A Genetic Algorithm for the Design of Passive Filters and a Distributed Amplifier

by

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Abstract

A genetic algorithm for the design of passive filters and a distributed amplifier is presented. The characteristics of the algorithm are studied through simple design examples that include a resistor divider, a T-filter, and a 5^{th}-order low-pass filter. In each case, the algorithm is able to meet all of the performance requirements. Preliminary results from the design of a distributed amplifier are presented. While the algorithm does not design an amplifier that meets the performance requirements within a reasonable design time, analysis of the evolution indicates that the design does improve with each generation. This demonstrates that the algorithm is able to evolve the circuit to meet the specification.

Acknowledgements

I would first like to thank Professor Anthony Chan Carusone for his guidance and support over this past year. My experience within his group has left a tremendously positive impact on me and has shaped my future ambitions. I would also like to thank my family for their ever-present support, and Diana for always believing in me. Finally, I would like to thank the classmates with whom I have shared the past four years.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1.1 Motivation

The use of a Genetic Algorithm (GA) in this work was inspired by recent work in electrical engineering and computer science. GAs are often used to solve a problem where there is little knowledge of what the solution will look like. These solutions may be non-intuitive or not easily derivable by the designer. GAs can also be used to fine tune a solution once the parameters have been roughly set. In this work, a GA is applied to the design of passive filters and a distributed amplifier. While simple filters are easily analyzed using hand calculations, in order to meet more demanding specifications, filters of higher order are required. The design of such filters is not easily accomplished by hand, especially for a less-experienced designer.

One area where GAs have found a number of applications is in the field of electromagnetics and Radio Frequency (RF) communications. In 2004, a computer scientist at the NASA Ames Research Center presented a monopole wire antenna that was designed to meet a specification for the Space Technology 5 spacecraft. The significance of this contribution lies not only in the fact that an evolutionary algorithm was used, but also that the final design (Figure 1.1) is extremely non-intuitive and would likely never have been developed by a human engineer [1]. The antenna met all of the specifications perfectly. In [2], a GA was used in real-time to electronically tune the radiation pattern and gain of an antenna array. In this case, the hardware-based testing helped the optimal solution to be found very quickly even for a large solution space.

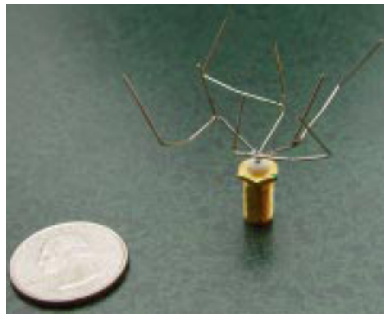


Fig. 1.1: The wire monopole antenna designed at NASA's Ames Research Center using an evolutionary algorithm [1].

In this work, a GA is applied towards research in Integrated Circuit (IC) design. With the emergence of broadband communication standards such as Ultra-wideband (UWB), the work is centered around the optimization of passive filters and a distributed amplifier, a wideband amplifier well-suited to use as a receiver front end.

1.2 Background

1.2.1 Genetic Algorithms

The convergence of a GA to an optimal solution is facilitated through the evolution of a large population of potential solutions. Each solution can be broken down into some number of parameters, or *genes*. This set of genes makes up the *genome*. Associated with each genome is a *cost* which describes how well-suited to the design specification that genome. A genome with a low cost meets the specifications well, while one with a high cost does not. By mutating the genes of each member of the population, the solution space of problem is traversed. GAs arrive at optimal solutions—minima of the cost function—by favouring the low-cost models during the evolution. This can be accomplished through:

- Selection: only the best models of each generation are allowed to reproduce.
- Crossover: the genes of more than one of the best models are used to create the gene(s)

of a descendent.

The algorithm used in this work only uses selection (not crossover), and can be described by the following steps:

1. Create random initial population
2. Test each individual and assign to it a *cost*
3. Select the B individuals with the lowest *cost*
4. Mutate each characteristic of the B best individuals
5. Return to Step 2

The number of generations that the evolution should go on for is not clearly defined. Where design time is a constraint, the designer may allow the algorithm to run for a fixed number of generations, and work with the best output at that time. Alternatively, the GA may be set to run until the the specification is met to within a certain error.

1.2.2 Wideband Communications

As stated above, there has been an emergence in broadband communications, the applications of which include the accomodation of multiple standards as well as bidirectional communications. UWB communication (also known as *impulse radio*) has recently acquired a lot of interest as a means of short-range communications [3]. UWB signals encode information in short pulses that are greatly separated in time, making the bandwidth very large. Since the power of the signal is distributed over such a large range, the amount of power from the signal in any given narrowband channel is small, minimizing interference to other information being sent on the channel. If the signal's power were allowed to increase, however, then it would interfere with information in the band. UWB is thus limited to a short-range.

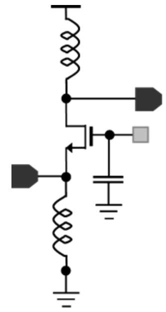


Fig. 1.2: CMOS Common-gate Input Stage

In order to accomodate these new communication systems, circuit blocks, such as receivers, must be developed. There exist many challenges, however, in designing wideband receiver front-ends. As the frequency of operation increases, the parasitics of the compenents become more and more detrimental. These effects can be assuaged through tuning using on-chip load inductors such as presented in [4], but the problem with this is that the tuning is only effective at the design frequency. An alternative approach is to employ a common-gate topology (Figure 1.2). Since the impedance at the input is dominated by $1/g_m$ seen looking up the source of the transistor, a good broadband match can be achieved by choosing $1/g_m$ to be 50Ω. The downside to this approach is that a good noise figure with large gain is difficult to achieve, although cascode stages can be added to help the latter problem.

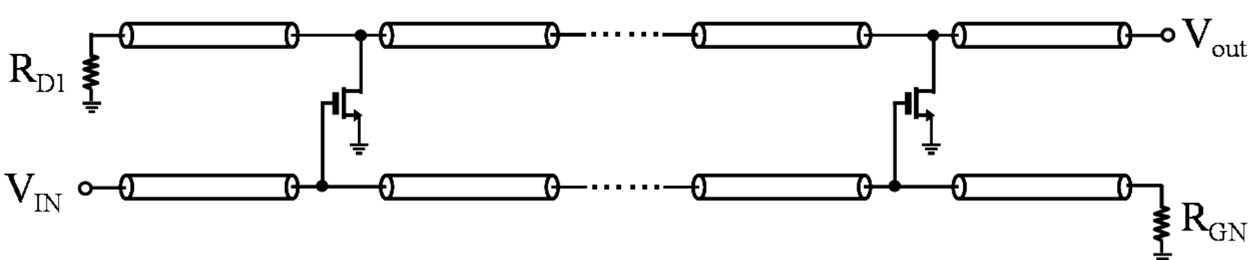


Fig. 1.3: Transmission Line Implementation of a Distributed Amplifier

Another approach, and the one that is examined in this thesis, is the use of a distributed amplifier (Figure 1.3). A distributed amplifier is an RF circuit topology that implements an additive transconductance gain, rather than a multiplicative one often seen in other amplifiers. Traditional designs consist of a series of amplifier stages (in this case, common-source stages) connected together by transmission lines. The voltage signal at the gate of each transistor produces a current signal at the drain. As long as the signals travelling along the gate and drain lines of the amplifier remain in phase, the current signal in the drain will be amplified through superposition.

2. GENETIC ALGORITHM IMPLEMENTATION

The main idea behind GAs is that potential solutions to the problem posed can be broken down into a series of parameters. From these parameters, a cost function can be evaluated to determine how well the model meets the performance criteria.

In this project, however, we are not as interested in finding some abstract relationship between the the genes of each solution and the resulting performance, as we are in creating a program that can easily optimize any circuit able to be described by network parameters. The circuit model is built from its parameters (such as inductances and capacitances), that constitute the *genome* of that model. Each parameter represents a *gene* that can be inherited or mutated from one generation to the next.

2.1 Cost Function

One of the main challenges in the implementation of a genetic algorithm is being able to quickly evaluate and rank the performance of each model in the population. This rank is generated by the *cost function* of the algorithm, and describes how well a particular model meets the design specifications.

For this thesis, the cost function used was a weighted average of the percent differences of the models' performance and the desired specification. A weighting system was used in the event that a particular specification needed to be emphasized. The cost function is given by

$$Cost = \frac{\sum_k w_k \%_{diffk}}{\sum_k w_k} \tag{2.1}$$

where w_k is the weight for specification k, and both summations are over the total number of specifications.

It should be noted that the cost function is often implemented to generate a number normalized to between 0 and 1. In this case, if the percent difference of a specification is large enough, the cost could exceed one. This was not found to be a problem.

2.2 Mutation

As with the cost function, there are many ways to implement the mutation of the models in a GA. Usually, the mutation incorporates the cost function in a way that emphasizes better solutions (ones with lower cost). For example, the number of offspring of a particular model could be made to be inversely proportional to its cost, increasing the number of offspring of the best-performing models.

In this investigation, however, the size of the population was kept constant. Rather, the *magnitude* of the mutation was made to be proportional to the cost function. The formula used to set the parameter P in generation N from generation N-1 is given by

$$P_N = P_{N-1} + 2(Cost)(\chi - \frac{1}{2})min[P_{MAX} - P_{N-1}, P_{N-1} - P_{MIN}] \tag{2.2}$$

where P_{MAX} and P_{MIN} are the maximum and minimum allowable values of P, and χ is an independent and identically distributed (IID) random variable between 0 and 1. From Equation 2.2 it is clear that as long as the value of the cost function remains between 0 and 1, the new parameter is ensured to lie within the acceptable range of values.

For each of the B best performing genomes in each generation, M descendents are produced, where M is the number of genes. A different gene in each descendent is then mutated according to Equation 2.2. In this way, the best models are emphasized, and the solution space is traversed in a manner that takes into account the cost function (through selection of the best genomes). Another common approach is to randomly mutate each of the genes in the descendents. The former method was selected as it was easier to implement and also allows the algorithm to run faster.

3. FILTER EXAMPLES

Before a distributed amplifier design is attempted, it must be shown that the genetic algorithm is able to converge to an optimal solution for simple circuits. This will be accomplished through the analysis of a resistor divider, a low-pass T-filter, and a 5^{th} order low-pass filter design example taken from [5]. Each design will be treated in its own section. Also note that from this point onwards, the circuit parameters will be referred to as *genes*, emphasizing the evolution of the model.

3.1 Resistor Divider

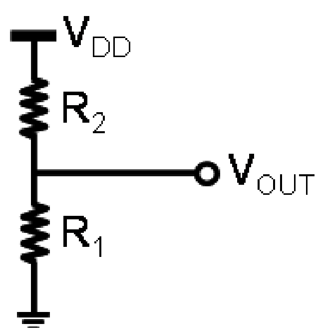


Fig. 3.1: Resistor Divider

The first circuit that will be designed by the GA is the simple resistor divider shown in 3.1. The genes for this circuit are the resistances R_1 and R_2. These values will be determined for a specified output voltage and power dissipation, given by Equations 3.1 and 3.2.

V_{DD}	5V
Generations	20
Population	30
$P_{dissipated}$	0.1W
V_{out}	3V
$R_{1,2}$ (min,max)	(0, 1kΩ)

Tab. 3.1: The setup for a resistor divider optimization.

$$V_{out} = \frac{R_1}{R_1 + R_2} V_{DD} \tag{3.1}$$

$$P_{dissipated} = \frac{V_{DD}^2}{R_1 + R2} \tag{3.2}$$

From Equations 3.1 and 3.2, it is clear that there is a strong connection between the genes (the resistance values) and the circuit's specifications (V_{out} and $P_{dissipated}$). This makes the GA implementation easy for two reasons:

1. The knowledge required by the designer is extremely minimal.

2. There is no ambiguity in the specification—they are a direct result of the main design formulae.

As will become clear through the filter design exampls of Sections 3.2 and 3.3, if either of these two points are not satisfied, the difficulty of the design can be greatly increased.

3.1.1 Experiment

The experimental setup of the resistor divider is summarized in Table 3.1. The algorithm was set to produce an output voltage of 3V while dissipating 0.1W. Both R_1 and R_2 were given an allowable range from 0Ω to $1k\Omega$. Since the solution space – being a function of only two variables – is so small, a population of 30 models with only 20 generations of mutation was implemented. With the algorithm running, the 5 top-performing genomes of each generation

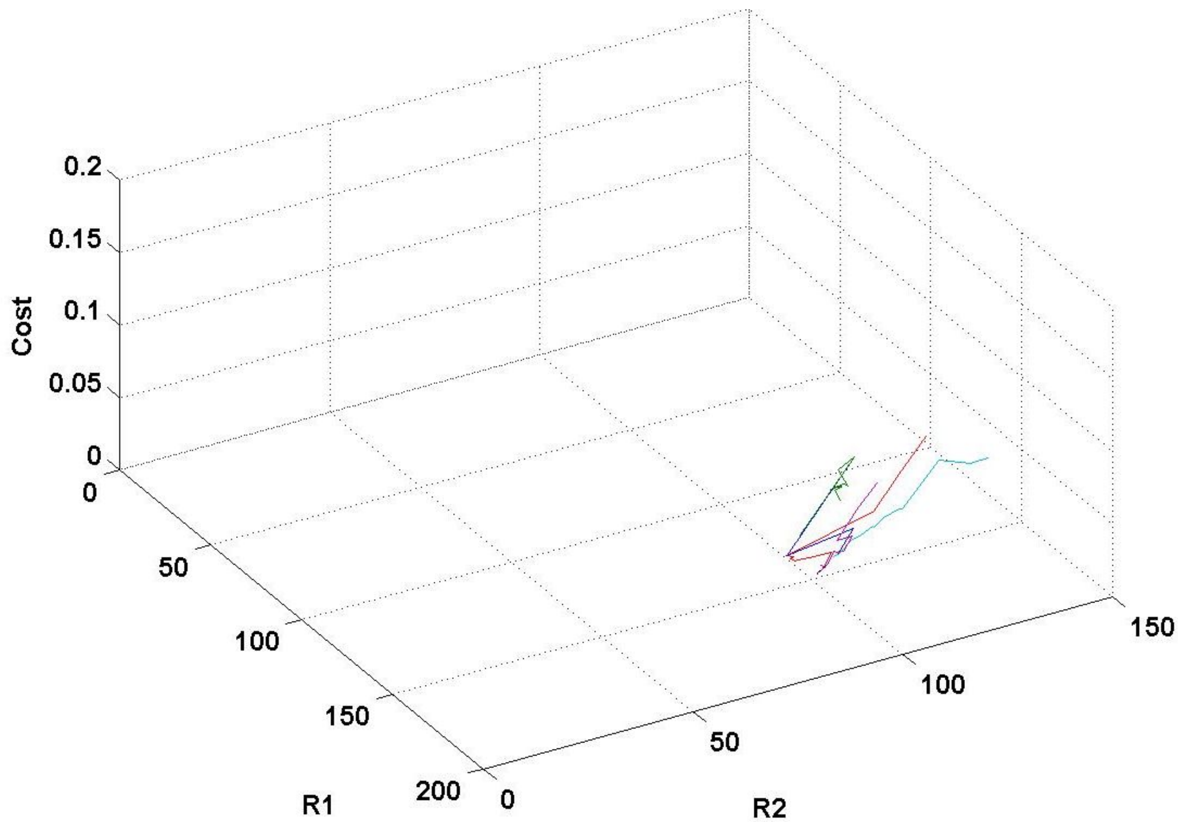


Fig. 3.2: The evolution of the resistances in order to produce and output voltage of 3V with a power dissipation of 100mW

are followed in Figure 3.2. Note that the starting points of the models are quite random, resulting in a relatively large cost. As the models evolve, only mutations that reduce the cost result in offspring that survive. In this manner, all of the best performing circuits eventually find their way to the solution R_1=150Ω, R_2=100Ω (which is the analytical solution).

The effect of the cost function weights was also investigated. Figures 3.3, 3.4 show the effect of changing the weight ratio on the models' ability to meet specification. The data presented is based on a total population of 1500 models that were allowed to evolve for 20 generations. This increase in population was made in order to obtain more consistent results.

From Figure 3.3, it is evident that when one of the specifications is heavily favoured, the

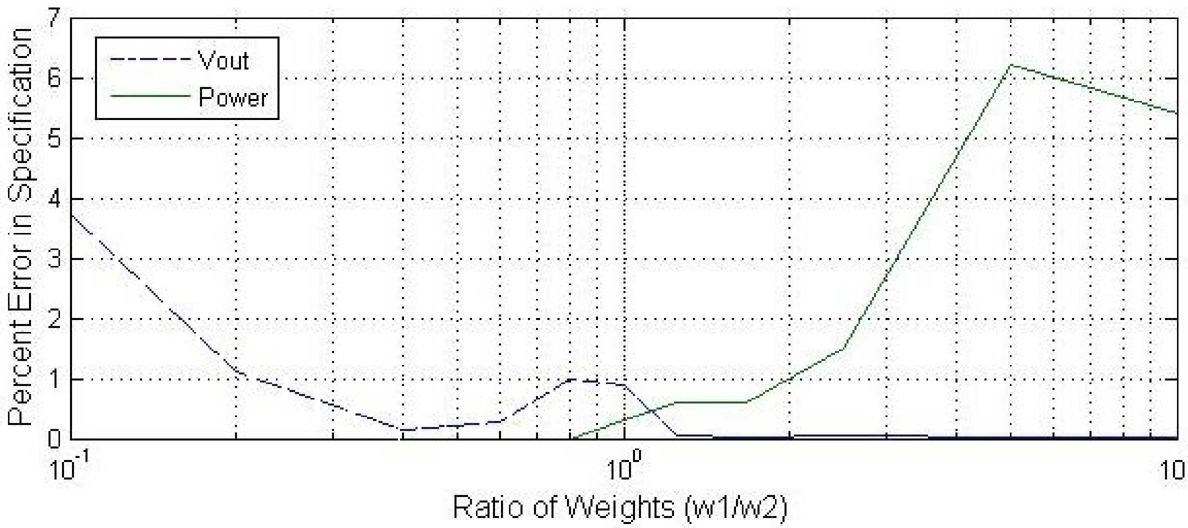


Fig. 3.3: The effect of changing the resistor divider specification weights on the performance.

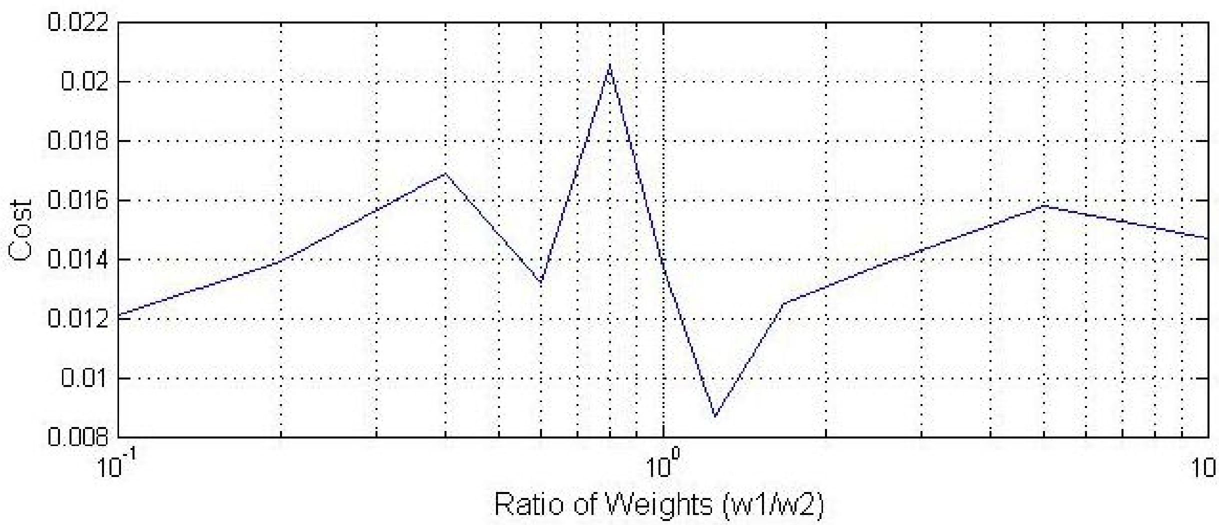


Fig. 3.4: The effect of changing the resistor divider specification weights on the performance.

percent error in that specification is driven to zero, while the other specification remains relatively high. Figure 3.4 indicates that when both specifications are to be met through equal weighting, the cost function is not driven to zero as easily.

While these results may seem intuitive, they are important in that they confirm that the algorithm is in fact mutating the genomes to produce an optimal design. When the population size was increased from 30 to 1500, there was concern that the resistances would not need to evolve, as the probability of hitting a near-optimal solution in the first generation would become very high for such a small solution space. However, if this were the case, then one would expect the cost function's weights to have relatively little impact on the ability to meet *all* of the specifications. From Figure 3.3, we see that this is not the case.

3.2 T-Filter

Having verified that the algorithm is capable of analyzing a simple, frequency-independent circuit, the analysis of the low-pass T-filter shown in Figure 3.5 was attempted. The filter is terminated with a 50Ω impedance and driven with a 50Ω source. The ABCD-matrix and S-parameter analysis of the filter are provided in Appendix B.

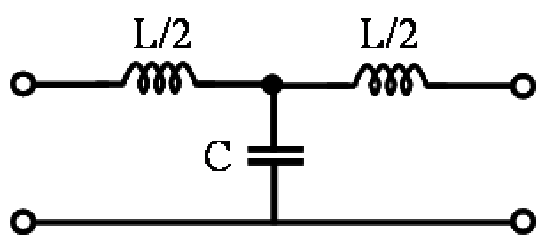


Fig. 3.5: Low-pass T-filter

Ultimately it it will be shown that given a cascade of networks, where the ABCD matrix of each of the networks is easily derived, we can set the algorithm to first build a model of the system as a whole, and then perform the optimization given. Before we proceed with an S-parameter analysis of this T-filter, the ability to perform the optimization based on design equations is verified. From [5], the cutoff frequency and characteristic impedance of the filter are given by Equations 3.3 and 3.4, respectively.

Generations	20
Population	2000
f_{-3dB}	40GHz
Z_o	50Ω
L (min,max)	(0, 10nH)
C (min,max)	(0, 1pF)

Tab. 3.2: The experimental setup for analysis of the T-filter.

$$\omega_c = \frac{2}{\sqrt{LC}} \tag{3.3}$$

$$Z_o = \sqrt{\frac{L}{C}} \tag{3.4}$$

3.2.1 Experiment

The genome for this problem is then made up of one inductance and one capacitance. As in [6], a filter with a cutoff frequency of 40GHz will be designed for a characteristic impedance of 50Ω. Table 3.2 summarizes the simulation setup and genome.

Equation-Based Analysis

Similar to the resistor divider, the analysis was first performed by means of design Formulae 3.3, 3.4, from which the solution can be calculated as L=398pH and C=159fF.

The results of varying the cost function weights are shown in Figures 3.6, 3.7. Similar to the resistor divider, when one of the design specifications is heavily favoured in weighting, its error is driven to zero while the other error remains quite large. The discussion from Section 3.1 regarding the ease with which the algorithm can perform the optimization given a set of design formulae again applies here.

One difference between this filter and the resistor-divider is that when both specifications

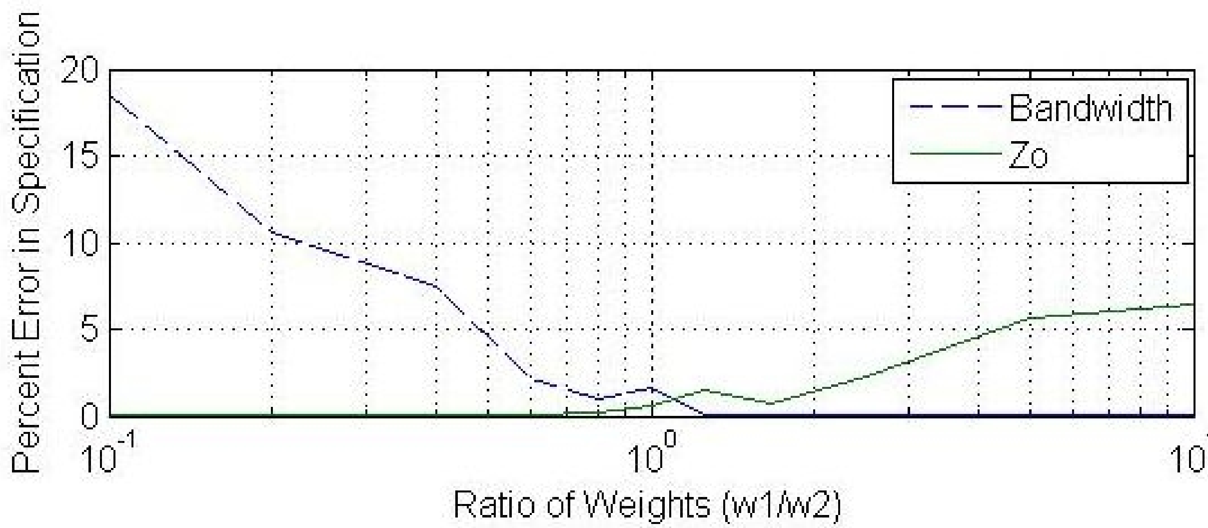


Fig. 3.6: The effect of changing the T-filter specification weights on the performance.

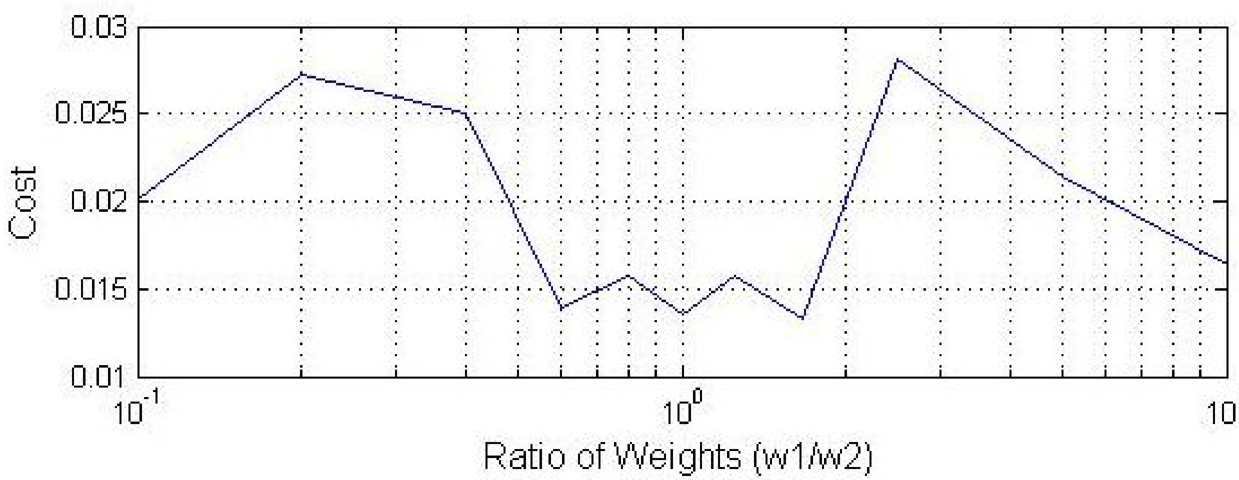


Fig. 3.7: The effect of changing the T-filter specification weights on the cost.

are attempted to be met, the cost function reaches a minimum. We also see that the percent errors are both low for equal cost weighting.

S-parameter Analysis

Having shown that the genetic algorithm can analyze the filter using design equations 3.3, 3.4, an analysis based on the S-parameters of the filter was attemped. Derivation of the S-parameters is shown in Appendix Section B.

With only the circuit's S-parameters available, the designer must decide how to define the performance specification in order for the algorithm to produce a genome with the desired solution. In the case of the T-filter, the optimal design depends heavily on the method by which the cutoff frequency and characteristic impedance are determined. Should the cutoff frequency be measured as the -3dB value from the peak or DC S_{21} value? How should a good input-match be achieved if both the input-impedance and S_{11} are both always changing over the band? The definitions that were found to be useful in this work are:

- *Bandwidth* - measured as the -3_{dB} bandwidth from the peak value of S_{21}.
- *Input Match* - Measured by calculating the magnitude of the average input impedance over the desired bandwidth.

It is interesting to note that the input match was originally set by requiring that the maximum value of S_{11} over the simulated band was -10dB. This often resulted in a good input match over a band far narrower than that desired – that is, this matching condition did not encourage the emergence of a wide bandwidth. Matching specifications based on S_{11} are complicated by the fact that, unlike S_{21}, the value changes significantly over the bandwidth. By driving the absolute value of the input impedance to 50Ω, we create a criteria that should be approximately the same over the bandwidth. The fact that the input impedance has both real and imaginary parts is not significant because for frequencies below the cutoff frequency, Z_{in} is almost entirely real, and is given by 3.4.

With regard to the bandwidth, taking the -3_{dB} points from the maximum value of S_{21} —as opposed to from the DC value—serves to reduce the amount of peaking in the filter's s-parameters. Peaking is generally considered undesirable, although in some designs, it can serve to increase the bandwidth.

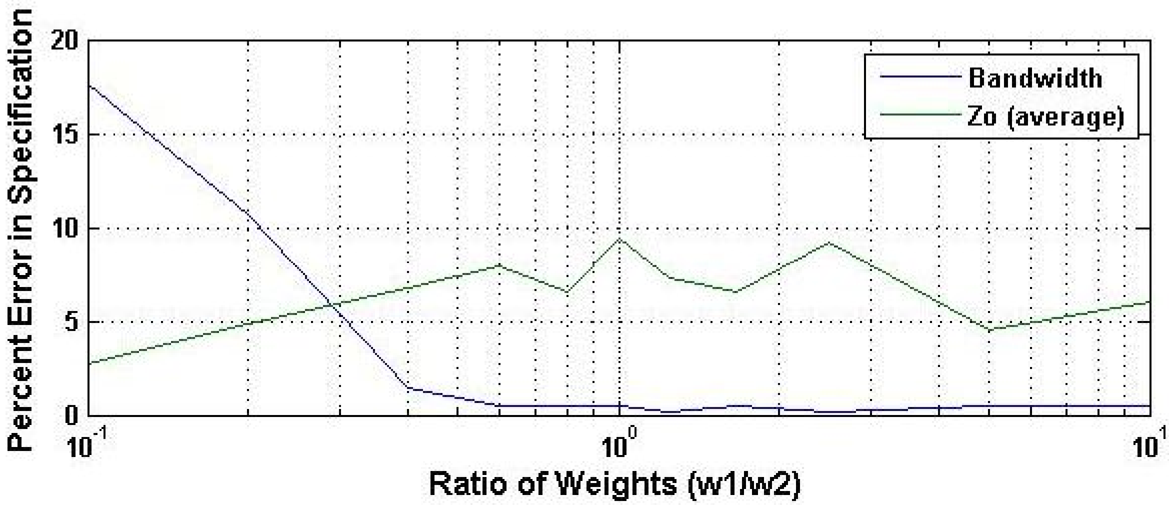


Fig. 3.8: The effect of changing the T-Filter specification weights on the performance.

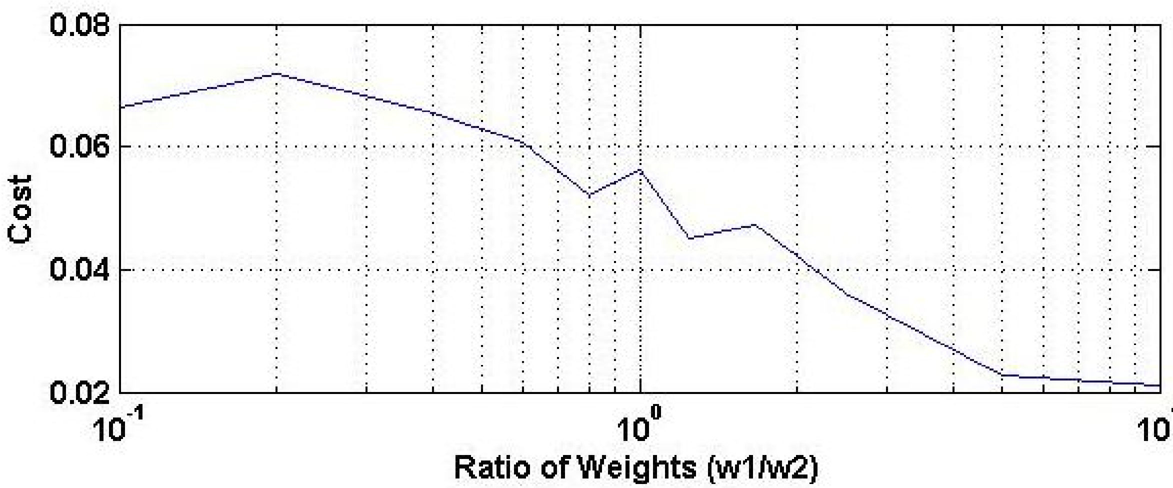
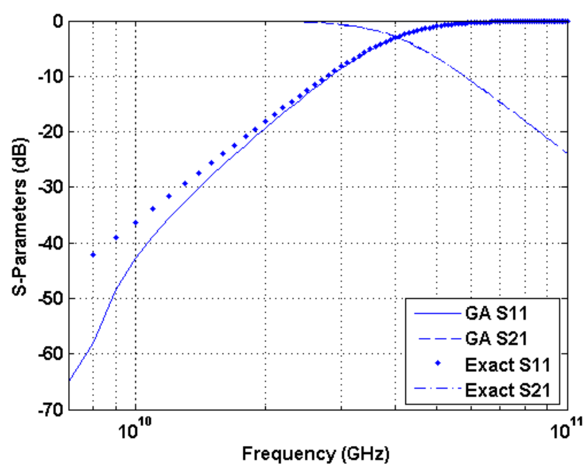


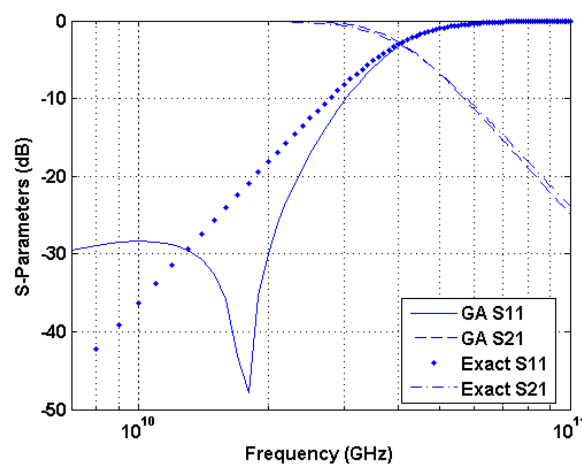
Fig. 3.9: The effect of changing the T-Filter specification weights on the cost.

Figure 3.8 shows the effect of changing the weights on the ability to meet the specification for the T-filter analyzed through its S-parameters. Comparing with Figure 3.6, it it clear that the performance of the algorithm has deteriorated through this method of analysis. This is likely due, at least in part, to the fact that the requirement that the magnitude of Z_{in} be driven to 50Ω is an artificial one, created by the designer to approximate a good broadband match. From this example, it is also evident that the designer requires a fair amount of knowledge and intuition in order to get the genetic algorithm to produce a desirable result.

Finally, Figure 3.10 compares the frequency responses of the filters designed by the GA using Equation-Based analysis and S-parameter analysis. It is clear that the algorithm was able to design the filter very close to specification.



a) Filter Designed by Equation-Based Analysis



b) Filter Designed by S-parameter Analysis

Fig. 3.10: S-parameter plots for the T-filters designed by the genetic algorithm.

3.3 Higher-Order Filter Design

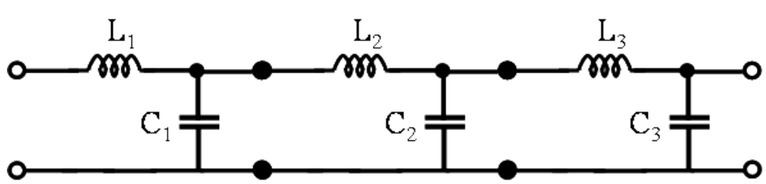


Fig. 3.11: Three-stage low-pass filter.

A more complex filter design will now be attempted. The example is taken from [5], and calls for the design of a maximally flat filter with a bandwidth of 2GHz, and a loss of at

Cutoff Frequency (f_{-3db})	2 GHz
Loss at 3 GHz	15dB
$Z_{in,avg}$	50

Tab. 3.3: Design specifications for the 5^{th} order filter design.

Example	Population	Generations	Inductance	Capacitance	S11 Spec	Number of Stages
1	1400	9	(10pH,10nH)	(10fF,10pF)	None	3
2	700	10	(10pH,10nH)	(10fF,10pF)	None	3
3	1400	6	(1pH,10nH)	(1fF,10pF)	None	3
4	1400	9	(1pH,10nH)	(1fF,10pF)	None	3
5	3500	14	(1pH,10nH)	(1fF,10pF)	-10dB	3

Tab. 3.4: Example setups for the design of the low-pass filter.

least 15dB at 3GHz. Note that, unlike the reference solution provide in [5], the solutions produced by the GA will not be maximally flat. In this design, we assume that we know that the number of elements needed is 5, and so three L-C filter stages are used, as shown in Figure 3.11. Inductance L_1 is set to zero. Also, all specifications will be weighted equally for the remainder of the experiments. In both of the previous examples, this was shown to yield genomes that meet the specification well.

The genome for this analysis is composed of the 3 inductances and 3 capacitances in the filter. The design specifications applicable to the cost function are given in Table 3.3.

3.3.1 Experiment

The performance of the algorithm will be explored through 5 experiments. The setup is summarized in Table 3.4.

Examples 1 and 2 are almost the same in the their setup, aside from the size of the population. Both setups were allowed to mutate for approximately the same number of generations, at which point the final percent errors are quite comparable. From Figure 3.12 and Figures 3.13a,b, it appears as though the larger population size of Example 1 causes its cost and specification percent errors to fall more quickly in early generations.

In Examples 3 and 4, the size of the design space was increased by an order of magnitude for each of the genes. This investigation is important, as it is questionable how accurately a designer should be able to estimate the range of values. Figures 3.12 and 3.13c,d show similar trends for the two experiments, even though Example 4 was allowed to mutate for more generations. It is encouraging that the algorithm seems to be relatively insensitive to this first investigation on increasing the range of allowable values, however future work should characterize this effect in much greater detail.

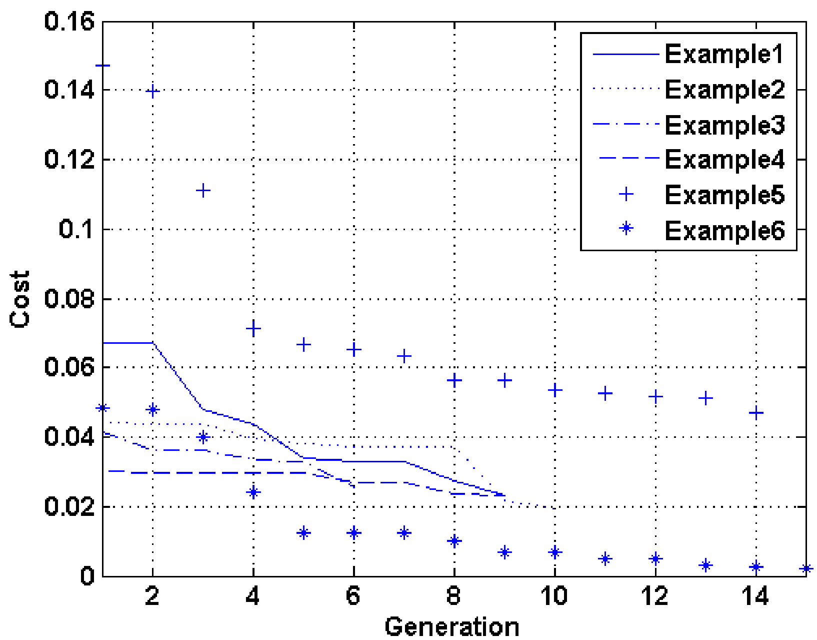


Fig. 3.12: The figure shows the effect of the generation on the cost function of the algorithm.

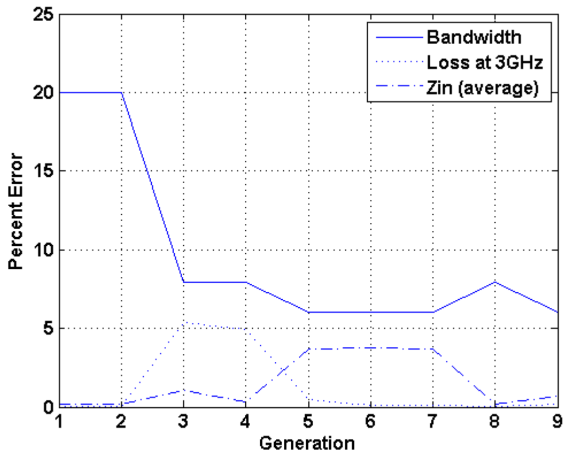
As discussed in Section 3.2, the broadband input match is achieved by driving the magnitude of the average input impedance to 50Ω. Example 5 introduces the additional constraint that the largest value of S_{11} over the band be limited to -10dB. In order to accomodate this, inductance L_1 was given the same range of values as the other inductance-genes. Figure 3.13e shows that although the percent errors of the initial population are far worse than in the previous generations, 3 of the 4 specifications are able to be met quite well by the 5th generation. After this time, the percent error of the worst specification continues to improve.

Example	L1	C1	L2	C2	L3	C3
1	0	1.4813 pF	0.4000 nH	81.3751 fF	5.5682 nH	3.2878 pF
2	0	2.7729 pF	7.1569 nH	2.2128 pF	2.5596 nH	0.3165 pF
3	0	2.2498 pF	1.1104 nH	0.8425 pF	5.7895 nH	2.5499 pF
4	0	0.3886 pF	3.5971 nH	1.8243 pF	6.0627 nH	3.2958pF
5	2.2246 pF	0.7492 pF	4.5295 nH	2.6218 pF	5.4467 nH	2.5102 pF

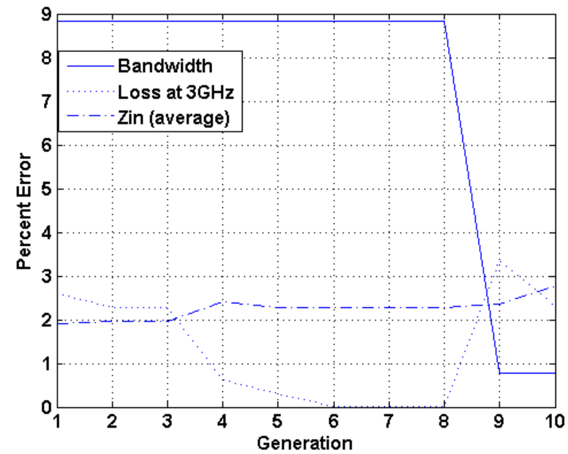
Tab. 3.5: Final design values for each of the higher-order filter examples.

Figure 3.12 shows that the cost for this example falls more steeply than any of the others. Figure 3.14 shows the higher order effects that have been mutated into the model to help keep S_{11} below -10dB over the band. Note that these effects are not nearly as pronounced in any of the other examples.

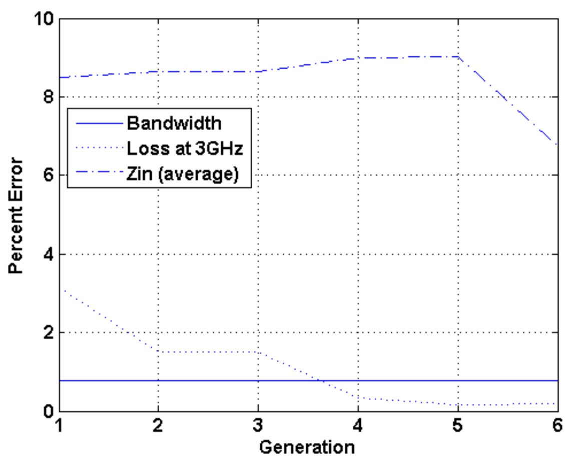
The final inductance and capacitance values for the filter are summarized in Table 3.5. Although the setup for each of the above experiments was different, there do appear to be trends in the results that are common to all of them. From Figure 3.13, it seems that by starting with such a large initial population, the algorithm will always be able to find a group of models that is able to meet all but one or two of the requirements quite well within the first generation. Through mutation and inheritance, the algorithm then attempts to meet the remaining specifications. This may be advantageous for the designer, because it means that very early on, a reasonable solution to the problem will emerge. It is only when all specifications must be met that the designer would need to wait for many generations.



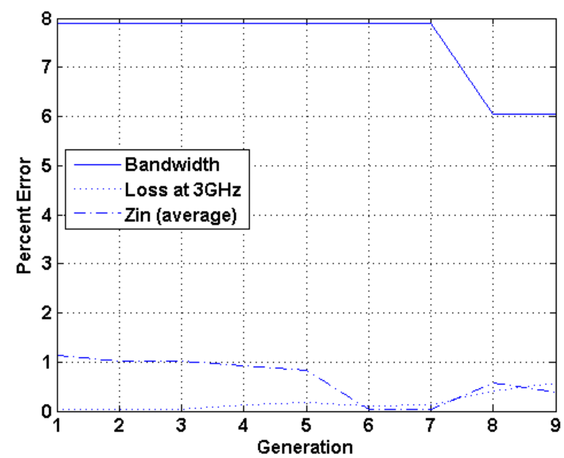
(a) Example 1



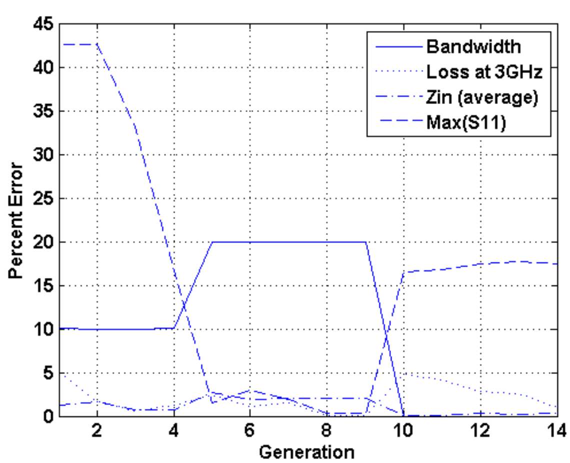
(b) Example 2



(c) Example 3

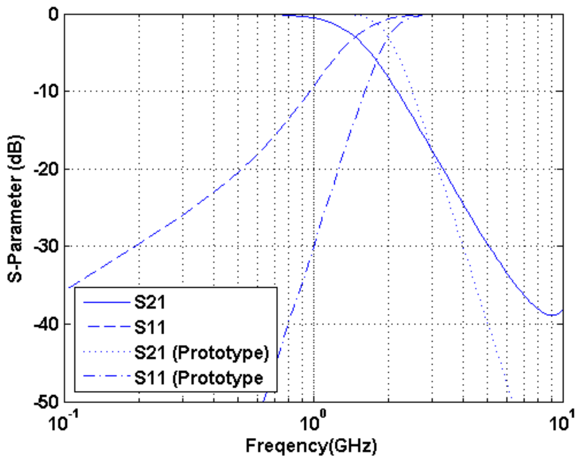


(d) Example 4

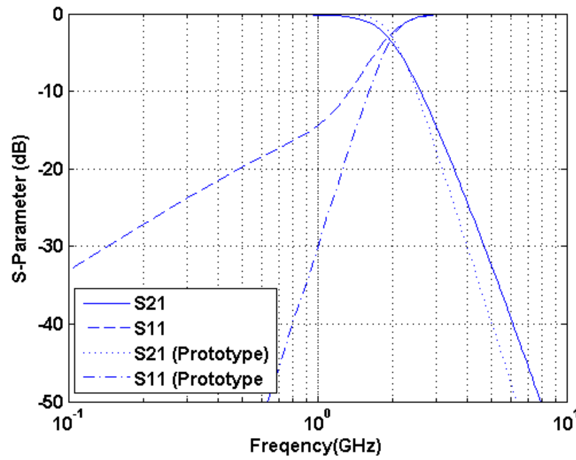


(e) Example 5 (6^{th} order)

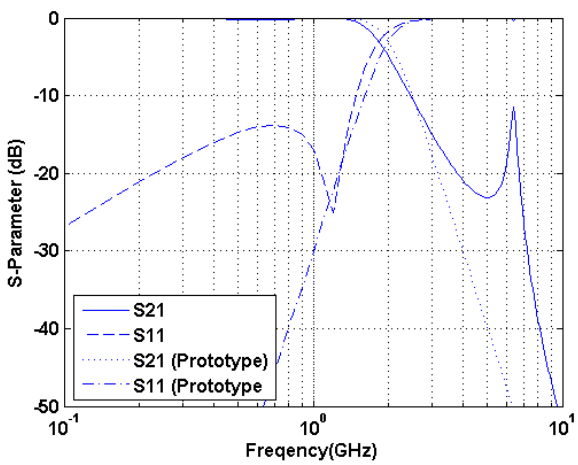
Fig. 3.13: The performance plots for each of the 5^{th} order filters designed by the GA.



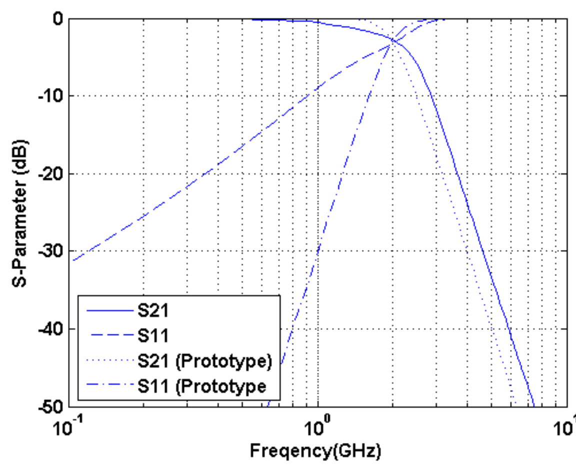
(a) Example 1



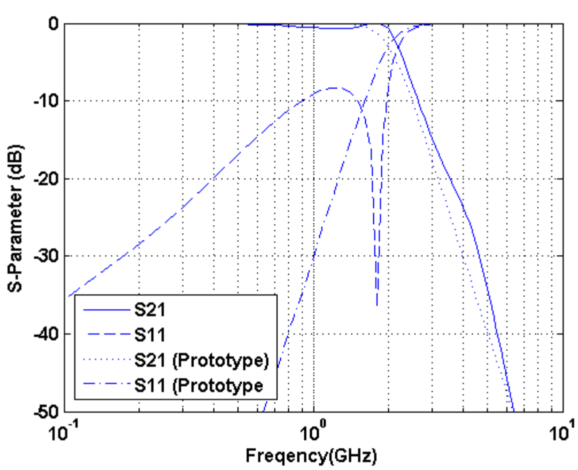
(b) Example 2



(c) Example 3



(d) Example 4



(e) Example 5 (6^{th} order)

Fig. 3.14: The S-parameter plots for each of the 5^{th} order filters designed by the GA.

4. GENETIC ALGORITHM FOR DISTRIBUTED AMPLIFIER DESIGN

Having demonstrated that the GA is able to design not only simple 2-3 element circuits, but also more complex 5^{th}– and 6^{th}–order filters, the design of a distributed amplifier was attempted. The distributed amplifier used in this work is similar to the one depicted in Figure 1.3, except that instead of transmission lines connecting the two stages, lumped inductances and capacitances are used. The exact model used is shown in Figure 4.1 and is based on the model presented in [6].

Some modifications have been made in the interest of simplicity. For example, the parallel capacitors in each of the gate and drain networks are not combined, for symmetry. Although beyond the scope of this work, these capacitors can also be thought to include the parasitic capacitances from the transistors. The transistor itself is modelled as an ideal voltage-controlled current source (VCCS), with the current being drawn from the network at the drain of the transistor. The transistors are implemented in a common-source configuration.

The motivation behind the examples in Chapter 3 was to show that the GA is able to effectively analyze 2-port networks through ABCD matrices and S-parameter analysis. While the distrubuted amplifier shown appears to be a 4-port network, by adding termination impedances R_{D1} and R_{GN} (Figure 1.3), the amplifier effectively becomes a 2-port network. This derivation is provided in Appendix D.

With a 2-port network, we can set up the design specifications and let the GA perform the optimization. For simplicity, we set the number of stages equal to two. The genome for this experiment is then given by

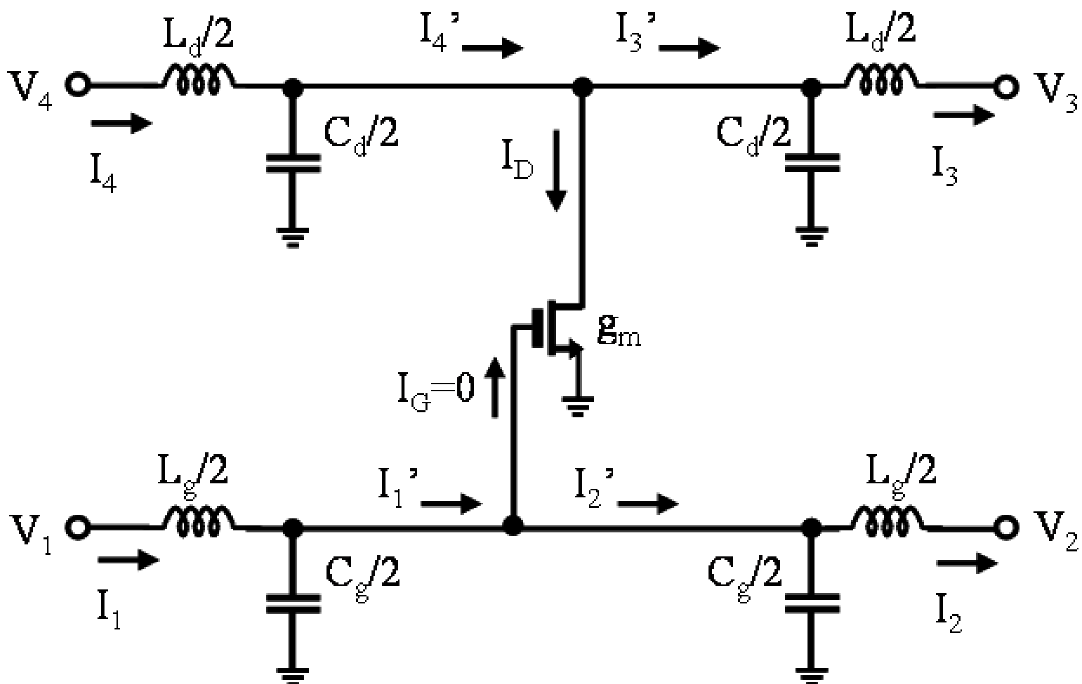


Fig. 4.1: The model for a single stage of the distributed amplifier.

L_{g1}	C_{g1}	L_{d1}	C_{d1}	g_{m1}	L_{g2}	C_{g2}	L_{d2}	C_{d2}	g_{m2}

Tab. 4.1: The genome for the distributed amplifier optimization.

4.1 Experiment

The specifications that contribute to the cost function are given in Table 4.2. Again, each specification was given equal weighting in the cost function (Equation 2.1).

Bandwidth	40 GHz
$Z_{in,avg}$	50 Ω
Gain	10 dB

Tab. 4.2: Specifications contributing to cost function for the distributed amplifier.

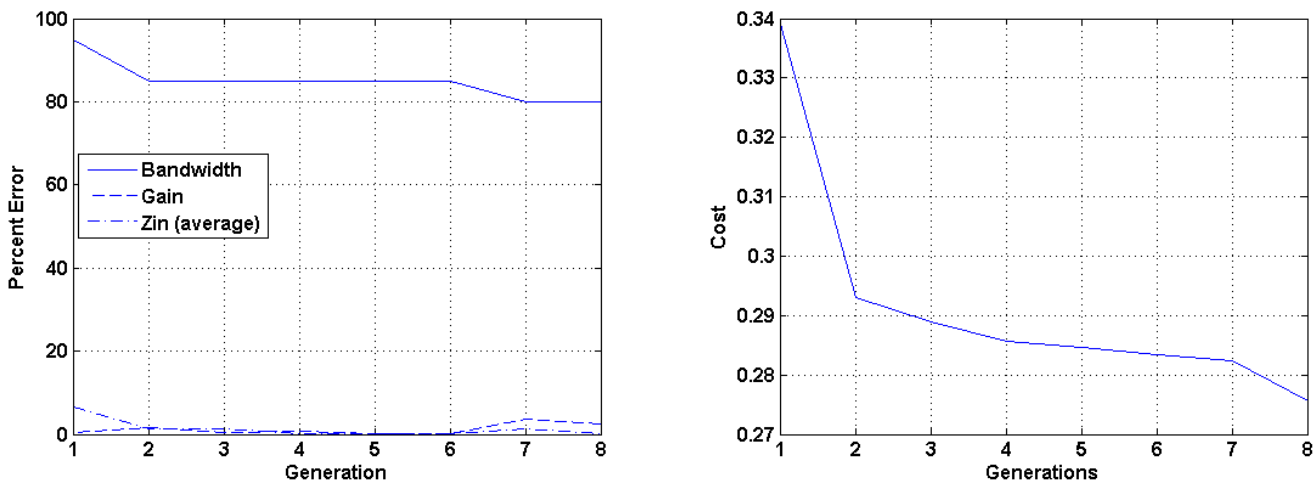
The experimental setup for the distibuted amplifier is summarized in Table 4.3.

Figure 4.2 shows the performance and cost of the distributed amplifier over the first few generations of evolution. Keeping in mind that the solution space of this problem is

Population	Generations	Inductance (min,max)	Capacitance (min,max)
6000	8	(1pH,10nH)	(1fF,1pF)

Tab. 4.3: The experimental setup for the disributed amplifier.

considerably larger than the examples of Chapter 3, these preliminary results are promising. From Figure 4.2(a), it is clear that the spread of the initial population yielded a genome that met both the input match and gain specifications well, but had very poor bandwidth. This ability to meet all but one ore two specifications early on is consistent with the results obtained in previous examples.



(a) (b)

Fig. 4.2: Performance of the genetic algorithm on the distributed amplifier.

It is also interesting to note that the cost begins to decrease quickly during the first 2-3 generations, but begins to taper off below the value of 1/3. This value is to be expected, as bandwidth is the specification lacking the most, and contributes a weighting of 1/3 to the cost.

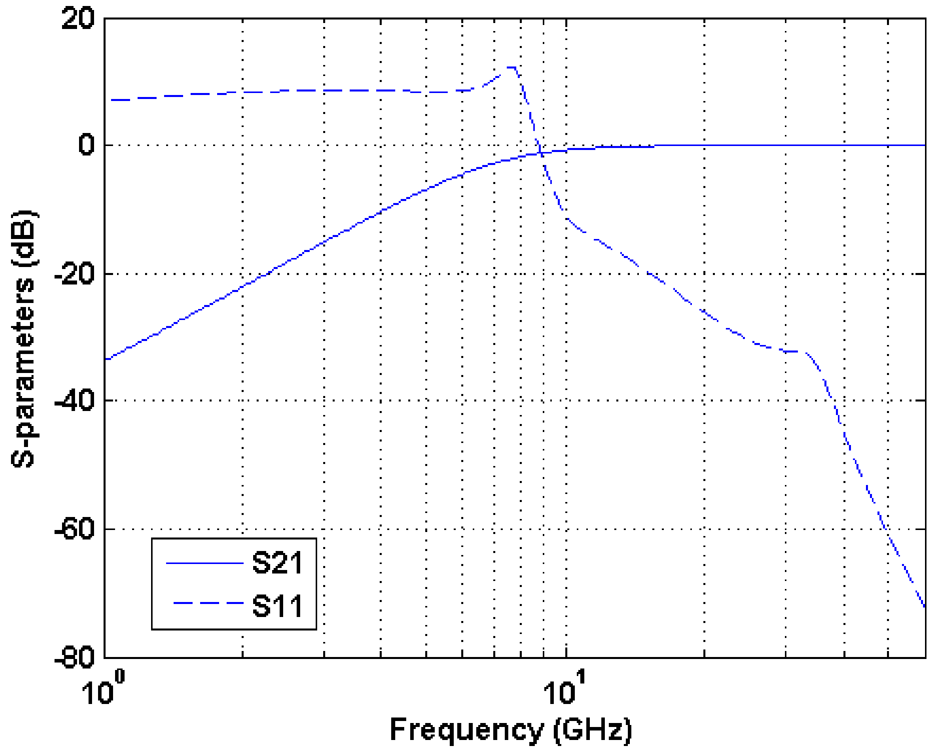
The 2-stage distributed amplifier design generated by the GA after 8 generations is summarized in Table 4.4. The output of the GA showed a maximum gain of 10.2dB and a bandwidth of 8GHz. Figure 4.3(a) shows the S-parameter plots of the GA-designed amplifier. From the figure, it is clear that neither of these specifications are particularly accurate. The

L_{g1}	458.1 pH
C_{g1}	397.7 fF
L_{d1}	6.81 pH
C_{d1}	119.5 fF
g_{m1}	34.2 mA/V
L_{g2}	655.0 pH
C_{g2}	25.35 fF
L_{d2}	395.9 pH
C_{d2}	335.6 fF
g_{m2}	10.2 mA/V

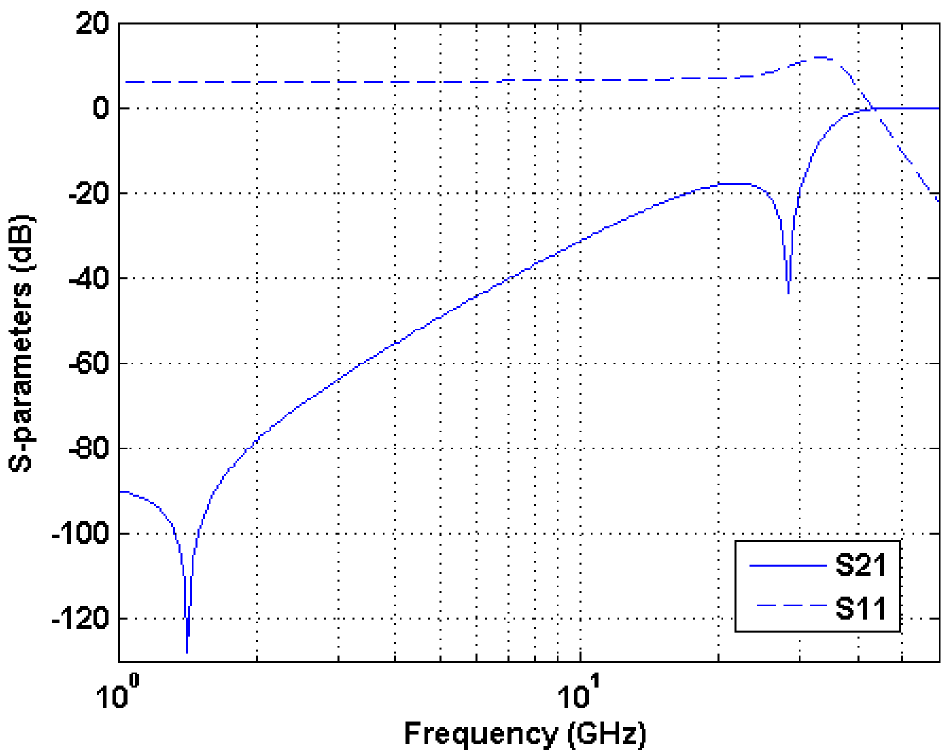
Tab. 4.4: The distributed amplifier genome after 8 generations of evolution.

reason for this is that the number of data points used by the algorithm in its S-parameter plots was limited to a much smaller number than that used to create Figure 4.3. This was a design choice that had to be made in order keep the simulation time of the algorithm within a reasonable time-frame. The result of this tradeoff was a design that suffers from peaking.

To compare, Figure 4.3 shows the S-parameter results for an amplifier designed by hand. The design is outlined in Appendix E, and is only intended as a quick comparison. It is clear that, although the human design also suffers from peaking, the it is likely that a design that meets the specifications reasonably well could be obtained with only minor adjustment. The use of a GA for the design of this 2-stage amplifier is then not easily justified.



a) Distributed amplifier designed by the GA



c) Distributed amplifier designed by Hand

Fig. 4.3: S-parameter plots for distributed amplifiers designed by the GA and by hand.

5. CONCLUSION

A genetic algorithm for the design of passive filters and a distributed amplifier was presented. Through the cost function, the algorithm was demonstrated to drive the best models of the population to meet the required specification. The algorithm successfully designed the passive filters and resistor divider, and preliminary results from the algorithm's distributed amplifier design show promise for future work.

The main problem found in this work was that in order for the algorithm to yield a useful solution, a substantially large amount of knowledge, experience, and intuition is required on the part of the designer in setting the cost specifications. This is perfectly exemplified in the design of the T-filter, where the input match specification needed to be changed several times. As a result, the algorithm, as presented, is useful only to an intermediate-level designer—one that does not have enough experience to design a circuit using more conventional, analytical means, but also one with enough intuition to troubleshoot specification definitions in order to achieve the desired result.

5.1 Future Work

Through the results of this first investigation, a substantial potential is seen for future work. The main areas of improvement lie in three main areas: the algorithm, the circuit models, and the implementatoin.

Future work for the GA itself includes implementation of more advanced forms of evolution, such as crossover and gradient descent. A more quantitative study of the performance characteristics is also necessary. In particular, more work needs to be done on determining how small an initial population the design can start with, and still converge to an optimal solution within a reasonable number of generations. Different cost and mutation functions should also be experimented with.

The circuit models themselves can also be improved by, for example, explicitly accounting for the parasitics of both the transistors and the lumped components. This step is crucial if the algorithm is ever to be used in either academia or industry for serious circuit design.

The implementation of the algorithm can be improved. In [2], hard-ware based genome testing significantly sped up the optimization time. A more viable option for this work is porting the algorithm to a faster programming language such as C. This would aid in both design and development time.

APPENDIX

A. ABCD MATRICES

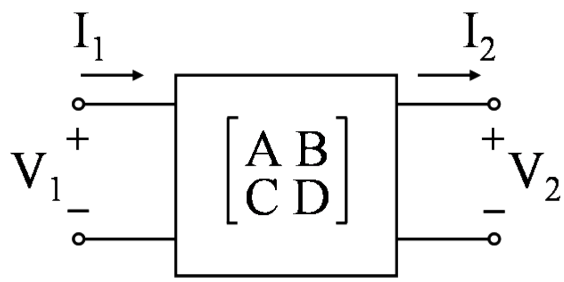


Fig. A.1: The ABCD matrix of a 2-Port Netork

One method of characterizing a 2-Port Network (Figure A.1) is through the network's ABCD matrix, which relates the voltages and currents of the two ports. The relationship is given by:

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \tag{A.1}$$

The benefit of this method is that the ABCD matrix of a cascade of networks is given by the product of the ABCD matrices of each of the individual networks, multiplied in order.

The S-parameters of the network, which relate incident and reflected voltage waves, can also be easily calculated for a given characteristic impedance Z_o using:

$$S_{11} = \frac{A + B/Z_o - CZ_o - D}{A + B/Z_o + CZ_o + D} \tag{A.2}$$

$$S_{12} = \frac{2(AD - BC)}{A + B/Z_o + CZ_o + D} \tag{A.3}$$

$$S_{11} = \frac{2}{A + B/Z_o + CZ_o + D} \tag{A.4}$$

$$S_{11} = \frac{-A + B/Z_o - CZ_o + D}{A + B/Z_o + CZ_o + D} \tag{A.5}$$

B. T-MODEL ANALYSIS

The ABCD-matrix for the T-Filter is derived from the T-equivalent circuit of a 2-Port network, shown in Figure B.1.

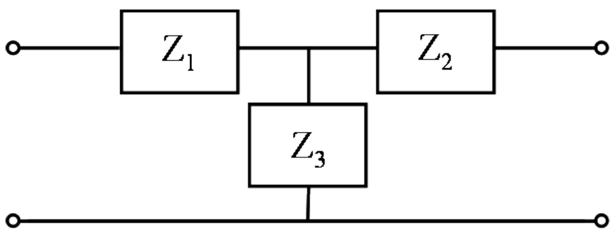


Fig. B.1: The T-equivalent Model of a 2-Port Network

From Figure 3.5, it is clear that for the T-Filter,

$$Z_1 = j\omega\frac{L}{2} \tag{B.1}$$

$$Z_2 = j\omega\frac{L}{2} \tag{B.2}$$

$$Z_3 = \frac{1}{j\omega C} \tag{B.3}$$

From [5], the ABCD matrices for the T-equivalent model are given by:

$$A = 1 + \frac{Z_1}{Z_3} \tag{B.4}$$

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \tag{B.5}$$

$$C = \frac{1}{Z_3} \tag{B.6}$$

$$D = 1 + \frac{Z_2}{Z_3} \tag{B.7}$$

and so the ABCD matrix for the T-Filter is given by:

$$A_{T-Filter} = 1 - \frac{\omega^2 LC}{2} \tag{B.8}$$

$$B_{T-Filter} = j\omega L - j\frac{\omega^3 L^2 C}{4} \tag{B.9}$$

$$C_{T-Filter} = j\omega C \tag{B.10}$$

$$D_{T-Filter} = 1 - \frac{\omega^2 LC}{2} \tag{B.11}$$

From these ABCD values, the S-parameters can be derived using Equations A.2- A.5.

C. MAXIMALLY FLAT DESIGN USING FILTER PROTOTYPE METHOD

The design problem of Section 3.3 is solved using the maximally flat filter prototype method presented in Example 8.3 of [5].

For a maximally flat filterr, the *Power Loss Ratio*, P_{LR}, is given by:

$$P_{LR} = 1 + k^2 (\frac{\omega}{\omega_c})^{2N} \tag{C.1}$$

where N is the order of the filter and ω_c is the cutoff-frequency. k is used to set the P_{LR} at the band-edge.

Now, from the problem statement, we know that the -3dB frequency is to be 2GHz. Therefore,

$$\begin{aligned} P_{LR} &= 1 + k^2 (\frac{\omega}{\omega_c})^{2N} |_{\omega=\omega_c} && \text{(C.2)} \\ &= 1 + k^2 && \text{(C.3)} \\ &= 2 \text{ (for -3dB attenutation)} && \text{(C.4)} \end{aligned}$$

Therefore k=1. The order of the filter will be set by the requirement that there be at least 15dB attenuation at 3GHz.

C_1	0.984 pF
L_2	6.438 nH
C_2	3.183 pF
L_3	6.438 nH
C_3	0.984 pF

Tab. C.1: Inductances and capacitances satisfying the maximally-flat specification.

$$15 \leq 10log(P_{LR}) \tag{C.5}$$

$$= 10log(1 + (\frac{f}{f_c})^{2N})|_{f=3GHz} \tag{C.6}$$

Solving for N we find that N must be greater than 4.22, so the minimum order of the filter is N=5. The element values for the maximally-flat prototype are obtained by looking up Table 8.3 from [5], and are given in Table C.1.

D. DISTRIBUTED AMPLIFIER S-PARAMETER DERIVATION

The following derivation follows that provided in [6] almost entirely, and is included for completeness. The derivation begins by using 4x4 ABCD matrices, and then reducing the size of the matrices with the terminating impedances. Voltage and current values are labeled in Figure 4.1. For the first set of Inductors and Capacitors (L_{d1},C_{d1},L_{g1},C_{g1}) the $4x4$ ABCD matrix is given by:

$$\begin{pmatrix} V_4 \\ I_4 \\ V_1 \\ I_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1-\omega^2 L_d C_d & j\omega^2 L_d & 0 & 0 \\ j\omega C_d & 1 & 0 & 0 \\ 0 & 0 & 1-\omega^2 L_g C_g & j\omega L_g \\ 0 & 0 & j\omega C_g & 1 \end{pmatrix}}_{S_1} \begin{pmatrix} V_4' \\ I_4' \\ V_1' \\ I_1' \end{pmatrix} \tag{D.1}$$

The transistor of the middle stage is modeled as a VCCS with transconductance g_m. The $4x4$ matrix for the middle section is then given by:

$$\begin{pmatrix} V_4' \\ I_4' \\ V_1' \\ I_1' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & g_m & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{S_2} \begin{pmatrix} V_3' \\ I_3' \\ V_2' \\ I_2' \end{pmatrix} \tag{D.2}$$

Finally, the $4x4$ matrix for the third section of each stage is given by:

$$\begin{pmatrix} V_3' \\ I_3' \\ V_2' \\ I_2' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & j\omega^2 L_d & 0 & 0 \\ j\omega C_d & 1-\omega^2 L_d C_d & 0 & 0 \\ 0 & 0 & 1 & j\omega L_g \\ 0 & 0 & j\omega C_g & 1-\omega^2 L_g C_g \end{pmatrix}}_{S_3} \begin{pmatrix} V_3 \\ I_3 \\ V_2 \\ I_2 \end{pmatrix} \tag{D.3}$$

Using the fact that the ABCD matrix for a cascade of networks is given by the ABCD matrices for each of the matrices multiplied by each other, the ABCD matrix for each stage is given by:

$$d_{4x4} = S_1 S_2 S_3 \tag{D.4}$$

The ABCD matrix for an N-stage amplifier is then given by:

$$D_{4x4} = \prod_{k=1}^{N} d_{4x4,n} \tag{D.5}$$

$$\begin{pmatrix} V_{D1} \\ I_{D1} \\ V_{G1} \\ I_{G1} \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{pmatrix} \begin{pmatrix} V_{DN} \\ I_{DN} \\ V_{GN} \\ I_{GN} \end{pmatrix} \tag{D.6}$$

The task now is to reduce the size of this $4x4$ matrix representing the entire amplifier to the common $2x2$ ABCD matrix for which there are well-known conversion formulae to S-sparameters and other network characteristics.

We begin by adding the terminating impedances to the first stage of the drain line, and the N^{th} stage of the gate line. The relationships that result are:

$$I_{D1} = -\frac{V_{D1}}{R_{D1}} \tag{D.7}$$

$$I_{GN} = \frac{V_{GN}}{R_{GN}} \tag{D.8}$$

D can be reduced to a $3x4$ matrix by dividing the first row by R_{D1} and adding it to the second. It can be further reduced by dividing the fourth column by R_{GN} and adding it to the third column. These operations yield:

$$\begin{pmatrix} 0 \\ V_{G1} \\ I_{G1} \end{pmatrix} = \begin{pmatrix} D_{21} + \frac{D_{11}}{R_{D1}} & D_{12} + \frac{D_{12}}{R_{D1}} & D_{13} + \frac{D_{13}}{R_{D1}} + \frac{D_{24}}{R_{GN}} + \frac{D_{14}}{R_{GN}R_{D1}} \\ D_{31} & D_{32} & D_{33} + \frac{D_{34}}{R_{GN}} \\ D_{41} & D_{42} & D_{43} + \frac{D_{44}}{R_{GN}} \end{pmatrix} \begin{pmatrix} V_{DN} \\ I_{DN} \\ V_{GN} \end{pmatrix} \tag{D.9}$$

Hand analysis can then be used in the first line to represent V_{GN} as a function of V_{DN} and I_{DN}. The ABCD matrix for the distributed amplifier is then given by:

$$\begin{pmatrix} V_{G1} \\ I_{G1} \end{pmatrix} = \begin{pmatrix} D_{31} - \frac{(D_{21}+\frac{D_{11}}{R_{D1}})(D_{33}+\frac{D_{34}}{R_{G1}})}{D_{23}+\frac{D_{13}}{R_{D1}}+\frac{D_{24}+\frac{D_{14}}{R_{D1}}}{R_{GN}}} & D_{32} - \frac{(D_{22}+\frac{D_{12}}{R_{D1}})(D_{33}+\frac{D_{34}}{R_{G1}})}{D_{23}+\frac{D_{13}}{R_{D1}}+\frac{D_{24}+\frac{D_{14}}{R_{D1}}}{R_{GN}}} \\ D_{41} - \frac{(D_{21}+\frac{D_{11}}{R_{D1}})(D_{43}+\frac{D_{44}}{R_{G1}})}{D_{23}+\frac{D_{13}}{R_{D1}}+\frac{D_{24}+\frac{D_{14}}{R_{D1}}}{R_{GN}}} & D_{42} - \frac{(D_{22}+\frac{D_{12}}{R_{D1}})(D_{33}+\frac{D_{34}}{R_{G1}})}{D_{23}+\frac{D_{13}}{R_{D1}}+\frac{D_{24}+\frac{D_{14}}{R_{D1}}}{R_{GN}}} \end{pmatrix} \begin{pmatrix} V_{DN} \\ I_{DN} \end{pmatrix} \tag{D.10}$$

The S-parameters of the amplifier can then be determined using Formulae A.2- A.5.

E. HAND DESIGN OF A DISTRIBUTED AMPLIFIER

A quick hand analysis of the 2-stage distributed amplifier is presented in this section as a means for comparison with the output of the GA.

Observing that gate and drain networks of each stage of the distributed amplifier (Figure 4.1) form a T-filter, we borrow from the 40GHz design presented in Section 3.2, and try setting $C_{d1,2}$ and $C_{g1,2}$ both equal to 159fF and $L_{d1,2}$ and $L_{g1,2}$ equal to 398pH. Intuitively, since the inductances and capacitances in the gate and drain networks are the same, the signals propagating through them will experience the same delay and so will continue to be amplified and add constructively. Further, the characteristic impedance of both the gate and drain networks given by Equation 3.4 are the same. The g_m values were set to be 40mA/V after minimal trial and error. The resulting S-parameters, shown in Figure 4.3, while not perfect, are good for a prototype.

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