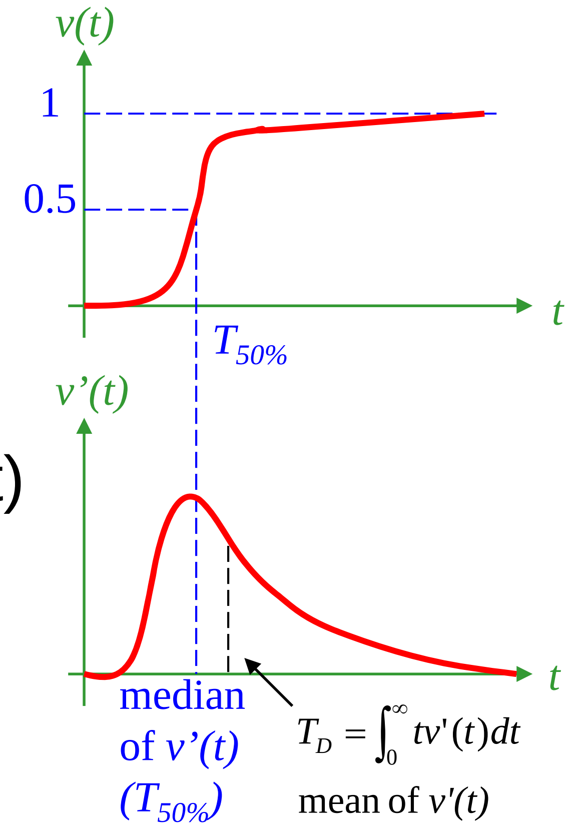
Elmore Delay for Monotonic Responses

- Assumptions:
  - Unit step input
  - Monotone output response

- Basic idea: use of mean of v'(t) to approximate median of v'(t)

$v(t)$: output response (monotone)

$v'(t)$: rate of change of $v(t)$



Elmore Delay for Monotonic Responses

- $T_{50\%}$: median of $v'(t)$, since

$$\int_0^{T_{50\%}} v'(t)dt = \int_{T_{50\%}}^{+\infty} v'(t)dt$$
$$= \text{half of final value of } v(t) \quad \text{(by def.)}$$

- Elmore delay T_D = mean of $v'(t)$

$$T_D = \int_0^{\infty} v'(t)\,t\,dt$$

Why Elmore Delay?

- Elmore delay is easier to compute analytically in most cases
  - Elmore's insight [Elmore, J. App. Phy 1948]
  - Verified later on by many other researchers, e.g.
    - Elmore delay for RC trees [Penfield-Rubinstein, DAC'81]
    - Elmore delay for RC networks with ramp input [Chan, T-CAS'86]
    - .....

- For RC trees: [Krauter-Tatuianu-Willis-Pileggi, DAC'95]

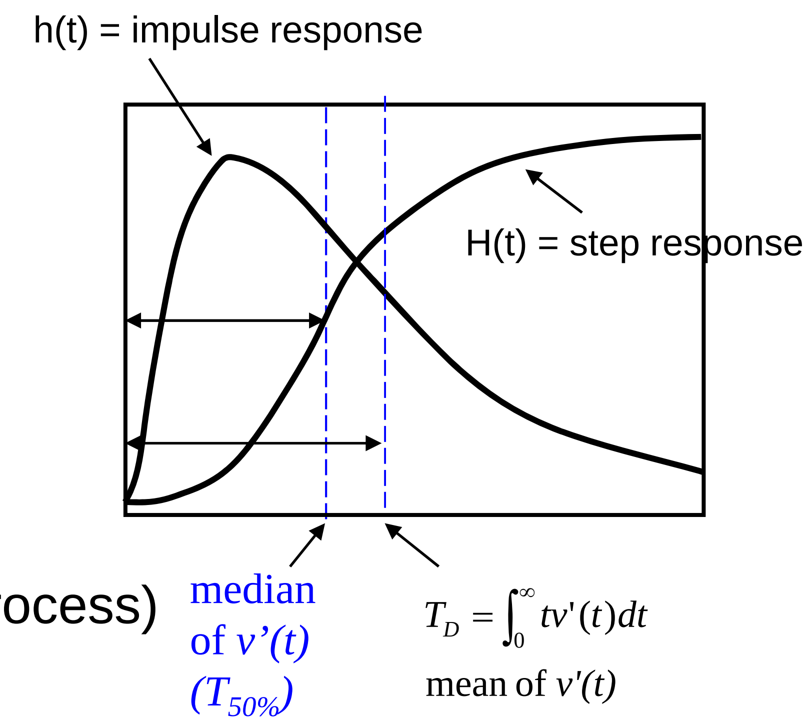
  $T_{50\%} \leq T_D$

- Note: Elmore delay is not 50% value delay in general!

Elmore Delay for RC Trees

- Definition
  - h(t) = impulse response
  - T_D = mean of h(t)

$$= \int_0^\infty h(t) \cdot t \, dt$$



- Interpretation
  - H(t) = output response (step process)
  - h(t) = rate of change of H(t)
  - $T_{50\%}$= median of h(t)
  - Elmore delay approximates the median of h(t) by the mean of h(t)

Elmore Delay of a RC Tree

[Rubinstein-Penfield-Horowitz, T-CAD'83]

Lemma: when a step input is applied to a RC tree

$v_i(t)$ is monotonic in t for every node i in tree

Proof: $\Leftrightarrow v'_i(t) \geq 0$ at every node i $(v'i(t) = h_i(t))$

$\Leftrightarrow$ impulse response $h_i(t) \geq 0$ at every node i

Let $h_{\min}(t)$ be the min. voltage of any node at t

Apply impulse func. at t=0:

$h_{\min}(0+) \geq 0$

Assume that $h_{\min}(t_0) < 0$

Then, $\exists t_1 < t_0$ s.t. $h'_{\min}(t_1) < 0$

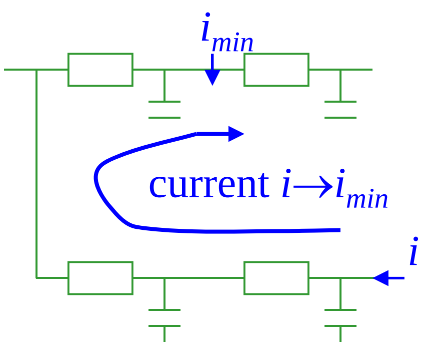
Let node $i_{\min}$ achieve $h_{\min}(t_1)$ at t_1

Then, the current from any node i to $i_{\min}$ is ≥ 0 at t_1

Since $h_i(t_1) \geq h_{\min}(t_1)$ & i connects $i_{\min}$ via resistors

Since all currents $i \rightarrow i_{\min}$ charge the capacitor at $i_{\min}$

$h'_{\min}(t_1) \geq 0 \Rightarrow$ contradiction!

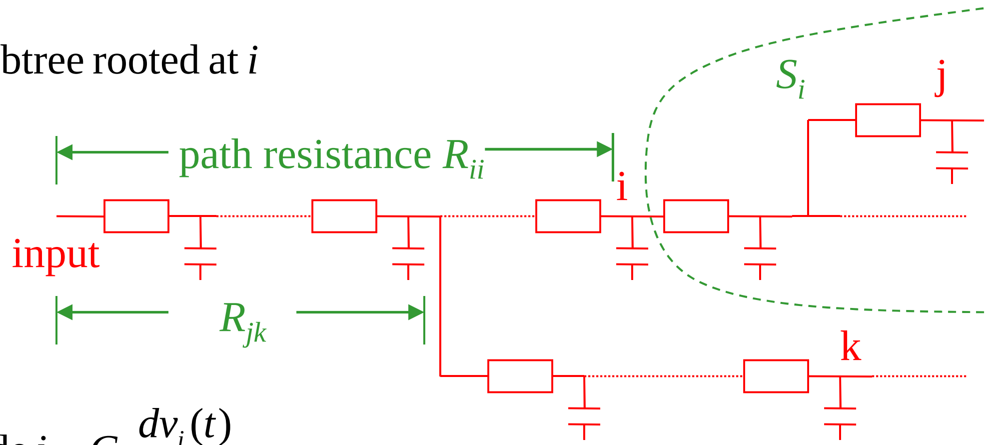


Elmore Delay in a RC Tree (cont'd)

P_i : path from input to node i ; s_i :subtree rooted at i

R_{jk} : resistance of common path

$P_j \cap P_k$ from input to j & k



Theorem : Elmore delay to node i

$$T_{D_i} = \sum_k R_{ki} C_k$$

Proof : The current to cap. of node $i = C_i \dfrac{dv_i(t)}{dt}$

$$1 - v_i(t) = \text{The voltage drop on } P_i = \sum_{k \in P_i} R_k \cdot (\text{current to all cap's in } S_i)$$

$$= \sum_k (\text{current to cap } k) \cdot (\text{common path res. between } P_i \text{ and } P_k)$$

$$= \sum_k C_k \frac{dv_k(t)}{dt} \cdot R_{ki} = \sum_k R_{ki} C_k \frac{dv_k(t)}{dt}$$

$$T_{D_i} = \int_0^\infty v'_i(t) t \cdot dt = v_i(t) \cdot t \,\Big|_0^\infty - \int_0^\infty v_i(t) dt$$

$$= \lim_{T \to \infty} [v_i(T) \cdot T - \int_0^T v_i(t) dt] = \lim_{T \to \infty} (v_i(T) - 1) \cdot T + \int_0^\infty (1 - v_i(t)) dt$$

Elmore Delay in a RC Tree (cont'd)

- We shall show later on that $\lim_{T\to\infty}(1-v_i(T))\cdot T = 0$
  i.e. $1-v_i(T)$ goes to 0 at a much faster rate than $1/T$ when $T\to\infty$

- Let $f_i(t) = \int_0^t [1-v_i(x)]dx$

$$f_i(t) = \int_0^t \sum_k R_{ki}C_k \frac{dv_k(x)}{dx}dx$$

$$= \sum_k R_{ki}C_k v_k(t)$$

$$= \sum_k R_{ki}C_k - \sum_k R_{ki}C_k[1-v_k(t)]$$

$$f_i(\infty) = \sum_k R_{ki}C_k$$

$$\therefore\ T_{D_i} = \lim_{T\to\infty}(1-v_i(T))T + \int_0^\infty [1-v_i(t)]dt$$

$$= f_i(\infty) = \sum_k R_{ki}C_k$$