Differential Kalman Filtering for Tracking Rayleigh Fading Channels

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Abstract—The performance of the estimator used in the tracking of a fading channel plays an essential role in many wireless receivers. The conventional Kalman filter is an optimum estimator; however, it is computationally demanding and complex for real-time implementation. In this paper a new approach is proposed for the implementation of the Kalman filter based on differential channel states. This leads to a robust differential Kalman filtering algorithm that can be simplified further to ease the implementation without any major loss in performance. It is also shown that the simplifications made to the differential Kalman filter lead to the Least Mean Squares (LMS) algorithm, identifying it as a special case of the Kalman filter.

1. INTRODUCTION

Maximum Likelihood Sequence Estimation (MLSE) is the optimum detection technique for digital signals corrupted by intersymbol interference (ISI) and additive Gaussian noise. Generally all adaptive versions of MLSE receivers require estimation algorithms for identifying time varying channels. In most wireless systems, the quality of the channel estimation method has a strong impact on the overall Bit Error Rate (BER) performance of the receiver. Therefore, a key factor in the receiver design is the estimation of the fading channel with high accuracy [1].

Many types of estimation algorithms require some information about the time varying parameters of the fading channel. This information is often in the form of state space model parameters of the channel. The optimum Kalman filter requires the exact parameters of the state space model and the second order statistics of the random model-parameters. The RLS algorithm, as shown by Sayed and Kailath [2], is a special case of the Kalman filter where the required information about the state space model are simply replaced by constant values. This means that the RLS algorithm is model dependent and since the actual model parameters of the system are replaced with constants we would expect a degradation in the performance due to a model mismatch [3].

In practice, a wide sense stationary uncorrelated scattering (WSSUS) model is used for data detection over fading channels [1][4]. The assumption of wide sense stationarity is somewhat controversial, since any change in the vehicle speed will affect the maximum Doppler frequency, and this changes the statistics of the channel. However, in practical situations the WSSUS model can be adopted,

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assuming constant vehicle speed for the duration of one or a few data frames.

An inherent difficulty associated with applying the estimation methods for tracking the channel is that the unknown transmitted data is required for the estimator adaptation. This information can be available to the estimator in one of three forms: (1) as training symbols inserted in a data frame (symbol-aided channel estimation)[5], (2) the detected data stream (decision directed mode) [6], (3) using the joint data and channel estimation techniques [7][8], and this can be performed in a per-survivor processing (PSP) fashion [9].

A bank of estimators are required to implement PSP, one for each survivor sequence, for channel tracking and branch metric computation. By choosing the Kalman filter for channel tracking, the complexity of the receiver can be prohibitive, particularly for channels with a long impulse response duration. The Kalman filter is computationally demanding and very sensitive to round-off errors. The efforts to overcome the design complexity of the Kalman based PSP receiver can be focused in two major directions. The first approach is to employ simpler channel estimators, and this can be in form of seeking suboptimal alternatives to the Kalman filter [3][2]. A second direction is to take advantage of recent advances in VLSI technology in parallel information processing and proposing parallel algorithms and structures for the implementation of Kalman filter [10][11][12].

The purpose of this paper is to introduce a new approach to define the Kalman filtering algorithm. Here we propose a different method to define the state space model of the channel from what has been reported in the literature so far. To derive the ARMA model of the channel impulse response, usually the consecutive instances of the impulse response are used as the basis [12][13]. We show that by choosing the impulse response and its time derivatives as an equivalent set of basis, the Kalman filter algorithm remains unchanged and only new parameters are used. With the new definition of states, the Kalman filter becomes more robust against the simplifications made to reduce the implementation complexity. The complex covariance matrix can be simplified to a reduced size real matrix to mitigate the complexity. Also the state transition matrix can be rounded to have only *one* and *zero* elements. The simplifications are aimed towards obtaining an LMS-type algorithm from the optimal Kalman filter.

In section 2 the mathematical model considered for the problem will be described. In section 3 the Kalman filter and its variants will be introduced for channel tracking. Simulation results are explained in section 4 and the conclusion is presented in section 5.

2. MATHEMATICAL MODELING

The MPSK complex data sequence $\{d_i \in \mathbb{C}\}$ with the symbol period T is transmitted and received via a frequency selective fading channel. The equivalent discrete time model for the adaptive receiver is shown in Fig. 1. The sampled signal, z_k, can be written as

$$z_k = \sum_{i=0}^{q-1} d_{k-i} x_{k,i} + n_k \tag{1}$$

where $x_{k,i}$ is the channel impulse response (CIR) at time k due to an impulse that

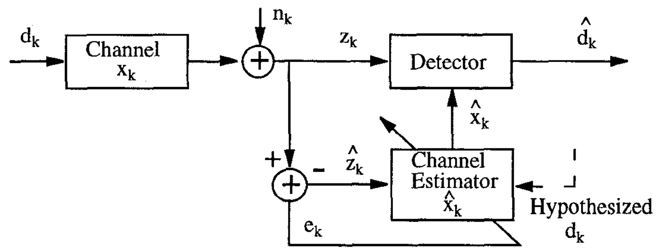

Fig. 1: The adaptive receiver model for joint data and channel estimation.

was applied at time *k-i*, and the total length of the CIR is assumed to be q. The additive white Gaussian noise, n_k is a circularly symmetric complex Gaussian process [6] with power spectral density N_o. In the channel estimator a hypothesized value for d_k is convolved with an estimate of the channel, resulting in $\hat{z}_k$, which is an estimate of the received signal. The difference between z_k and $\hat{z}_k$ is the error e_k, and it is used to update the channel estimates via the estimation algorithm. The hypothesized transmitted data sequence can be available to the channel estimator as in the PSP method [9], where a different hypothesized data sequence is assumed along each branch of the Viterbi algorithm trellis diagram.

The channel is considered as WSSUS and its, $x_{k,i}$, is a wide-sense stationary Gaussian random signal and can be modeled with a third order AR representation [7] as

$$x_{k,i} = c_1 x_{k-1,i} + c_2 x_{k-2,i} + c_3 x_{k-3,i} + c_4 w_{k,i} \tag{2}$$

The circularly symmetric complex Gaussian noise process $w_{k,i}$ is the driving noise of the AR model. The CIR at time k can be defined in vector form as

$$\boldsymbol{x}_k = [x_{k,0}, x_{k,1}, \ldots, x_{k,q-1}]^T \tag{3}$$

where $(.)^T$ denotes the matrix transposition operator. Using (2) and (3) we obtain

$$\boldsymbol{x}_k = c_1 \boldsymbol{x}_{k-1} + c_2 \boldsymbol{x}_{k-2} + c_3 \boldsymbol{x}_{k-3} + c_4 \boldsymbol{w}_k \tag{4}$$

where the vector $\boldsymbol{w}_k = [w_{k,0}, w_{k,1}, \ldots, w_{k,q-1}]^T$ is a zero mean white complex Gaussian process with the diagonal covariance matrix $\boldsymbol{Q}$ defined as $E\{\boldsymbol{w}_k \boldsymbol{w}_l^H\} = \boldsymbol{Q}\delta(k-l)$. Here, $\delta(k-l)$ is the Kroneker delta function, and $(.)^H$ denotes Hermitian transposition. From (4) it is clear that the state of the channel at time k can be expressed based on its 3 past consecutive values and we can write

$$\begin{bmatrix} \boldsymbol{x}_{k+1} \\ \boldsymbol{x}_k \\ \boldsymbol{x}_{k-1} \end{bmatrix} = \begin{bmatrix} c_1\boldsymbol{I} & c_2\boldsymbol{I} & c_3\boldsymbol{I} \\ \boldsymbol{I} & \underline{\boldsymbol{0}} & \underline{\boldsymbol{0}} \\ \underline{\boldsymbol{0}} & \boldsymbol{I} & \underline{\boldsymbol{0}} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_k \\ \boldsymbol{x}_{k-1} \\ \boldsymbol{x}_{k-2} \end{bmatrix} + \begin{bmatrix} c_4\boldsymbol{I} \\ \underline{\boldsymbol{0}} \\ \underline{\boldsymbol{0}} \end{bmatrix} \boldsymbol{w}_{k+1} \tag{5}$$

where $\boldsymbol{I}$ and $\underline{\boldsymbol{0}}$ are $q \times q$ -identity matrix and $q \times q$ -zero matrix, respectively.

Let's define the $3q$-dimensional vectors of the channel state and the input data respectively as

$$X_k = [x_k^T, x_{k-1}^T, x_{k-2}^T]^T \quad (6)$$

$$H_k = \left[d_k, d_{k-1}, d_{k-2}, \ldots, d_{k-q+1}, \overbrace{0, 0, \ldots, 0}^{2q \text{ zeros}} \right] \quad (7)$$

$$= \left[D_k, \overbrace{0, 0, \ldots, 0}^{2q \text{ zeros}} \right] \quad (8)$$

where D_k represents the nonzero part of the data vector. From (1) and (5) the state space model of the fading channel can be described by the channel state equation and the channel output equation as:

$$X_{k+1} = FX_k + Gw_{k+1} \quad (9)$$

$$z_k = H_k X_k + n_k \quad (10)$$

where F is the $3q \times 3q$ transition matrix and G is the $3q \times q$ measurement coupling matrix.

3. ADAPTIVE TRACKING

The state space model of (9) and (10) represents a linear time-variant system described by a Markov model of order three. In the following, first we introduce the standard Kalman filtering solution for estimating the state vector of this system or in other words for estimating the CIR of the fading channel. Then we will introduce the idea of using differential states for this model and will study the practical effects on performance and implementation.

3.1. The Conventional Kalman filter

The Kalman filter can recursively estimate the states of the linear system defined in (9) and (10). The recursion equations of the Kalman filter are:

$$R_k = H_k P_k H_k^H + N_o \quad (11)$$

$$e_k = z_k - H_k \hat{X}_k \quad (12)$$

$$\hat{X}_{k+1} = F\hat{X}_k + FP_k H_k^H R_k^{-1} e_k \quad (13)$$

$$P_{k+1} = F(P_k - P_k H_k^H R_k^{-1} H_k P_k) F^H + GQG^H \quad (14)$$

where $\hat{X}_k$ is the state estimate and P_k is the state estimation error covariance matrix. The Kalman filter is an optimum estimator that minimizes the mean squared estimation error.

The Kalman filter is computationally demanding, and this limits its use in real-time applications. The conventional Kalman filter is also very sensitive to round-off errors and to maintain its stability, the algorithm has to be implemented with a rather long digital word-length. In order to obtain a numerically accurate and stable algorithm square-root solutions have been proposed for implementation of the Kalman filter [14]. With recent advances in VLSI technology parallel information processing has become more and more feasible, allowing for the implementation of dedicated systolic structures for square-root Kalman filtering [10]. However, It is usually common practice to make a trade-off between cost and performance and look for suboptimal versions of the Kalman filter trying to keep the performance acceptable while reducing the implementation cost.

Another problem with the implementation of a Kalman filter for data detection over fading channels is that it requires some information about the channel model, such as $\boldsymbol{F}$, $\boldsymbol{G}$, $\underline{\boldsymbol{Q}}$, and N_o. This information is not easily available at the receiver and it has to be extracted from the received signal which leads to state estimation with model uncertainty. In practice, it is reasonable to assume that the parameters of the channel model are *almost* constant compared to the state variations over a certain time interval. In this way constant channel parameters can be used in the Kalman filter as long as they are valid, say, over one or a few data blocks. Some receivers avoid this problem totally or partially by employing estimation algorithms that require a minimum amount of a priori information about the channel, e.g. the LMS algorithm only considers the received signal.

In the following we propose a solution to mitigate the above problems. This is in the form of a change of basis in the state space model, and this leads to a simpler implementation of the Kalman filtering algorithm.

3.2. The Differential Kalman Filter

We will show that the implementation of the channel estimation and tracking process will be enhanced if we use another set of states in the state space model. This is equivalent to choosing a new set of basis in the same space. The elements of the new set of basis at time k are $\boldsymbol{x}_k$ and its first and second order time derivatives. Let's define the new $3q$-dimensional channel state vector as

$$\underline{\boldsymbol{X}}_k = [\boldsymbol{x}_k^T, \dot{\boldsymbol{x}}_k^T, \ddot{\boldsymbol{x}}_k^T]^T \tag{15}$$

$$=[\boldsymbol{x}_k^T, \boldsymbol{x}_k^T - \boldsymbol{x}_{k-1}^T, \dot{\boldsymbol{x}}_k^T - \dot{\boldsymbol{x}}_{k-1}^T]^T \tag{16}$$

It is easy to verify that

$$\underline{\boldsymbol{X}}_k = \boldsymbol{T}\boldsymbol{X}_k \tag{17}$$

where the $3q \times 3q$ transformation matrix is

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{I} & \underline{\boldsymbol{0}} & \underline{\boldsymbol{0}} \\ \boldsymbol{I} & -\boldsymbol{I} & \underline{\boldsymbol{0}} \\ \boldsymbol{I} & -2\boldsymbol{I} & \boldsymbol{I} \end{bmatrix} \tag{18}$$

and $\boldsymbol{T} = \boldsymbol{T}^{-1}$. In general for an nq dimensional state vector the components of this matrix can be expressed as

$$\boldsymbol{T}_{i,j} = \begin{cases} (-1)^{j-1}\binom{i-1}{j-1}\boldsymbol{I} & j \le i \\ \underline{\boldsymbol{0}} & j > i \end{cases} \tag{19}$$

By applying the above transformation to (9) and (10) the state space model can be described with

$$\underline{X}_{k+1} = \underline{F}\underline{X}_k + \underline{G}w_{k+1} \tag{20}$$

$$z_k = H_k\underline{X}_k + n_k \tag{21}$$

and the Kalman filtering algorithm of (11)-(14) will be transformed to

$$R_k = H_k\underline{P}_kH_k^H + N_o \tag{22}$$

$$e_k = z_k - H_k\hat{\underline{X}}_k \tag{23}$$

$$\hat{\underline{X}}_{k+1} = \underline{F}\hat{\underline{X}}_k + \underline{F}\underline{P}_kH_k^HR_k^{-1}e_k \tag{24}$$

$$\underline{P}_{k+1} = \underline{F}(\underline{P}_k - \underline{P}_kH_k^HR_k^{-1}H_k\underline{P}_k)\underline{F}^H + \underline{G}\underline{Q}\underline{G}^H \tag{25}$$

where $\underline{F} = TFT$, $\underline{G} = TG$, and $\underline{P}_k = TP_kT^H$. Also note that $H_kT = H_k$. Hence, we realize that by using the above transformation the algorithms for conventional Kalman filter and differential Kalman filter will be the same. In the following we will see that in practice the performance of the differential method is superior due to its robustness against numerical errors. Also we show that its implementation can be simplified without a major loss in performance.

I) Rounding $\underline{F}$ to make an upper triangular matrix

The matrix $\underline{F}$ can be computed as

$$\underline{F} = TFT = \begin{bmatrix} (c_1+c_2+c_3)I & (-c_2-2c_3)I & c_3I \\ (c_1+c_2+c_3-1)I & (-c_2-2c_3)I & c_3I \\ (c_1+c_2+c_3-1)I & (-c_2-2c_3-1)I & c_3I \end{bmatrix} \tag{26}$$

We can show that in practical situations the components of this matrix can be rounded to obtain an upper triangular matrix with all nonzero elements equal to one. This is very appealing for the digital implementation of the algorithm. Fig. 2 shows the variation of three coefficients in the first row of $\underline{F}$ versus vehicle speed. To obtain the coefficients Rayleigh fading channel, and a symbol rate of 25 ksymbol/sec in the 900 MHz band is assumed. As we can see all three coefficients can be rounded to one for a wide range of vehicle speeds with a very small approximation error, while in turn the matrix $\underline{F}$ is converted to an upper triangular matrix as described above.

II) Forcing $\underline{F}$ to be an upper triangular matrix

The state transition equation of (20) can be replaced by another equation where the $\underline{F}$ matrix is upper triangular without any approximation. Based on the definition of differential states we can write

$$x_{k+1} = x_k + \dot{x}_{k+1} = x_k + \dot{x}_k + \ddot{x}_{k+1} = x_k + \dot{x}_k + \ddot{x}_k + \dddot{x}_{k+1} \tag{27}$$

then it is easy to verify

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \\ \ddot{x}_{k+1} \end{bmatrix} = \begin{bmatrix} I & I & I \\ \underline{0} & I & I \\ \underline{0} & \underline{0} & I \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} + \begin{bmatrix} \dddot{x}_{k+1} \\ \dddot{x}_{k+1} \\ \dddot{x}_{k+1} \end{bmatrix} \tag{28}$$

or

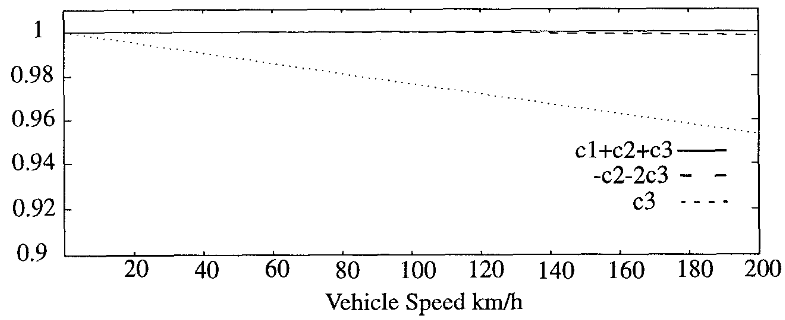

Fig. 2: Variation of the coefficients of matrix $\underline{F}$ with the vehicle speed. (Rayleigh fading channel, Symbol rate = 25 ksymbol/sec, 900 MHz band, c1 $\approx$ 3, c2 $\approx$ -3, c3 $\approx$ 1)

$$\underline{X}_{k+1} = \begin{bmatrix} I & I & I \\ \underline{0} & I & I \\ \underline{0} & \underline{0} & I \end{bmatrix} \underline{X}_k + \begin{bmatrix} I \\ I \\ I \end{bmatrix} \ddot{x}_{k+1} \tag{29}$$

where $\ddot{x}_{k+1}$ is a zero mean white Gaussian process. Equation (29) describes the state transitions and can replace (20) in the definition of the channel state space model. This forces the $\underline{F}$ matrix to be upper triangular with only ones and zeros. In (29), $\ddot{x}_{k+1}$ is a random process and to implement the Kalman filtering algorithm its diagonal covariance matrix, $E\{\ddot{x}_k \ddot{x}_l^H\} = \underline{Q}\delta(k-l)$, is required. It can be shown that

$$\ddot{x}_k = x_k - 3x_{k-1} + 3x_{k-2} - x_{k-3} \tag{30}$$

and to obtain $\underline{Q}$ we note that from (30)

$$\underline{Q} = E\{\ddot{x}_k \ddot{x}_k^H\} = 20R(0) - 30R(1) + 12R(2) - 2R(3) \tag{31}$$

where $R(j) = E\{x_k x_{k+j}^H\}$ is the autocorrelation matrix of the channel impulse response vector. To calculate $R(j)$ for different j values, one can multiply both sides of (4) by x_j^H and get the expected value to obtain the following equation that can be solved to find the desired values.

$$\begin{bmatrix} -I & c_1 I & c_2 I & c_3 I \\ c_1 I & (c_2 - 1)I & c_3 I & \underline{0} \\ c_2 I & (c_1 + c_3)I & -I & \underline{0} \\ c_3 I & c_2 I & c_1 I & -I \end{bmatrix} \begin{bmatrix} R(0) \\ R(1) \\ R(2) \\ R(3) \end{bmatrix} = \begin{bmatrix} -c_4 Q \\ \underline{0} \\ \underline{0} \\ \underline{0} \end{bmatrix} \tag{32}$$

III) Simplifying the covariance matrix $\underline{P}_k$

To reduce the computational burden of the differential Kalman filtering algorithm (22)-(25), we can realize that after simplifying the $\underline{F}$ matrix, most of the computational power is spent in computing $\underline{P}_k$. This is a $3q \times 3q$ complex matrix with Hermitian symmetry. We show that it is possible to consider a simpler form for $\underline{P}_k$ to simplify the calculations with a small effect on performance. The covariance matrix can be considered as

$$\underline{P}_k = \begin{bmatrix} \mu_{1,k}I & \alpha_{1,k}I & \beta_{1,k}I \\ \alpha_{1,k}I & \alpha_{2,k}I & \beta_{2,k}I \\ \beta_{1,k}I & \beta_{2,k}I & \beta_{3,k}I \end{bmatrix} \tag{33}$$

where $\mu_{i,k}$, $\alpha_{i,k}$, and $\beta_{i,k}$ are real scalers. This will simplify the computation of the Kalman filtering equations drastically as q^2 complex values are replaced by one real scalar. To interpret the above simplification one can consider the state estimation of (24) as two stages. First is the measurement update equation as

$$\hat{\underline{X}}_{k|k} = \hat{\underline{X}}_k + \underline{P}_k H_k^H R_k^{-1} e_k \tag{34}$$

where the channel estimate is updated based on the information of the received signal and then the time update equation based on our knowledge of the system model as

$$\hat{\underline{X}}_{k+1} = \underline{F}\hat{\underline{X}}_{k|k} \tag{35}$$

Using (8) and (33) the measurement update equation of (34) can be viewed as three LMS type update equations for the states and the differential states of different orders

$$x_{k|k} = x_k + R_k^{-1}\mu_{1,k}D_k^H e_k \tag{36}$$

$$\dot{x}_{k|k} = \dot{x}_k + R_k^{-1}\alpha_{1,k}D_k^H e_k \tag{37}$$

$$\ddot{x}_{k|k} = \ddot{x}_k + R_k^{-1}\beta_{1,k}D_k^H e_k \tag{38}$$

where $R_k^{-1}\mu_{1,k}$, $R_k^{-1}\alpha_{1,k}$, and $R_k^{-1}\beta_{1,k}$ serve as the step-size parameter of the LMS algorithm. However, these parameters are not fixed as in a regular LMS method, and will be updated adaptively by the covariance equation of (25). It is possible to simplify the computation of (25) one step further by reducing the matrix I to one in $\underline{F}$, $\underline{P}_k$ and $\underline{G}$ and also approximating $D_k^H D_k$ with the constant value of $\|d_k\|^2$. In this case the size of the matrices in (25) reduces from $3q \times 3q$ to 3×3 leading to a simpler computation. Similar to the approach of Sayed and Kailath in [2] we can see that the LMS algorithm, like RLS, is a simplified variant of the Kalman filter. This can be verified if we let $\underline{F}$ be the identity matrix and in (33) have all of the parameters but $\mu = \mu_{1,k}$ equal to zero, and μ can be constant as in LMS.

The conventional Kalman filter is very sensitive to numerical errors. The simplification of (33) leads to very poor BER performance and occasional divergence of the adaptive algorithm in our simulations of a channel estimator with conventional Kalman filter. However the differential Kalman filter allows for the above simplifications with a small degradation in performance as will be shown in the next section.

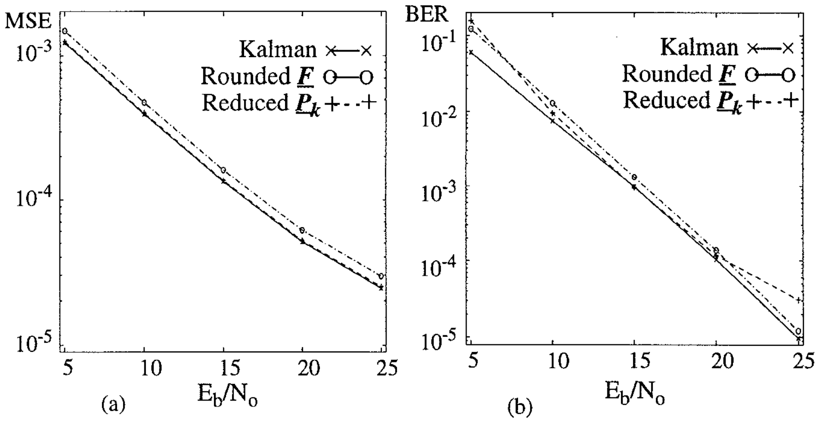

Fig. 3: (a) MSE in estimation and (b) receiver's BER results for QPSK modulation with Differential Kalman filter, Differential Kalman filter with rounded $\underline{F}$, and Differential Kalman filter with reduced $\boldsymbol{P}_k$. and normal $\underline{F}$. (Vehicle speed = 100 km/h, and PSP is used for detection)

4. PERFORMANCE RESULTS

In the computer simulations, the modulation scheme employed is differentially coherent QPSK, with a symbol rate of 25 ksymbol/s. As in the IS-136 standard, the differentially encoded data sequence is arranged into 162 symbol frames. The first 14 symbols of each frame is a training preamble sequence to help the adaptation of the channel estimator. The fading channel is simulated as a symbol-spaced two-path model with time varying complex coefficients. The two fading paths are independent with equal strength. The ISI at the receiver is due to the multipath nature of the channel. The total length of the channel impulse response is 2 symbol intervals and there are four possible states in the trellis diagram of the Viterbi algorithm at the receiver. For each state in the trellis there are four possible transitions to the four states in the next stage.

Fig. 3(a) shows the steady state average mean squared error (MSE) in the estimation of the channel impulse response. It is clear that by simplifying $\underline{P}_k$ to a real matrix with reduced size as described above, the change in MSE is negligible. Also by rounding the $\underline{F}$ matrix there is one dB degradation in performance. Fig. 3(b) shows the bit error rate performance (BER) of a Kalman-based Viterbi algorithm to detect the transmitted data. Also in this case the result of the above simplifications is a degradation of only 1-2 dB in the BER performance.

5. CONCOLUSION

The differential Kalman filter was introduced using a new basis in the state space model. The algorithm for this filter is similar to that of the conventional Kalman filter. However, it proves to be more robust against simplifications made to mitigate the implementation problems. The complex error covariance matrix was approximated by a real matrix of reduced size. The transition matrix was also modified to contain ones and zeros which simplifies matrix multiplications. Finally, we showed that the LMS algorithm can be obtained along with these simplifications. This is similar to the approach of [2] and proves that LMS also belongs to the family of the Kalman filter variants.

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