations in channel length and temperature.

If the channel can be characterized, the correct equalizer settings can be determined. Unfortunately, straightforward attempts to observe the channel response introduce some additional load [1], thereby altering the response. Instead, this work proposes a method to infer the channel response indirectly without loading the channel by observing signals already present at the output of the system.

Amplifiers used for binary data streams are nonlinear, which means that only signals that have already passed through a nonlinearity can be observed at the receiver output without loading the channel. Fortunately, these nonlinearities do not change the relative zero-crossing times of the received waveform. Hence, this paper proposes a technique that requires only information about the relative zero-crossing times, quantified as jitter. A snapshot of the system, highlighting the focus of this project, is shown in Fig. 1.

## II. Channel Characterization

Much past work has been done to predict received jitter from knowledge of the channel response [2], [3]. This work

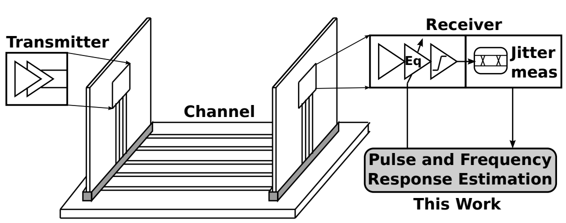


Fig. 1. By examining the measured jitter at the output of a receiver's front-end, this work proposes a method to determine the pulse and frequency response of the system without loading the receiver input.

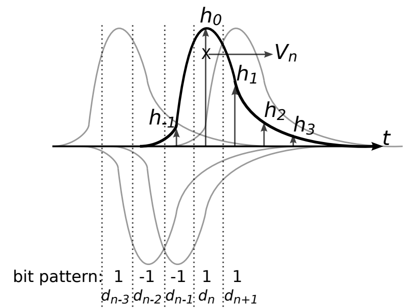


Fig. 2. Previous and subsequent bits interfere with the received voltage causing $V_n$ to differ from its ideal value at $h_0$.

addresses the inverse problem: how to infer the channel response by looking only at received jitter. This section describes how this can be done.

### A. Channel Estimation using Intersymbol Interference

When a random binary sequence is subjected to frequency-dependent losses, this causes the transmitted pulses to spread across multiple symbol periods, resulting in intersymbol interference (ISI). This spreading means that the received voltage at bit $n$, $V_n$, is determined not only by the value of the current bit, $d_n$, but also by contributions from some number of previous and subsequent bits. This concept is illustrated in Fig. 2 where 3 previous bits ($d_{n-1}$ to $d_{n-3}$) each contribute some amount of "postcursor" interference, given by $h_1$ to $h_3$. This figure also shows how 1 subsequent bit ($d_{n+1}$) contributes some amount of "precursor" interference, given by $h_{-1}$.

This same idea can be represented by the equation

$$V_n = d_{n+1}h_{-1} + d_n h_0 + d_{n-1}h_1 + d_{n-2}h_2 + d_{n-3}h_3 + \ldots \quad (1)$$
$$= \sum_k d_{n-k}h_k \quad (2)$$

where negative values of $k$ represent the precursor terms and positive values represent the postcursor terms considered. Therefore the number of pre and postcursor terms considered can be modified by changing the range of $k$. The resulting system of equations that can be used to determine the received voltage resulting from any sequence of bits can be represented in matrix form as

$$[V] = [d]\,[h] \quad (3)$$

$$\begin{bmatrix} V_n \\ V_{n-1} \\ V_{n-2} \\ \vdots \end{bmatrix} = \begin{bmatrix} d_{n+1} & d_n & d_{n-1} & \cdots \\ d_n & d_{n-1} & d_{n-2} & \\ d_{n-1} & d_{n-2} & d_{n-3} & \\ \vdots & & & \end{bmatrix} \begin{bmatrix} h_{-1} \\ h_0 \\ h_1 \\ \vdots \end{bmatrix} \quad (4)$$

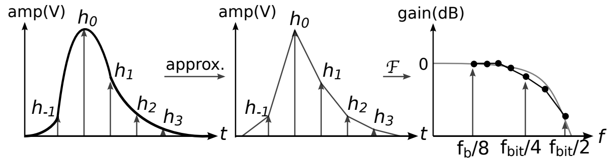


Fig. 3. ISI terms can be used to create an estimate of the pulse response, which can in turn be converted to the frequency response of the channel.

where the dimensions of each matrix are:

- $[V]$ is an $r$ x 1 matrix where $r$ is the number of received voltages considered.
- $[h]$ is a $c$ x 1 matrix where $c$ is the total number of ISI terms that contribute to each received voltage.
- $[d]$ is an $r$ x $c$ matrix.

Therefore, for any known sequence of received bits and corresponding voltage levels, it is possible to determine the ISI terms, $h$, by rearranging matrices such that

$$\begin{bmatrix} h_{-1} \\ h_0 \\ h_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} d_{n+1} & d_n & d_{n-1} & d_{n-c+2} \\ d_n & d_{n-1} & d_{n-2} \cdots & d_{n-c+1} \\ d_{n-1} & d_{n-2} & d_{n-3} & d_{n-c} \\ \vdots & & \ddots & \vdots \\ d_{n-r+2} & d_{n-r-1} & d_{n-r} \cdots & d_{n-r-c+2} \end{bmatrix}^{-1} \begin{bmatrix} V_n \\ V_{n-1} \\ V_{n-2} \\ \vdots \end{bmatrix} \quad (5)$$

Note that, in order for a solution to Equation (5) to exist, $r$ must be $\geq c$. The ISI terms determined from this equation can then be used to reconstruct an estimate of the pulse response, which can be converted to an approximation of the channel response by taking the Fourier transform, as shown in Fig. 3.

### *B. Pulse Response Estimation using Measured Jitter*

When the received signal passes through nonlinearities in the receiver, the voltage levels are distorted and the relationship described by Equation (5) no longer holds. This is illustrated by Fig. 4, which shows that when a pseudo-random binary sequence (PRBS) is passed through the receiver's transfer function, the variations in the received voltage levels, $V_n$, are reduced. Since the nonlinearities in the transfer function are found near the limits of the input voltage swing, the voltages near the zero crossings of the signal experience linear amplification in the receiver, represented by the highlighted region of Fig. 4. This preserves the effects of ISI.

Therefore, this work proposes using samples of the received signal during data transitions (i.e. $V_{n+0.5}$) to reconstruct the pulse response, and thus the frequency response of the channel, as shown in Fig. 5. However, since the precise voltage of the received signal at the ideal transition time is difficult to obtain, this work instead proposes to use the jitter, given by $\Delta t_{n+0.5}$ in Fig. 4, which can be readily obtained by monitoring the receiver output. If we assume that the slope of the signal, $m$, is constant in the small region near the zero crossing, then the voltage $V_{n+0.5}$ can be found to be

$$V_{n+0.5} = m\Delta t_{n+0.5}. \quad (6)$$

Note that, due to the dependence on measured jitter, this

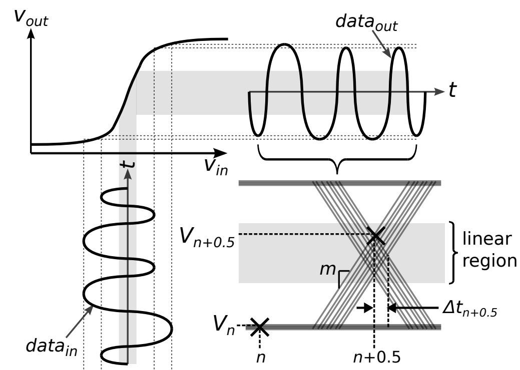


Fig. 4. When PRBS data passes through the nonlinearities of the receiver transfer function, voltages at the centers of the bit periods are distorted, while those near the zero crossings are preserved. The measured jitter can therefore be used to recreate the pulse response of the system.

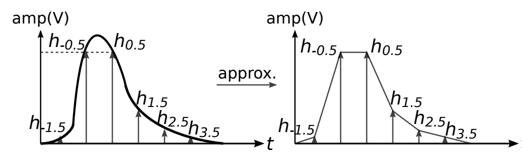


Fig. 5. Samples of the data taken near the zero-crossings can also be used to approximate the pulse response of a system.

equation holds only when there is a transition between bits $n$ and $n+1$.

By shifting the sampling time by a half of one unit interval (UI), the precursor and postcursor ISI contributions must also shift by this same amount and Equation (4) becomes

$$V_{n+0.5} = d_{n+2}h_{-1.5} + d_{n+1}h_{-0.5} + d_n h_{0.5} + d_{n-1}h_{1.5} + ... \quad (7)$$

Note that since this equation only holds when there is a transition in the data, this implies that

$$d_{n+1} = -d_n. \quad (8)$$

In addition, since $h_{0.5}$ and $h_{-0.5}$ are defined by the rising and falling crossings of a single threshold level, as shown in Fig. 5, we see that

$$h_{0.5} = h_{-0.5}. \quad (9)$$

By substituting Equations (6), (8) and (9) into Equation (11) the expression for $V_{n+0.5}$ becomes

$$m\Delta t_{n+0.5} = d_{n+2}h_{-1.5} + d_{n-1}h_{1.5} + d_{n-2}h_{2.5} + ... \quad (10)$$

and similar equations can be formulated for each transition in any arbitrary bit sequence.

To illustrate this procedure, Equation (10) is expanded into a system of equations in matrix form for the example case of the received bits shown in Fig. 6. This results in

$$\begin{bmatrix} m\Delta t_{n+0.5} \\ m\Delta t_{n-3.5} \\ m\Delta t_{n-4.5} \\ \vdots \end{bmatrix} = \begin{bmatrix} d_{n+2} & d_{n-1} & d_{n-2} & \\ d_{n-1} & d_{n-4} & d_{n-5} & \cdots \\ d_{n-2} & d_{n-5} & d_{n-6} & \\ \vdots & & & \end{bmatrix} \begin{bmatrix} h_{-1.5} \\ h_{1.5} \\ h_{2.5} \\ \vdots \end{bmatrix} \quad (11)$$

which can be rearranged to obtain the coefficients of the pulse

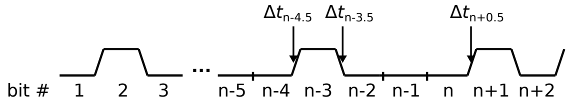


Fig. 6. An example sequence of bits used to illustrate the proposed pulse response estimation.

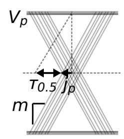


Fig. 7. Close-up of the transitions of an eye diagram. The zero crossings of the pulse response are determined using the peak jitter, $j_p$, and the slope $m$.

response as

$$\begin{bmatrix} \tau_{-1.5} \\ \tau_{1.5} \\ \tau_{2.5} \\ \vdots \end{bmatrix} = \begin{bmatrix} d_{n+2} & d_{n-1} & d_{n-2} & \\ d_{n-1} & d_{n-4} & d_{n-5} & \cdots \\ d_{n-2} & d_{n-5} & d_{n-6} & \\ \vdots & & & \end{bmatrix}^{-1} \begin{bmatrix} \Delta t_{n+0.5} \\ \Delta t_{n-3.5} \\ \Delta t_{n-4.5} \\ \vdots \end{bmatrix} \quad (12)$$

where $\tau$ is defined as

$$\tau_n = \frac{h_n}{m} \quad (13)$$

Note that, for the purpose of determining the frequency response of the channel, scaling the $h$ coefficients by a constant $m$ in this way has no effect.

In order to complete the reconstruction of the pulse response, term $\tau_{0.5} = \tau_{-0.5}$ must also be determined. If we define the peak jitter, $j_p$, to be

$$j_p = \sum_{k \neq 0.5} |\tau_k| \quad (14)$$

then the picture of an example zero crossing of an eye diagram in Fig. 7 shows that

$$\tau_{0.5} = \frac{V_p}{m} - j_p. \quad (15)$$

Since $m$ is unknown, however, $\tau_{0.5}$ cannot be determined directly. Instead a reasonable approximation can be obtained if we assume that

$$\frac{V_p}{m} \approx \frac{\mathrm{UI}}{4} \quad (16)$$

and therefore

$$\tau_{0.5} \approx \frac{\mathrm{UI}}{4} - j_p. \quad (17)$$

Equations (13) and (17) can then be used to reconstruct an estimate of the pulse response as shown in Fig. 8. To verify the accuracy of these estimates, simulations using PRBS7 data were performed using Matlab to determine the pulse responses for a variety of channel models. The output eye diagrams and pulse responses of these simulations are shown in Fig. 9 for (a) 10 Gb/s over a 10 m length of coaxial cable, (b) 30 Gb/s over a second order low-pass channel response with a 3-dB frequency ($f_{3dB}$) of 7.5 GHz and (c) 30 Gb/s over a third order low-pass response with $f_{3dB}$=11.1 GHz. Although the characteristics of these channels vary significantly, each result shows good agreement with the estimate produced using the method described above. It should be noted that in each case, a value is chosen for $m$ in order to convert the pulse response coefficients from $\tau$ in units of seconds to $h$ in units of volts, as described by Equation (13).

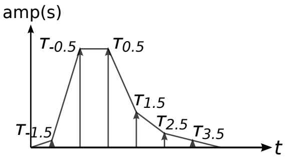


Fig. 8. The pulse response can be reconstructed using the $\tau$ values instead of voltage coefficients.

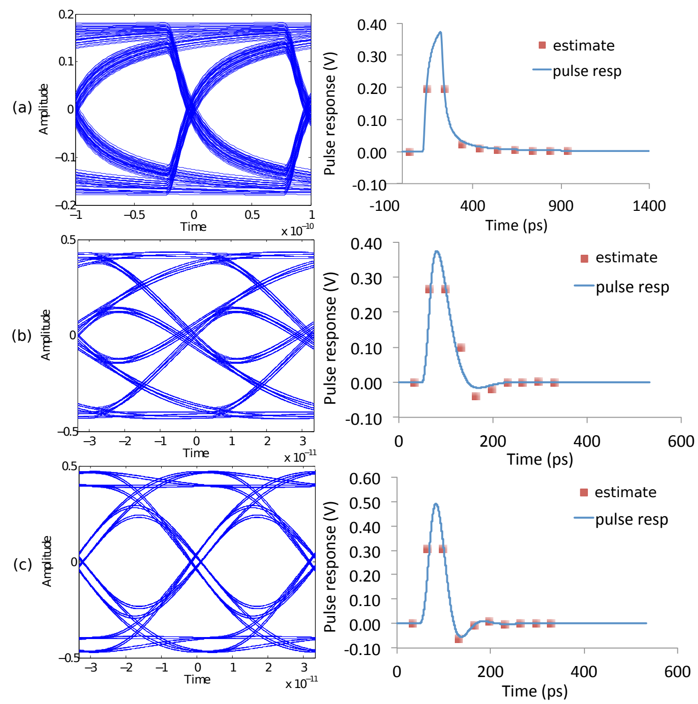


Fig. 9. The eye diagrams and pulse responses are compared to the pulse response reconstructions for (a) 10 Gb/s over a 10 m coaxial cable channel, (b) 30 Gb/s over a second-order low-pass filter channel with $f_{3dB}$=7.5 GHz and (c) 30 Gb/s over a third-order low-pass filter channel with $f_{3dB}$=11.1 GHz.

### C. Frequency Response Estimation

Having achieved a good approximation of the pulse response, we next attempt to determine the frequency response of the channel by taking the Fourier transform of the reconstructed pulse response. Since this pulse consists of points spaced at intervals of the bit period, $T_b$, the resulting frequency response contains information for frequencies at $1/(2NT_b)$ where $N$ is an integer $\geq 1$. Using this technique, the frequency responses of the three channels used to create the pulse responses shown in Fig. 8 are plotted in the left-hand column of Fig. 10.

From these results, it is apparent that the accuracy suffers when the channel loss is low. This is due in part to the fact

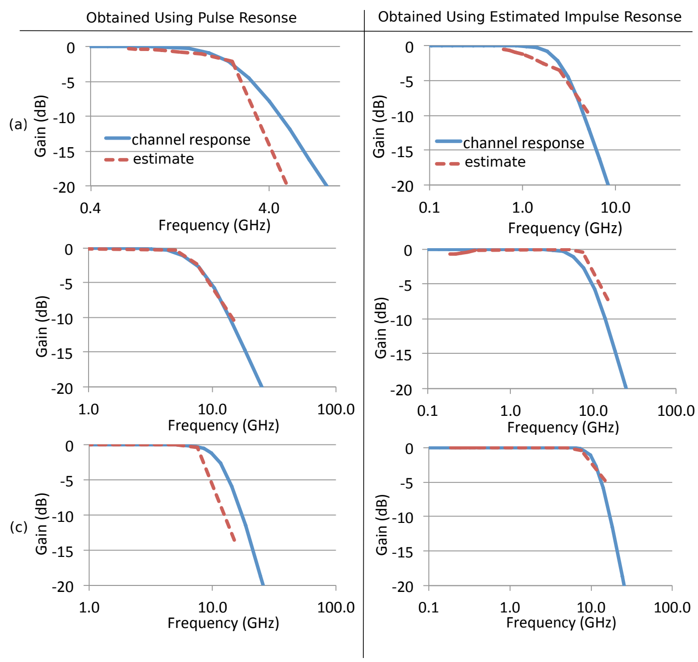


Fig. 10. The frequency responses of the channels reported in Fig. 9 are compared to the estimates obtained using the Fourier transform of the pulse response (left column) or impulse response (right column).

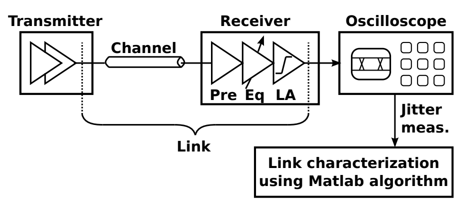


Fig. 11. Test setup used to verify the performance of the proposed link characterization technique.

that the pulse response should be converted to the impulse response in order to obtain accurate channel information. This requires deconvolution of the pulse response with an ideal pulse of width equal to the bit period $T_b$. However, due to the limited number of samples available in the reconstructed pulse, this deconvolution is difficult to perform accurately. Instead, an approximation of the deconvolution can be achieved by removing $\tau_{0.5}$ from the pulse response. The estimates of the channel response produced in this way are shown in the right-hand column of Fig. 10.

## III. Measurement Results

To demonstrate the proposed algorithm, jitter measurements of a physical wireline link, operating as shown in Fig. 11, were used to reconstruct the pulse and frequency responses of the link. The results are shown in Fig. 12. Since in this case the link includes the transmitter output, the channel and the receiver front-end, which includes equalization (Eq) and limiting amplifiers (LA), neither the pulse nor frequency response of the physical system can be determined in any other way. While this means that the measured results cannot be compared to any known data, it also highlights usefulness of

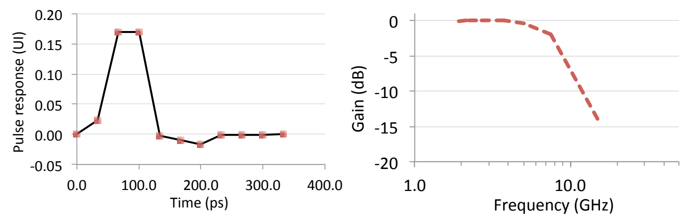


Fig. 12. Reconstructed pulse and frequency response of a measured wireline communication link.

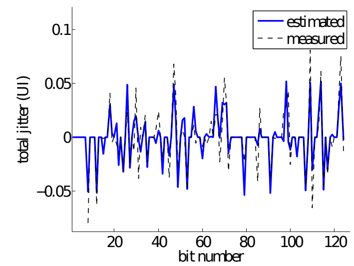


Fig. 13. The accuracy of the reconstructed pulse response can be tested by using it to predict the jitter of a sequence of known bits.

the proposed characterization technique.

As an alternate means of verification, the pulse response can be used to predict the jitter at each transition of a known sequence of bits. To accomplish this, the jitter generated at each transition in a sequence of 127 bits sent across this link were measured and compared to those predicted for the same sequence of bits using the reconstructed pulse response. The measured and predicted jitter for each of these bits are plotted in Fig. 13. The agreement in this figure indicates that the pulse response is an accurate representation of the physical system. Differences between the measured and simulated data are due in part to random noise in the measured results.

## IV. Conclusions

This paper has presented a technique to determine the pulse and frequency responses of a wireline link. This method overcomes the issue of nonlinearities in the receiver circuitry since it requires only the measured jitter values at the receiver output. Since the method avoids loading the signal path, it has no impact on the received signal quality and allows the system to operate at its maximum possible speed.

Simulated and measured results validate the accuracy of this technique. Although it was beyond the scope of this work, the resulting pulse or frequency responses could be used to adapt gain or equalization controls.

## References


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