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A Statistical Modeling Approach for FEC-Encoded High-Speed Wireline Links

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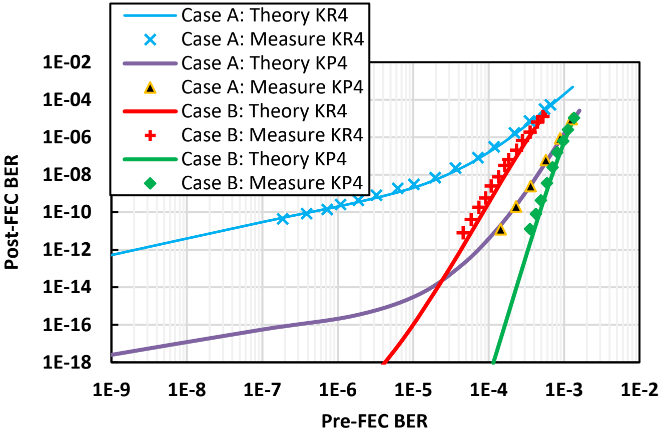
Outline

1. Motivation
2. Statistical Model for BER Estimation
    a. Modeling DFE Error Propagation in 2-PAM
    b. 4-PAM Statistical Model
    c. Post-FEC BER Estimation for Non-Binary Linear Block Codes
3. Common Coding Techniques in Wireline Links
    a. Interleaved FEC Code
    b. MOD4 Precoding
4. Modeling Other Type of Noise Sources
    a. Residual ISI
    b. Jitter
5. Experimental Verification
6. Conclusion

Motivation

- **We want to confirm post-FEC BERs in simulation down to 10^{-15} – 10^{-21} quickly and accurately**
- **To be accurate, the method must capture the statistics of errors**
- **Bit or symbol error occurrences are correlated; they sometimes occur in bursts due to DFE error propagation, low-frequency clock jitter, supply noise, etc.**
- **Error statistics strongly affect the performance of FEC**

Example: Cases A & B are two different channels and DFE tap weights resulting in very different post-FEC BER for the same pre-FEC BER



Signal Integrity Analysis Paradigms

Monte Carlo

- *Simulate with random data, random noise, and track the state of the transmitter, channel, and receiver, including FEC encoder/decoder, as the simulation progresses*
  - Captures how "memory" in the link ultimately effects the error statistics
  - Impractical to capture post-FEC BER of $10^{-15} – 10^{-21}$

Statistical

- *Determine the probability of pre-FEC errors*

  - Typical techniques consider ISI and other statistical correlations in the transceiver and channel

  - Accurate even for low probabilities

  - The results generally do not capture the time-correlation of error events

- *Apply the FEC-limit paradigm*

  - e.g. using a particular code, a pre-FEC BER of 10^{-5} produces a post-FEC BER of 10^{-18}

  - Does not account for the fact that FEC performance depends on the time-correlation of errors

This Work

This Work

- *This work seeks Statistical methods to address these two shortcomings:*

- Capture time-correlation of error events (focusing on DFE errors)

- Capture how those error statistics impact FEC performance

Statistical

- *Determine the probability of pre-FEC errors*

  - Typical techniques consider ISI and other statistical correlations in the transceiver and channel
  - Accurate even for low probabilities
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- *Apply the FEC-limit paradigm*

  - e.g. using a particular code, a pre-FEC BER of 10^{-5} produces a post-FEC BER of 10^{-18}
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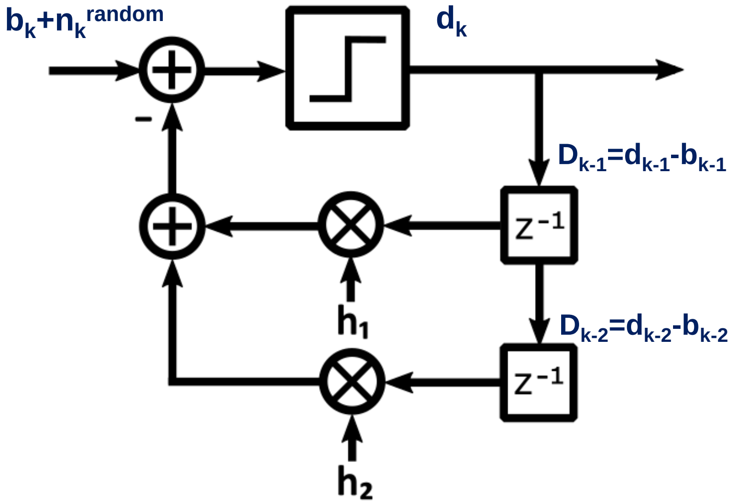
Markov Model for DFE Error Propagation

- 2-Tap DFE Example

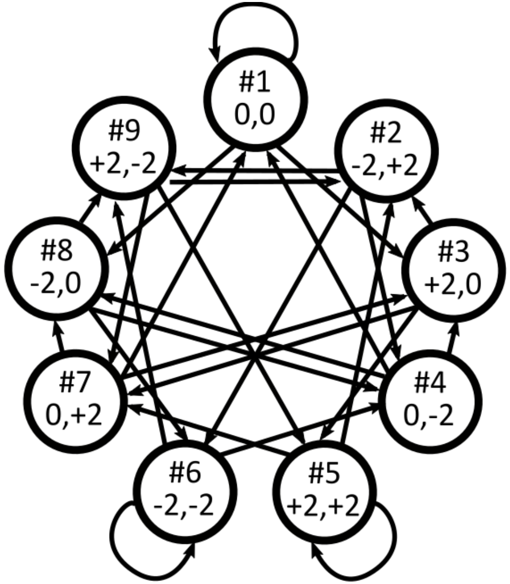


2-tap DFE, 2PAM [1 -1]

Error distance: $D_k \in [2, 0, -2]$

9 possible states: (0,2) (0,-2) (2,0) (-2,0)

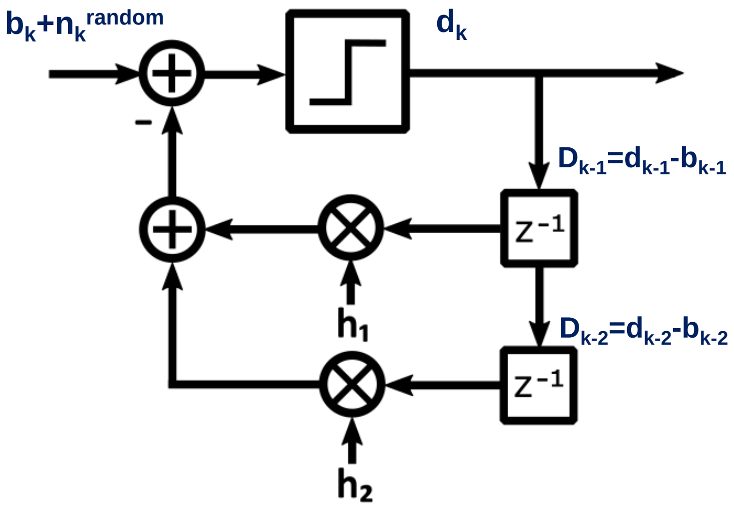
(2,2) (2,-2) (-2,2) (-2,-2) (0,0)



States are defined as possible combinations of error distance (D_{k-1}, D_{k-2})

Markov Model for DFE Error Propagation

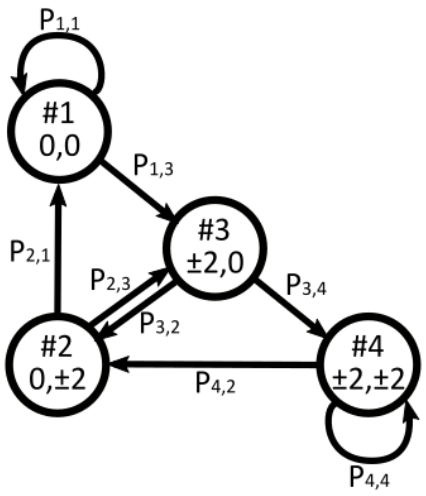
- Simplified Model using State Lumping



2-tap DFE

4 simplified states:

(0,0) (0, ±2) (±2,0) (±2, ±2)



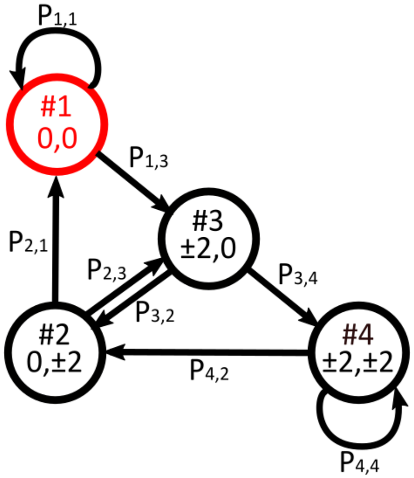
- Reduced complexity due to symmetry of the situation
- Fine if we don't care about the polarity of the bit error

Markov Model – State Transition Probability

<u>**Use conventional statistical analysis to find the branch probabilities, p_{ij}**</u>

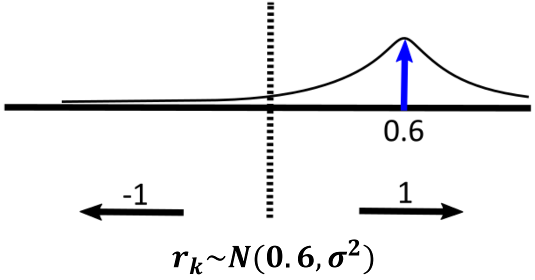
Example: Previous decisions were all correct

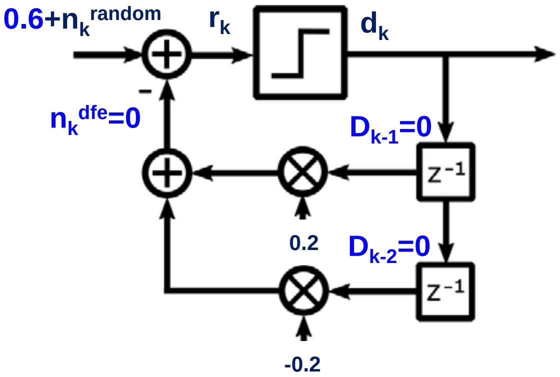
$\Rightarrow$ Current state #1: (0,0)



Markov chain state transitions
(2-tap DFE)

Transmit b_k = +1
2PAM [-1 1]
Slicer Level=0





Channel: [0.6 0.2 -0.2]
b_k Transmit: [1 -1 -1]
d_k Detect: [X -1 -1]
$n^{DFE}=0$

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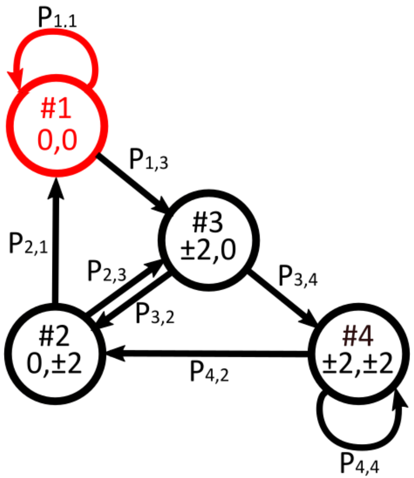

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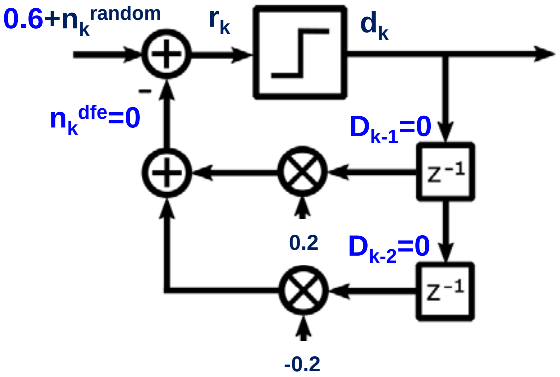

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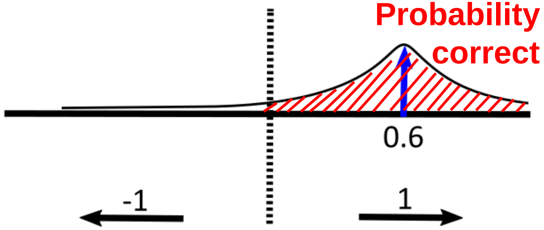


$r_k \sim N(0.6, \sigma^2)$

Channel: [0.6 0.2 -0.2]

b_k Transmit: [1 -1 -1]

d_k Detect: [1 -1 -1]

n^{DFE}=0

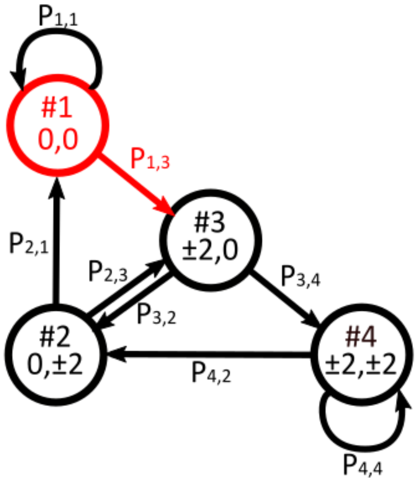
$\Rightarrow$ Must also consider what happens when we transmit b_k = -1 to calculate p_{11}

Markov Model – State Transition Probability

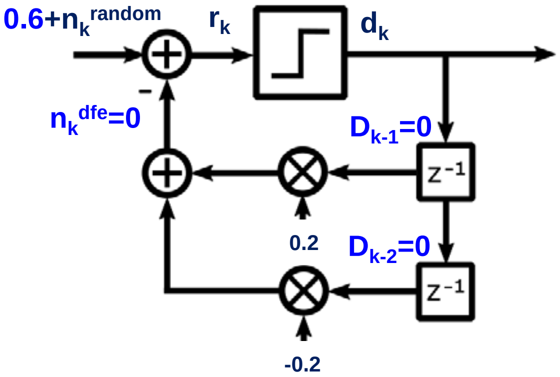
<u>Use conventional statistical analysis to find the branch probabilities, p_{ij}</u>

Example: Previous decisions were all correct

$\Rightarrow$ **Current state #1: (0,0)**



Markov chain state transitions (2-tap DFE)



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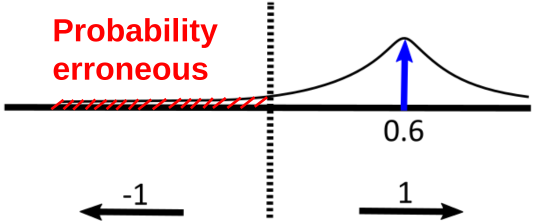
Slicer Level=0



$r_k \sim N(0.6, \sigma^2)$

Channel: [0.6 0.2 -0.2]

b_k Transmit: [1 -1 -1]

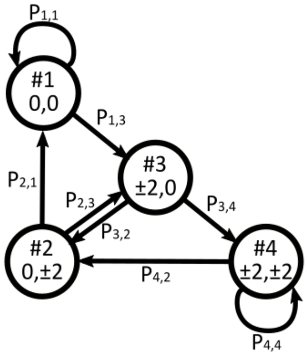
d_k Detect: [-1 -1 -1]

n^{DFE}=0

$\Rightarrow$ Must also consider what happens when we transmit b_k = -1 to calculate p_{13}

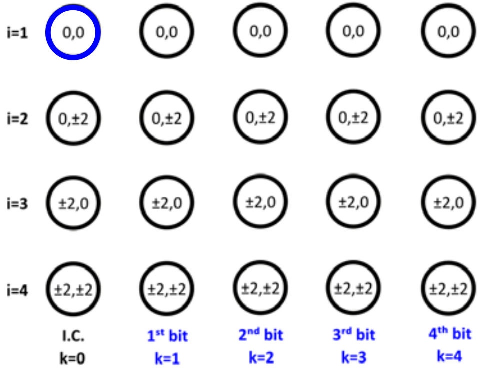
Finding Error Pattern Probability in PAM Trellis

- **Example: Finding the probability of a specified error pattern**



Transmit	1	1	-1	1	-1
Detect	1	-1	1	1	-1
Error	0	1	1	0	0

Error State $\boxed{1 \rightarrow 3} \rightarrow 4 \rightarrow 2 \rightarrow 1$



- **Error Patter Probability:**
  $P_{13421} = P(1) \cdot P_{1,3} \cdot P_{3,4} \cdot P_{4,2} \cdot P_{2,1}$

Probability we are initially in state #1

Finding Error Pattern Probability in PAM Trellis

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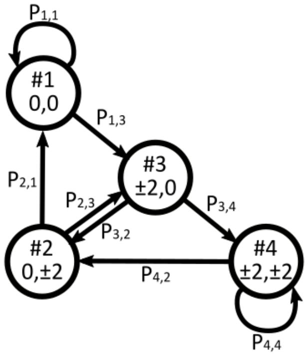

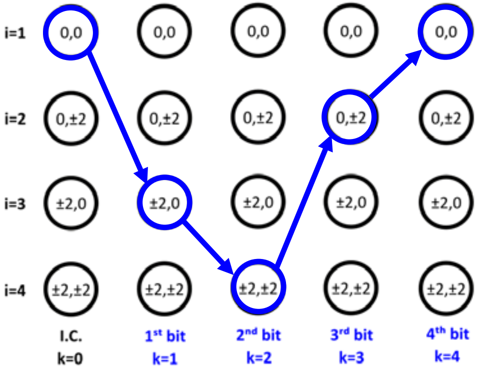

Transmit	1	1	-1	1	-1
Detect	1	-1	1	1	-1
Error	0	1	1	0	0
Error State	$\boxed{1 \rightarrow 3} \rightarrow 4 \rightarrow 2 \rightarrow 1$				

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  $P_{13421} = P(1) \cdot P_{1,3} \cdot P_{3,4} \cdot P_{4,2} \cdot P_{2,1}$

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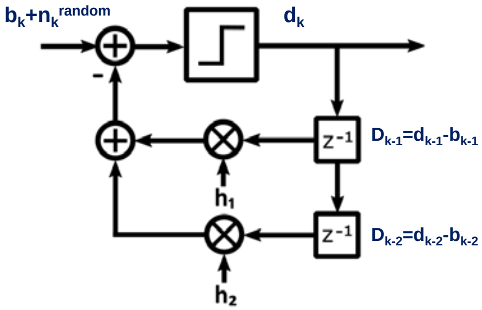
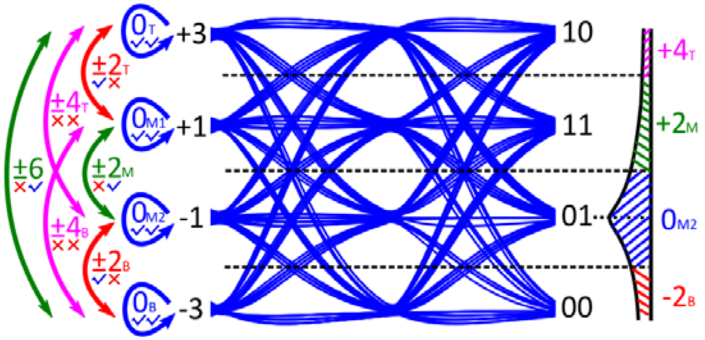
4-PAM Markov Model

- **All states having the same error magnitude are aggregated together by applying weak lumpability, $D_k \in \{0, \pm2, \pm4, \pm6\}$**

- **More DFE error states are needed in the Markov Model**

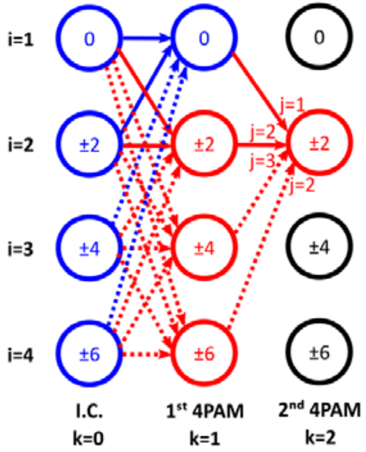


A receiver eye diagram indicating all possible symbol-detection outcomes for a link communicating Grey-coded 4-PAM symbols $b_k \in \{\pm3, \pm1\}$

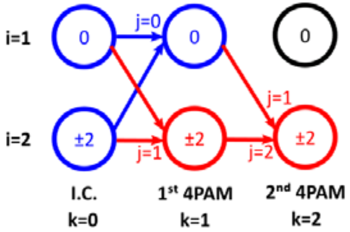


4-PAM Trellis Model

- ±4 and ±6 events are unlikely at 10^{-15} post-FEC BER



Trellis example of a 1-Tap DFE for a 4-bit codeword with all possible paths ending in state ±2 (i=2)



Simplified trellis by ignoring all the dotted paths that have unlikely ±4 and ±6 error events

Finding Pre-FEC BER

- **Over a sequence of n bits, the probability of:**
  - 1 bit error is $\mathrm{Pr}_n(1)$
  - 2 bit errors is $\mathrm{Pr}_n(2)$
  - etc…

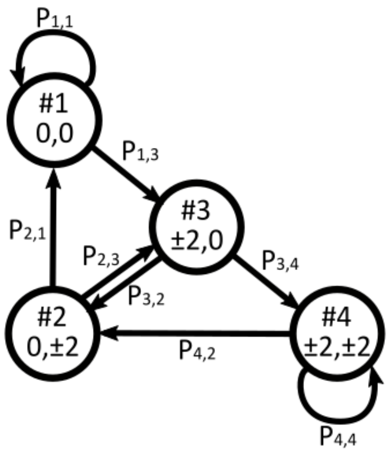
- **Then, we can calculate the BER over a *n*-bit codeword**

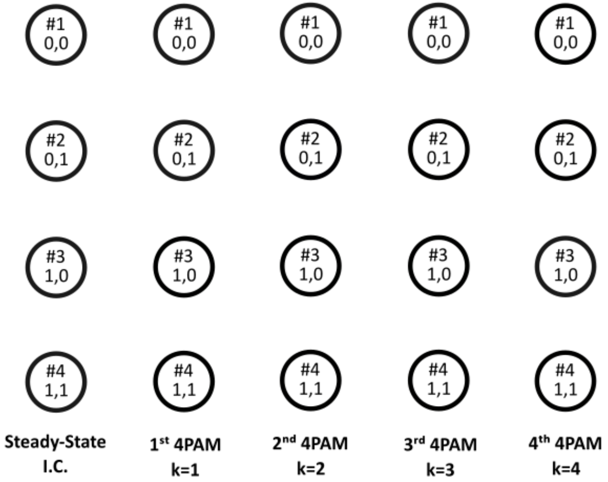
$$BER = \frac{1}{n}\sum_{j=1}^{n} j \cdot \mathrm{Pr}_n(j)$$

- **In general, for L-PAM and N-tap DFE, traversing a length-n trellis exhaustively requires computations that are $O(L^N L^n)$**

Example of Traversing Trellis

- **Example: 2-tap DFE, 8-bit codeword, 4PAM**

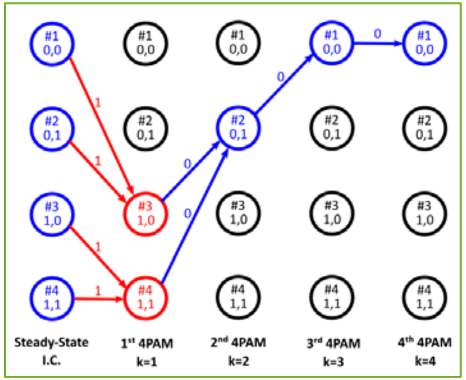




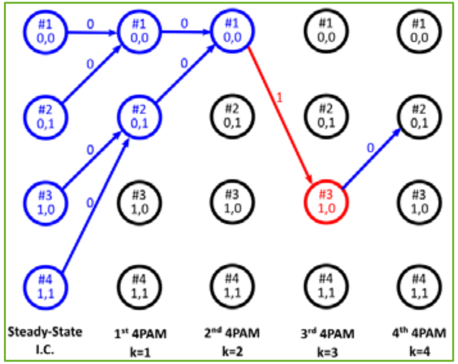
- **Finding $\mathrm{Pr}_n(1)$, the probability of all trellis paths having exactly 1 bit error**

Probability Model - Finding Pre-FEC BER

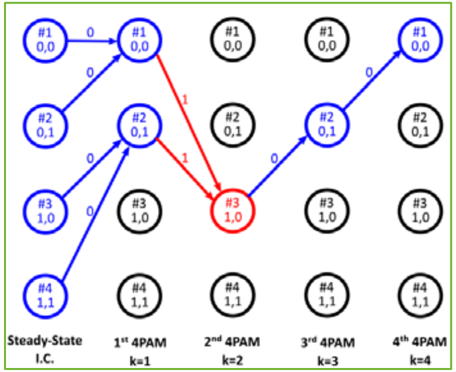
- **Example: 2-tap DFE, 8-bit codeword, 4PAM**



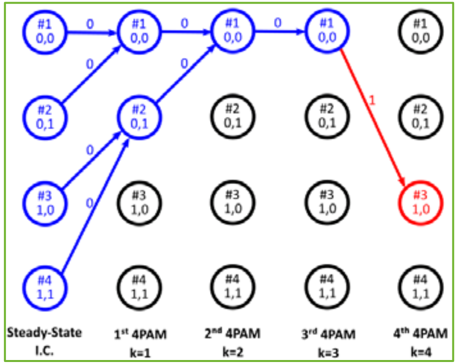
- **Case 1: error at 1st stage**
- $\Pr_n(1)$ = $p_{13}p_{32}p_{21}p_{11}+p_{23}p_{32}p_{21}p_{11}+p_{34}p_{42}p_{21}p_{11}+p_{44}p_{42}p_{21}p_{11}$



- **Case 3: error at 3rd stage**
- $\Pr_n(1)$ = $p_{11}p_{11}p_{13}p_{32}+p_{21}p_{11}p_{13}p_{32}+p_{32}p_{21}p_{13}p_{32}+p_{42}p_{21}p_{13}p_{32}$



- **Case 2: error at 2nd stage**
- $\Pr_n(1)$ = $p_{11}p_{13}p_{32}p_{21}+p_{21}p_{13}p_{32}p_{21}+p_{32}p_{23}p_{32}p_{21}+p_{42}p_{23}p_{32}p_{21}$



- **Case 4: error at 4th stage**
- $\Pr_n(1)$ = $p_{11}p_{11}p_{11}p_{13}+p_{21}p_{11}p_{11}p_{13}+p_{32}p_{21}p_{11}p_{13}+p_{42}p_{21}p_{11}p_{13}$

Inefficiency of Exhaustive Computations

Case 1: $Pr_n(1) = p_{13}p_{32}p_{21}p_{11}+p_{23}p_{32}p_{21}p_{11}+p_{34}p_{42}p_{21}p_{11}+p_{44}p_{42}p_{21}p_{11}+$

Case 2: $p_{11}p_{13}p_{32}p_{21}+p_{21}p_{13}p_{32}p_{21}+p_{32}p_{23}p_{32}p_{21}+p_{42}p_{23}p_{32}p_{21}+$

Case 3: $p_{11}p_{11}p_{13}p_{32}+p_{21}p_{11}p_{13}p_{32}+p_{32}p_{21}p_{13}p_{32}+p_{42}p_{21}p_{13}p_{32}+$

Case 4: $p_{11}p_{11}p_{11}p_{13}+p_{21}p_{11}p_{11}p_{13}+p_{32}p_{21}p_{11}p_{13}+p_{42}p_{21}p_{11}p_{13}$

- Computations required to repeat this for $\mathrm{Pr}_4(2)$, $\mathrm{Pr}_4(3)$, $\mathrm{Pr}_4(4)$ errors
- Pre-FEC BER = $\mathrm{Pr}_n(1) + 2\,\mathrm{Pr}_n(2) + 3\,\mathrm{Pr}_n(3) + 4\,\mathrm{Pr}_n(4)$
- Not practical to enumerate all error patterns for a long codeword
- Some multiplications are performed twice
- Trellis dynamic programming systematically stores these intermediate results so that the same multiplication is only performed once

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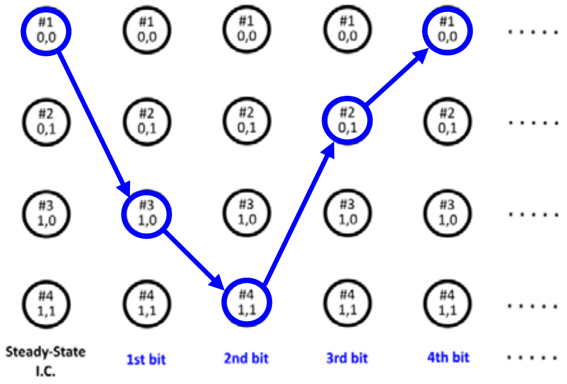
Finding Post-FEC BER of Long Block Codes

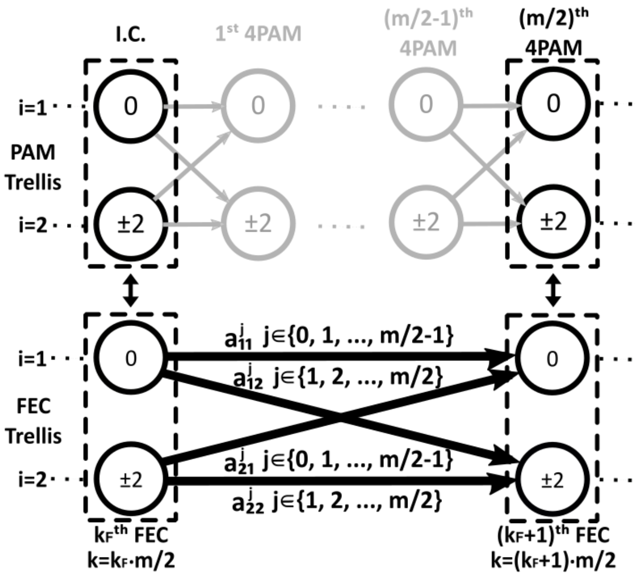
- **We wish to find the BER at the output of a FEC decoder operating on GF(2^m), $m > 1$**
  - e.g. many of the standard wireline codes are Reed Solomon codes of this type
- **Brute force approach would catalog all possible error patterns which are correctable**
- **Find the probability of these error patterns**

Example below corresponds to a 2-tap DFE; hence, 4-state PAM trellis



- **Example: RS(544, 514, 15) KP4 FEC on GF(2^{10})**
  - Each block is 5440 bits long
  - Can correct up to 15 FEC symbol errors
- **Number of trellis paths to compute is intractable**

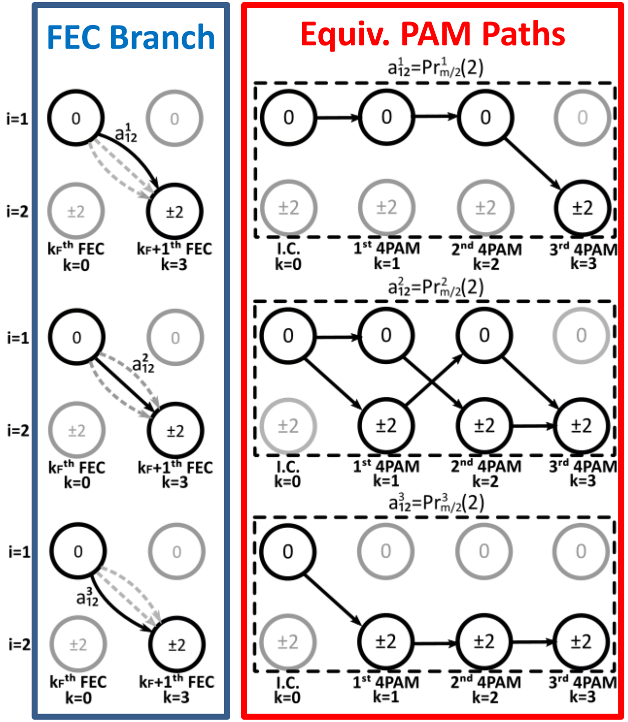
The "FEC Trellis"



Example above: 1-tap DFE

- Construct a new trellis where each stage corresponds to an entire FEC symbol rather than a PAM symbol
  - "Time aggregation" of a Markov model
  - ✓ Much shorter "FEC Trellis"
- Branch probabilities in the FEC Trellis can be found by analysis of the short length-*m*/2 trellis above

Finding Branch Probabilities in the FEC Trellis



Example above: 1-tap DFE, $m = 6$

Thus, each FEC symbol is 3 4-PAM symbols

- The FEC trellis has a higher radix if we need to keep track of the number of pre-FEC bit errors

- Example:

$$a_{12}^{1} = \mathrm{Pr}_{m/2}^{1}$$

≡ probability of going from state 1 (no error in DFE) to state 2 (error in DFE) traversing a FEC symbol (duration 3 PAM-4 symbols in this case) experiencing exactly one bit error

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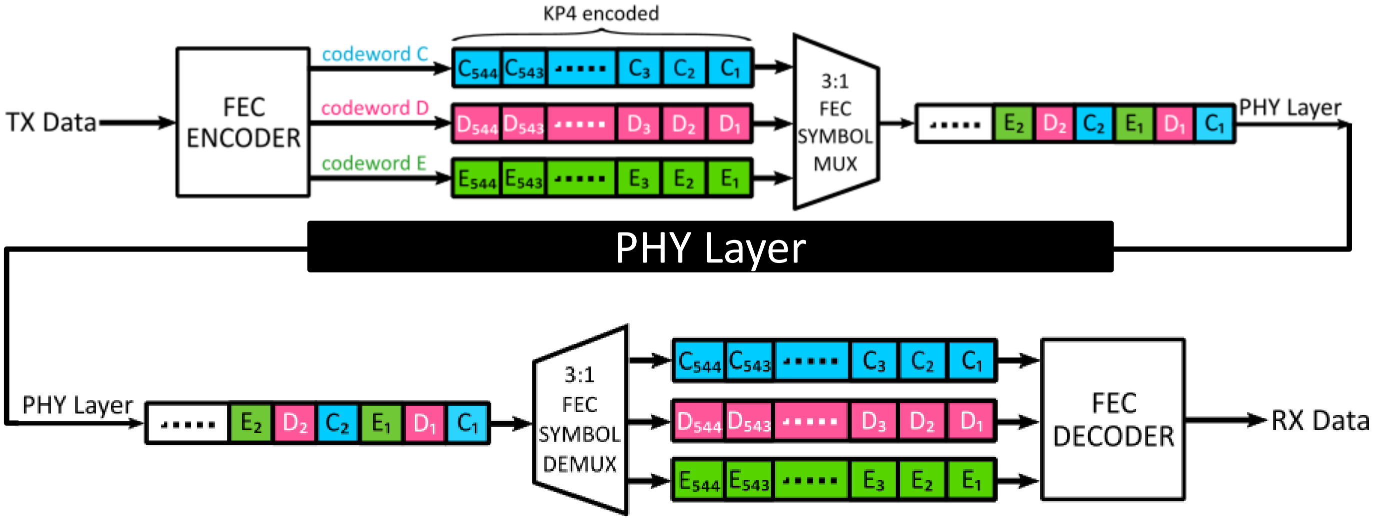
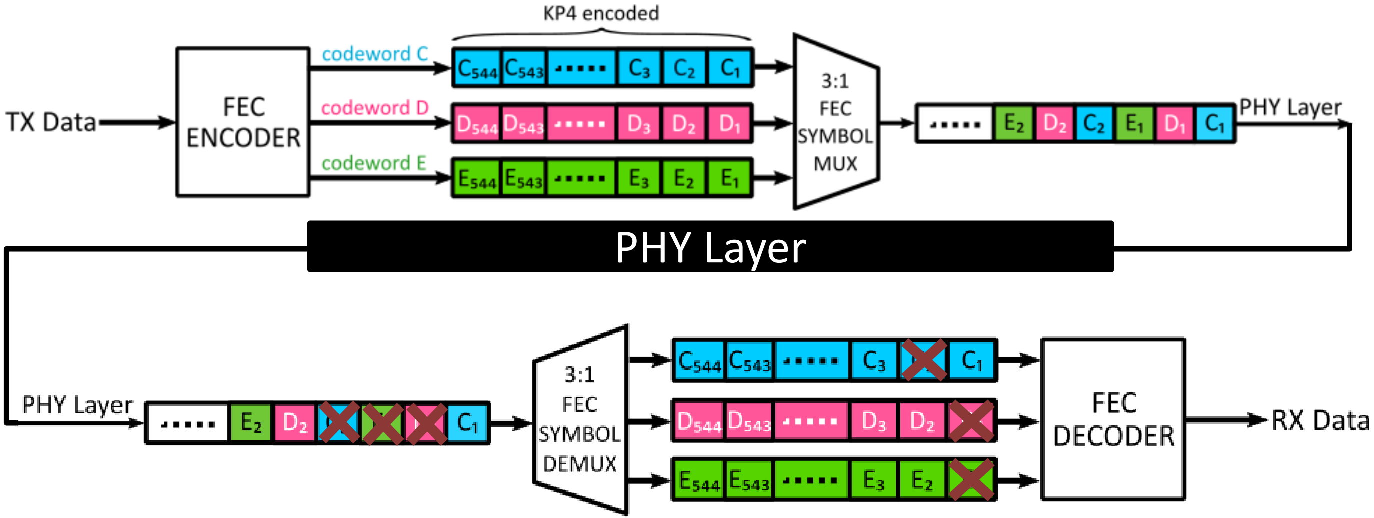
Interleaved FEC Code



- **Interleaving FEC code blocks is a simple way to spread bursts across multiple code blocks, and thereby improve burst-error-correction performance**

- **Cost is additional transceiver memory and latency**

Interleaved FEC Code



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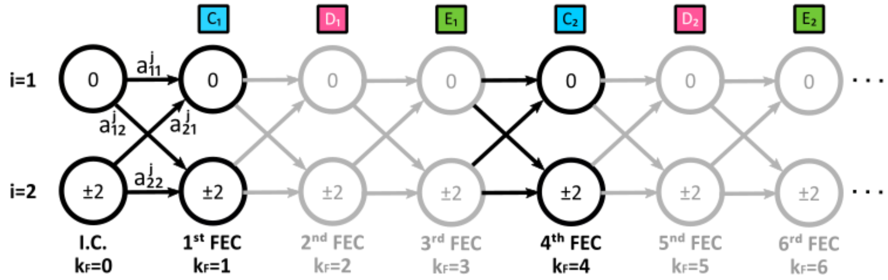
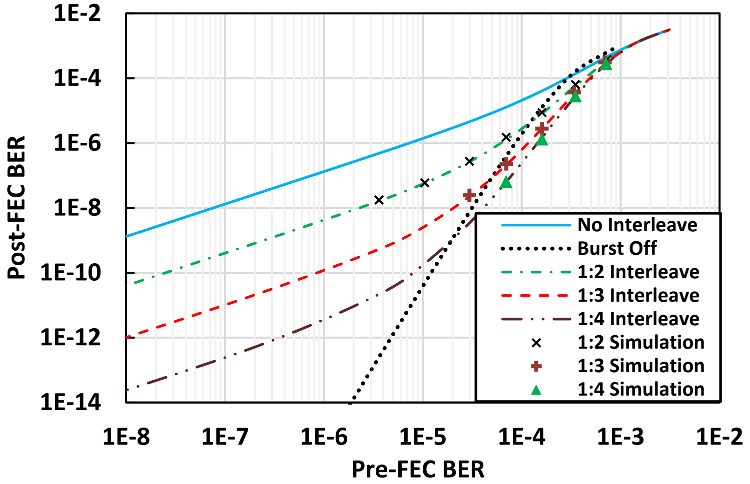
Statistical Analysis of Time Interleaved Codes



- **Analysis of a 3:1 interleaved code of length *n* requires analysis of a length 3*n* trellis**

- **Results confirm the improved burst-error tolerance offered by interleaving**



Pre-FEC vs post-FEC BER plot for interleaved RS(1000,992,4) codes with

$h = 0.5 + 0.25z^{-1} - 0.25z^{-2}$.

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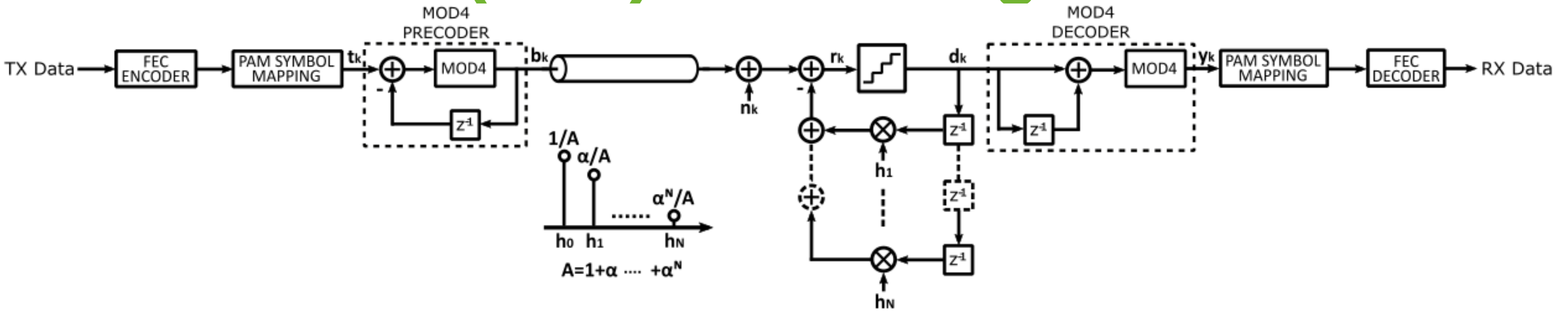
Impact of MOD4 (1+D) Precoding



- **(1+D) precoding converts error bursts into only 2 errors: one at the start and one at the end of the burst**
- **Very beneficial for long bursts**
  - Bursts spanning 3 or more FEC symbols turn into only 2 FEC symbol errors

Precoder Input t_k	0	2	3	1	1	0	2
Precoder Output b_k	0	2	1	0	1	3	3
DFE Output d_k	0	3	0	1	0	3	3
Error Value d_k-b_k	0	1	-1	1	-1	0	0
Decoder Output y_k	0	3	3	1	1	3	2

- **Unfortunately, this also applies to very short bursts**
- **Bursts of length 1 become 2 bit errors**
  - Some isolated random errors (without DFE error propagation) may corrupt 2 FEC symbols

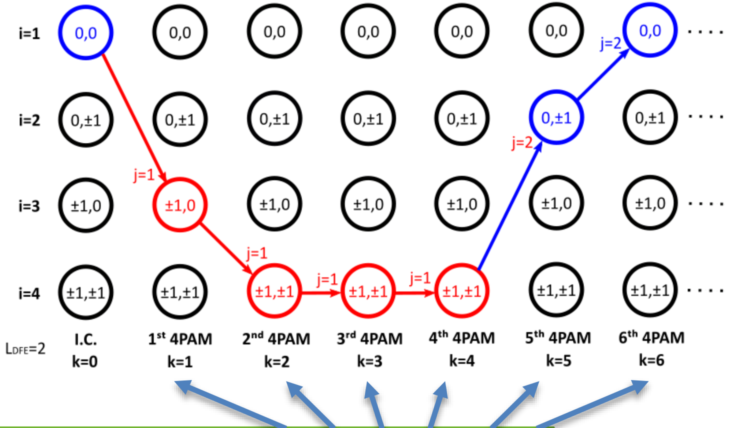
Precoder Input t_k	0	2	3	1	1	0	2
Precoder Output b_k	0	2	1	0	1	3	3
DFE Output d_k	0	3	1	0	1	3	3
Error Value d_k-b_k	0	1	0	0	0	0	0
Decoder Output y_k	0	3	0	1	1	0	2

Statistical Analysis of (1+D) Precoding

- **Statistical analysis method allows us to identify probability of all error patterns**

- **(1+D) precoding maps each error pattern to a different error patterns**

Example below corresponds to a 2-tap DFE; hence, 4-state PAM trellis



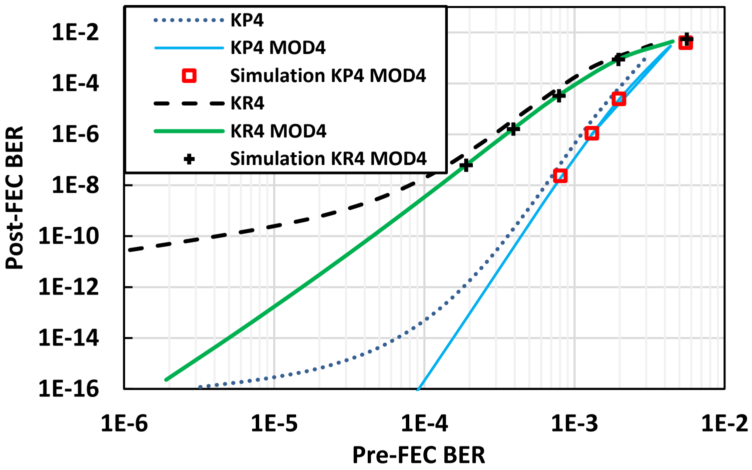
Precoder Input **t**$_k$	0	2	3	1	1	0	2
Precoder Output **b**$_k$	0	2	1	0	1	3	3
DFE Output **d**$_k$	0	3	0	1	0	3	3
Error Value **d**$_k$-**b**$_k$	0	1	-1	1	-1	0	0
Decoder Output **y**$_k$	0	3	3	1	1	3	2

Example Analysis Including (1+D) Precoding

- **Note that for the same SNR the pre-FEC BER is worse with precoding than without precoding**

- **However, precoding eliminates the error floor imposed by long burst errors**

Pre-FEC vs post-FEC BER plot for the RS(544,514,15) KP4 and RS(528,514,7) KR4 code with $h = 0.6 + 0.2z^{-1} - 0.2z^{-2}$



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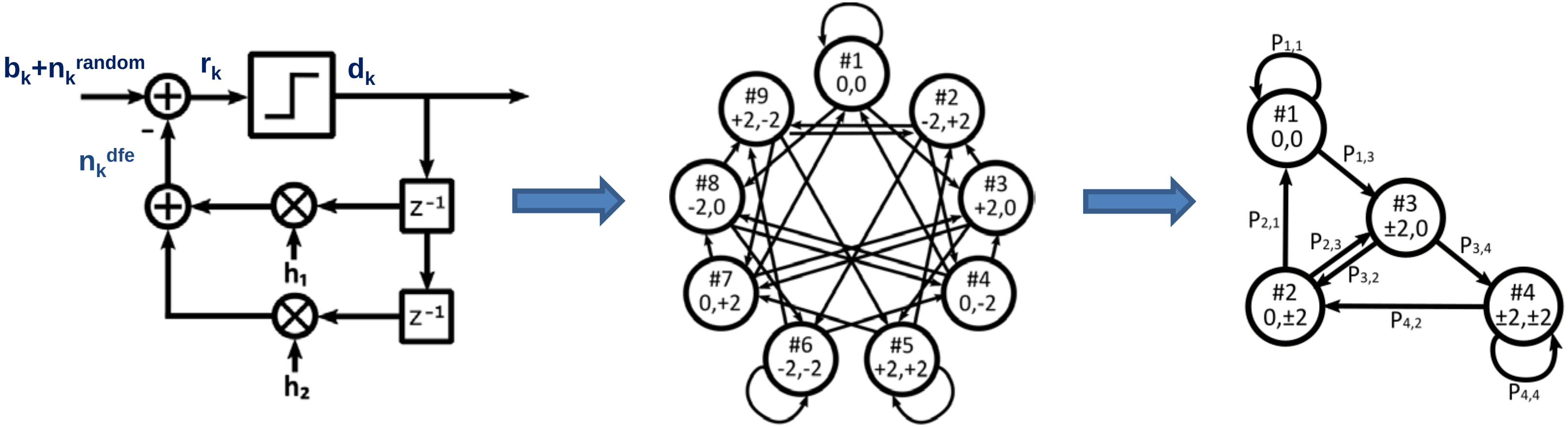
Consider Residual ISI

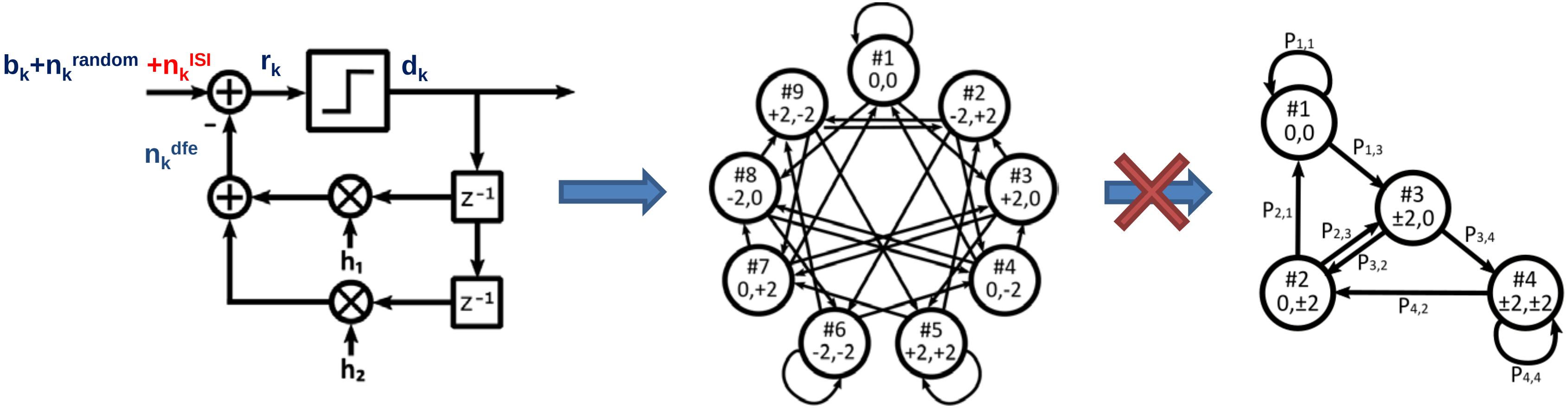


- **The statistical model discussed thus far assume perfect ISI equalization**
  - Only AWGN noise and DFE feedback error are considered

- **Allowing us to lump states having the same error values**

- **Certain conditions must be satisfied to perform state lumping [1], only true without residual ISI**

Consider Residual ISI



- **Residual ISI can be treated as an additive noise**
  - $r_k = b_k h_0 + n_k^{dfe} + n_k^{random} + n_k^{ISI}$

- **State lumping is no longer possible in the presence of residual ISI**
  - Can only work with the original Markov model

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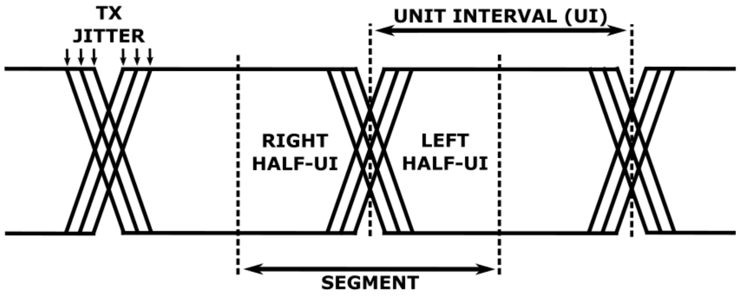
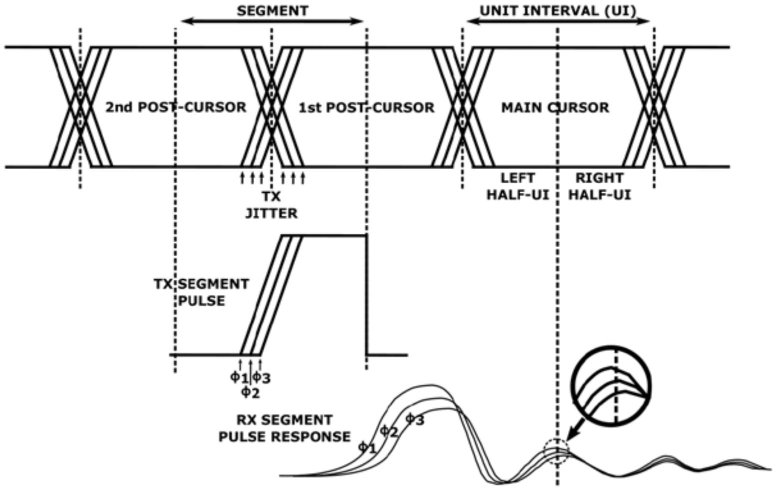
Jitter



- **Without TX jitter, total ISI distribution can be obtained by convolving the ISI pdf of each UI**
  - ✓ ISI pdf of each UI is independent of others, convolution allowed

- **TX jitter modulates the rising/falling edge of each data transition**
  - ISI distribution of each UI is dependent with the neighboring UI
  - Cannot use convolution to obtain total ISI

Jitter



- **Adapting segment-based analysis [3]**
  - Segments are defined as a jittery transition from the right half-UI of a symbol to the left half-UI of the subsequent symbol
- **Every data transition occurs in the middle of a segment**
  - ISI distribution of each segment now is independent with other segments
  - ✓ Convolution allowed

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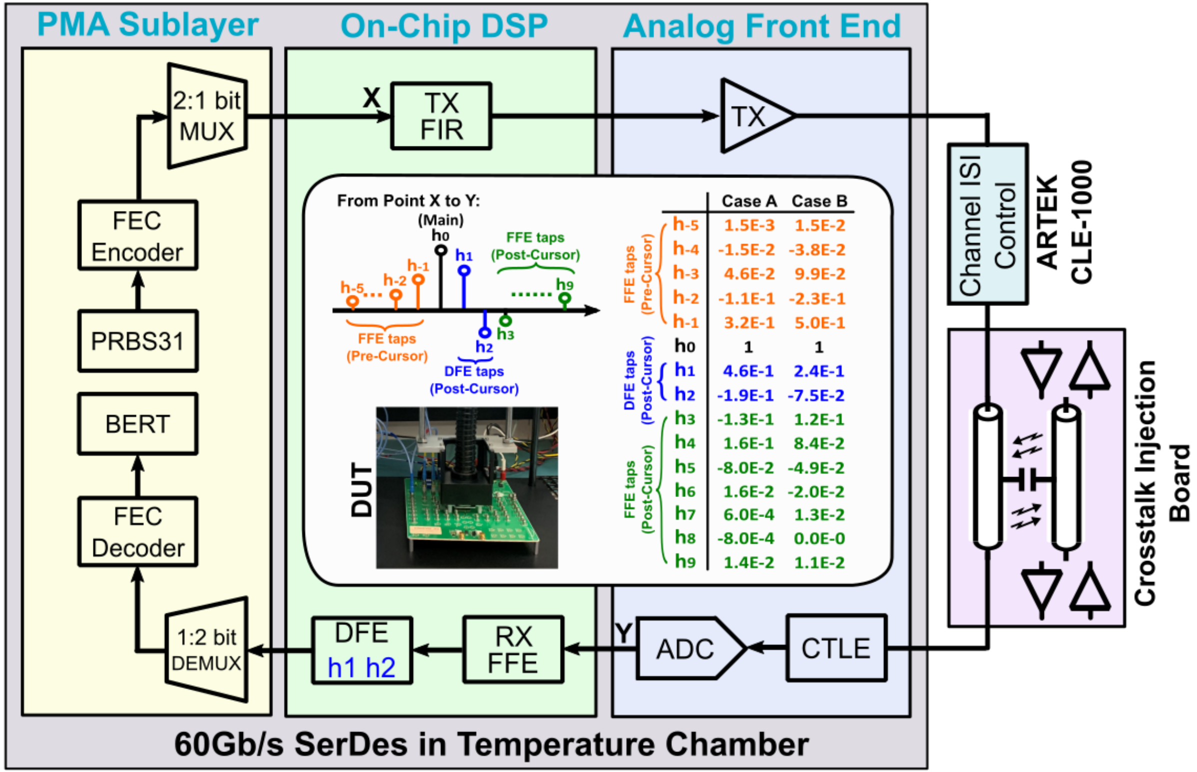
Test Bench Setup

- **A 4-PAM 60 Gb/s full transceiver fabricated in 7 nm FinFET [3]**
- **Two test cases:**
  - o Case A: 29 dB insertion loss
  - o Case B: 24 dB
- **Inject Gaussian-like crosstalk**
- **CDR phase locked after adaptive equalization to minimize random jitter**

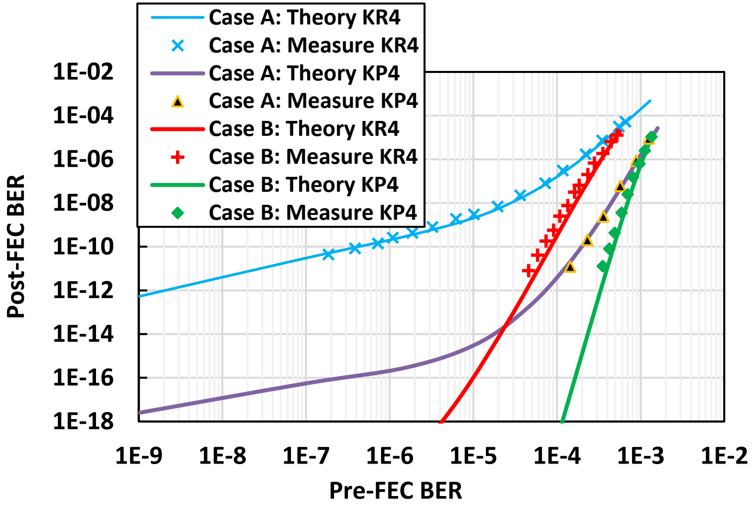


		Case A	Case B
FFE taps (Pre-Cursor)	h-5	1.5E-3	1.5E-2
	h-4	-1.5E-2	-3.8E-2
	h-3	4.6E-2	9.9E-2
	h-2	-1.1E-1	-2.3E-1
	h-1	3.2E-1	5.0E-1
	h0	1	1
DFE taps (Post-Cursor)	h1	4.6E-1	2.4E-1
	h2	-1.9E-1	-7.5E-2
FFE taps (Post-Cursor)	h3	-1.3E-1	1.2E-1
	h4	1.6E-1	8.4E-2
	h5	-8.0E-2	-4.9E-2
	h6	1.6E-2	-2.0E-2
	h7	6.0E-4	1.3E-2
	h8	-8.0E-4	0.0E-0
	h9	1.4E-2	1.1E-2

Measured Results

- Measured results for both the RS(544, 514, 15) KP4 and RS(528, 514, 7) KR4 code are reported
- Different data points are generated by varying the amount of Gaussian-like crosstalk injected to the channel
- A measurable floor is expected in the post-FEC BER where burst errors due to error propagation in the DFE dominate

Measured and theoretical pre-FEC vs post-FEC BER plot for RS(528, 514, 7) and RS(544, 514, 15) code



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Conclusion

- **We presented a statistical approach that accurately estimates post-FEC BER for high speed wireline links subject to DFE burst errors and other important noise sources**

- **Using this approach we can accurately predict post-FEC BER and observe:**
    - The “error floor” imposed by burst errors
    - The positive impact of time interleaving and (1+D) precoding on the burst-error-performance of codes

- **The method was validated using a prototype 60 Gb/s 4-PAM link with KP4 and KR4 standard Reed-Solomon codes**

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Thank you!

QUESTIONS?