

Chapter 5 - Problems

5.1)

a) Find $\omega_{-3dB} = \frac{1}{R_{out} C_{eq}} = \frac{1}{r_{DS2} // r_{DS4} \times C_L (1 + A_2)}$

where $r_{DS4} = \frac{8000L}{I_{D4}} = \frac{8000 \times 1.2}{0.05} = 192 \text{ k}\Omega$

$$r_{DS2} = \frac{12000L}{I_{D2}} = \frac{12000 \times 1.2}{0.05} = 288 \text{ k}\Omega$$

$$\therefore r_{DS4} // r_{DS2} = 115 \text{ k}\Omega$$

$$A_2 = g_{m7} (r_{DS6} // r_{DS7})$$

$$r_{DS6} = \frac{12000 \times 1.2}{0.1} = 144 \text{ k}\Omega$$

$$r_{DS7} = \frac{8000 \times 1.2}{0.1} = 96 \text{ k}\Omega$$

$$\therefore r_{DS6} // r_{DS7} = 58 \text{ k}\Omega$$

$$g_{m7} = \sqrt{2 \mu_n C_{ox} W/L I_{D5}} = (2 \times 92 \times 10^{-6} \times 300 / 1.2 \times 0.1 \times 10^{-3})^{1/2}$$
$$= 2.15 \text{ mA/V}$$

$$\therefore A_2 = 2.15 \times 10^{-3} \times 58 \times 10^3 = 124$$

$$\therefore \omega_{-3dB} = \frac{1}{115 \times 10^3 \times 10 \times 10^{-12} \times 125} = \underline{\underline{2\pi \times 1.1 \text{ kHz}}}$$

b) Find unity-gain frequency, ω_t .

$$\omega_t = g_{m1} / c_c \quad \text{where } g_{m1} = \sqrt{2 \times 30 \times 10^{-6} \times \frac{300}{1.2} \times 50 \times 10^{-6}}$$
$$= 0.866 \text{ mA/V}$$

$$\therefore \omega_t = \frac{0.866 \times 10^{-3}}{10 \times 10^{-12}} = \underline{\underline{2\pi \times 14 \text{ MHz}}}$$

c) Find slew rate.

$$SR = \frac{2I_{D1}}{C_c} = \frac{2 \times 50 \mu\text{A}}{10 \text{ pF}} = \underline{\underline{10 \text{ V}/\mu\text{sec}}}$$

5.2) With $C_c = 4 \text{ pF}$,

$$SR = \frac{2I_{D1}}{C_c} = \frac{2 \times 50 \mu\text{A}}{4 \text{ pF}} = \underline{\underline{25 \text{ V}/\mu\text{sec}}}$$

To double the slew rate while maintaining C_c , we need to double I_{D1} by doubling the width of Q_5 .

In order to prevent this from changing g_{m1} and g_{m2} and hence ω_t , we need to reduce the widths of Q_1 and Q_2 by half.

5.3) Referring to Problem 5.2, if we scale the widths of Q_1 and Q_2 by half, we need to maintain the equality

$$\frac{W/L_7}{W/L_4} = 2 \frac{W/L_6}{W/L_5} \quad (5.28)$$

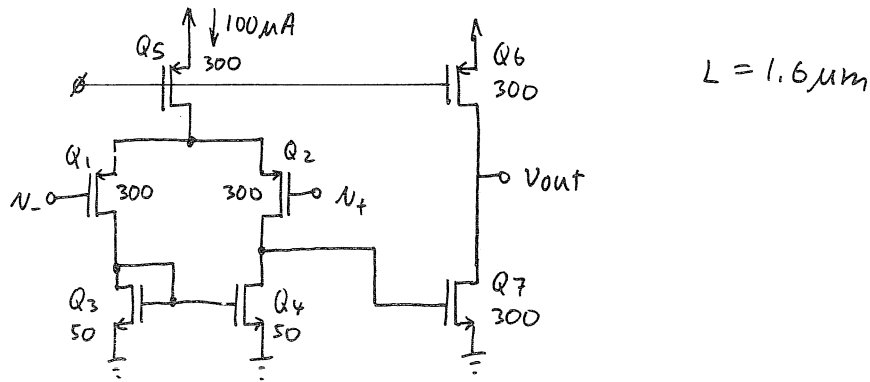
$$\therefore \frac{(W/L)_7}{(W/L)_6} = \frac{2 (W/L)_4}{(W/L)_5} = \frac{2 (150/1.2)}{(600/1.2)} = \underline{\underline{\frac{1}{2}}}$$

2 POSSIBILITIES ARE :

1) IF bias current at output stage remains unchanged $\Rightarrow (W/L)_6 = \frac{300}{1.2}$ & $(W/L)_7 = \frac{150}{1.2}$

2) IF bias current at output stage is doubled $\Rightarrow (W/L)_6 = \frac{600}{1.2}$ & $(W/L)_7 = \frac{300}{1.2}$

5.4)



To remove the inherent, systematic offset, the widths of Q_3 and Q_4 should become $150 \mu\text{m}$. In that case,

$$V_{\text{eff}7} = V_{\text{eff}4} = \sqrt{\frac{2I_{D7}}{\mu_n C_{ox} W/L_7}} = \sqrt{\frac{2 \times 100 \times 10^{-6}}{92 \times 10^{-6} \times 300/1.6}} = \underline{0.1077 \text{ V}}$$

However, with the current circuit,

$$V_{\text{eff}4} = \sqrt{\frac{2 \times 50 \times 10^{-6}}{92 \times 10^{-6} \times 50/1.6}} = 0.1865 \text{ V}$$

∴ An input offset voltage will have to be applied in order to decrease $V_{\text{gs}7}$ by

$$\Delta V_{\text{gs}7} = 0.1865 - 0.1077 = 78.8 \text{ mV}$$

For the input-referred offset voltage $V_{\text{i}off}$,

$$V_{\text{i}off} = \Delta V_{\text{gs}7} / A_1$$

where A_1 is the first stage's voltage gain

$$A_1 = g_{m1} \times r_{\text{os}2} // r_{\text{os}4}$$

$$g_{m1} = \sqrt{2 \times 30 \times 10^{-6} \times \frac{300}{1.6} \times 50 \times 10^{-6}} = 0.75 \text{ mA/V}$$

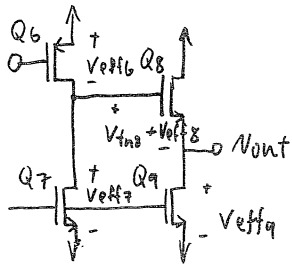
$$r_{\text{os}4} = 8000 \times 1.6 / 0.05 \text{ mA} = 256 \text{ k}\Omega$$

$$r_{\text{os}2} = 12000 \times 1.6 / 0.05 \text{ mA} = 384 \text{ k}\Omega$$

$$\therefore A_1 = 0.75 \times (256 // 384) = 115$$

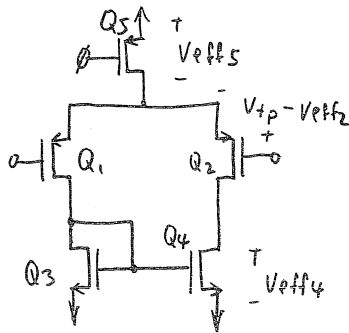
And $V_{\text{i}off} = \frac{78.8 \text{ mV}}{115} = \underline{0.7 \text{ mV}}$ (applied to v_+ input).

5.5) Find max. and min v_{out} and v_{in} (common mode)



$$v_{out-max} = V_{DD} - V_{eff6} - V_{tn8} - V_{eff8}$$

$$v_{out-min} = V_{SS} + V_{eff9}$$



$$v_{in-cm-max} = V_{DD} - V_{eff5} - V_{eff1} + V_{tp1}$$

$$v_{in-cm-min} = V_{SS} + V_{eff3} + V_{tn3} + V_{tp1}$$

Calculating all values,

$$V_{eff5} = V_{eff6} = \sqrt{\frac{2I_{D5}}{\mu_n \epsilon_{ox} W/L}} = \sqrt{\frac{2 \times 100 \times 10^{-6}}{30 \times 10^{-6} \times 300/1.6}} = 0.189V$$

similarly,

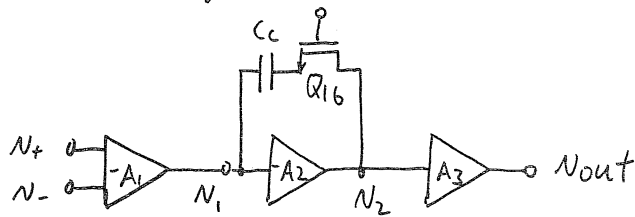
$$V_{eff1} = V_{eff2} = 0.133V$$

$$V_{eff3} = V_{eff4} = 0.108V$$

$$V_{eff8} = V_{eff9} = 0.108V$$

$$\left. \begin{aligned} v_{out-max} &= 5 - 0.189 - 0.8 - 0.108 = 3.9V \\ v_{out-min} &= -5 + 0.108 = -4.9V \\ v_{in-cm-max} &= 5 - 0.189 - 0.133 - 0.9 = 3.8V \\ v_{in-cm-min} &= -5 + 0.108 + 0.8 - 0.9 = -4.99V \end{aligned} \right\}$$

5.6) Bad design of compensation network:



In order to understand why oscillations occur on account of this arrangement, we have to keep in mind our use of Q_{16} , operated in the triode region, as a substitute for a feed-forward resistor. When v_{out} is large and positive, v_2 is similarly so. Should the voltage across C_c happen to be small, a large voltage would appear across the drain and source of Q_{16} . If this voltage were large enough to put Q_{16} into the active mode, capacitor C_c would effectively become disconnected from the output. The result would be an uncompensated op amp that is prone to oscillations.

By keeping Q_{16} connected to the input of the second stage, most of the large signal swings will appear across the capacitor instead. As such, Q_{16} will more reliably remain in the triode region of operation.

5.7) The body effect changes the threshold voltages of Q_8 , Q_1 and Q_2 . The change in V_{tn8} affects only $V_{out-max}$ while the changes in V_{tp1} and V_{tp2} affect both $V_{in-cm-max}$ and $V_{in-cm-min}$.

From Problem 5.5, $V_{out-min} = -4.9V$

The other values must be solved iteratively as the threshold voltages are dependent on V_{SB} 's.

For $V_{out-max}$:

Assume $V_{tn8} = 0.8V$ for first iteration

$$\begin{aligned} \therefore V_{SB8} &= V_{out-max} - V_{SS} = V_{DD} - V_{eff6} - V_{tn8} - V_{eff8} - V_{SS} \\ &= 5 - 0.189 - 0.8 - 0.108 - (-5V) \\ &= 8.9V \end{aligned}$$

$$\begin{aligned} \therefore V_{tn8} &= V_{tn0} + \gamma (\sqrt{V_{SB8} + 2\phi_F} - \sqrt{2\phi_F}) \\ &= 0.8V + 0.5 (\sqrt{8.9 + 0.7} - \sqrt{0.7}) \\ &= 1.93V \end{aligned}$$

1st iteration

$$\begin{aligned} \therefore V_{SB8} &= 5 - 0.189 - 1.93 - 0.108 - (-5V) \\ &= 7.77V \end{aligned}$$

$$V_{tn8} = 0.8 + 0.5 (\sqrt{7.77 + 0.7} - \sqrt{0.7})$$

$$\underline{V_{tn8} = 1.84V}$$

2nd iteration

$$\therefore V_{out-max} = V_{DD} - V_{eff6} - V_{tn8} - V_{eff8} = 5 - 0.189 - 1.84 - 0.108$$

$V_{out-max} = 2.9V$ which is almost 1 volt lower than our original result in P5.5.

For $V_{in-cm-max}$:

$$V_{SB1} = V_{eff5} = 0.189V \quad (\text{no iterations required})$$

$$\begin{aligned} V_{tp1} &= V_{tp0} - \gamma (\sqrt{V_{SB1} + 2\phi_F} - \sqrt{2\phi_F}) \\ &= -0.9 - 0.8 (\sqrt{0.189 + 0.7} - \sqrt{0.7}) \\ &= -0.98V \end{aligned}$$

$$\therefore V_{in-cm-max} = V_{DD} - V_{eff5} - V_{eff1} + V_{tp1} = 5 - 0.189 - 0.133 - 0.98$$

$$\underline{V_{in-cm-max} = 3.7V}$$

(cont.)

5.7) (cont.)

For $V_{in_{cm-min}}$:

Assume $V_{tp1} = -0.9V$

$$\begin{aligned} V_{SB1} &= V_{DD} - (V_{in_{cm-min}} + V_{eff1} - V_{tp1}) \\ &= V_{DD} - (V_{SS} + V_{eff3} + V_{tn3} + \sqrt{V_{TP1}} + V_{eff1} - \sqrt{V_{TP1}}) \\ &= V_{DD} - V_{SS} - V_{eff3} - V_{tn3} - V_{eff1} = 5 - (-5) - 0.108 - 0.8 - 0.133 \\ &= 8.96V \end{aligned}$$

$$\begin{aligned} \therefore V_{tp1} &= V_{tp0} - \delta (\sqrt{V_{SB1} + 2\phi_F} - \sqrt{2\phi_F}) \\ &= -0.9 - 0.8 (\sqrt{8.96 + 0.7} - \sqrt{0.7}) \\ &= \underline{\underline{-2.7V}} \end{aligned}$$

$$\begin{aligned} \therefore V_{in_{cm-min}} &= V_{SS} + V_{eff3} + V_{tn3} + V_{tp1} = -5 + 0.108 + 0.8 - 2.7 \\ &= \underline{\underline{-6.8V}} \end{aligned}$$

Note that the min common-mode input voltage is lower than the $-5V$ supply.

5.8) Show that $1/\omega_{eq} = \sum \frac{1}{\omega_{pi}} - \sum \frac{1}{\omega_{zi}}$.

We are given that

$$\angle [H(j\omega t)] \approx \angle [H_{app}(j\omega t)]$$

$$\begin{aligned} LS &= \angle [H(j\omega t)] \approx \angle \left[\frac{\prod (1 + j\omega t/\omega_{zi})}{\prod (1 + j\omega t/\omega_{pi})} \right] \\ &= \tan^{-1} \left(\frac{\omega t}{\omega_{z1}} \right) + \tan^{-1} \left(\frac{\omega t}{\omega_{z2}} \right) + \dots + \tan^{-1} \left(\frac{\omega t}{\omega_{zn}} \right) - \tan^{-1} \left(\frac{\omega t}{\omega_{p1}} \right) \\ &\quad - \dots - \tan^{-1} \left(\frac{\omega t}{\omega_{pn}} \right) \end{aligned}$$

But the Taylor Series expansion for $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$
 $\approx x$ for $x \ll 1$

\therefore If the op amp is compensated such that the unity gain frequency, ωt , is much lower than all higher order poles and zeros but much higher than ω_{p1} ,

$$LS \approx \frac{\omega t}{\omega_{z1}} + \dots + \frac{\omega t}{\omega_{zn}} - 90^\circ - \frac{\omega t}{\omega_{p2}} - \dots - \frac{\omega t}{\omega_{pn}} = \omega t \left(\sum \frac{1}{\omega_{zi}} - \sum \frac{1}{\omega_{pi}} \right) - 90^\circ$$

Similarly, $RS = \angle [H_{app}(j\omega t)] = \tan^{-1} \left(\frac{\omega t}{\omega_{eq}} \right) - 90^\circ \approx -\frac{\omega t}{\omega_{eq}} - 90^\circ$

$$\therefore \omega t \left(\sum \frac{1}{\omega_{zi}} - \sum \frac{1}{\omega_{pi}} \right) \approx -\omega t / \omega_{eq}$$

or $\underline{\underline{\frac{1}{\omega_{eq}} = \sum \frac{1}{\omega_{pi}} - \sum \frac{1}{\omega_{zi}}}}$ Q.E.D.

5.9) Given:

$$H(s) = \frac{K}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})(1 + s/\omega_{p3})(1 + s/\omega_{p4})}$$

where $\omega_{p1} = 2\pi \times 3 \text{ kHz}$, $\omega_{p2} = 2\pi \times 130 \text{ MHz}$
 $\omega_{p3} = 2\pi \times 160 \text{ MHz}$, $\omega_{p4} = 2\pi \times 180 \text{ MHz}$

$$\angle H(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega/\omega_{p1}}{130}\right) - \tan^{-1}\left(\frac{\omega/\omega_{p2}}{160}\right) - \tan^{-1}\left(\frac{\omega/\omega_{p3}}{180}\right)$$

Given that $\angle H(j\omega_{eq}) = -135^\circ$ then trial and error gives $\omega_{eq} \cong 2\pi \times 41.3 \text{ MHz}$

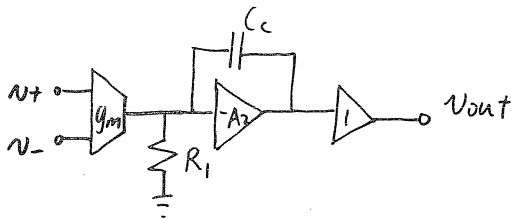
Using Eq. (5.45),

$$\frac{1}{\omega_{eq}} \cong \frac{1}{\omega_{p2}} + \frac{1}{\omega_{p3}} + \frac{1}{\omega_{p4}} = \frac{1}{2\pi} \left(\frac{1}{130 \times 10^6} + \frac{1}{160 \times 10^6} + \frac{1}{180 \times 10^6} \right)$$

$$\therefore \omega_{eq} \cong 2\pi \times 51.3 \text{ MHz}$$

This estimate is about 24% above the true value.

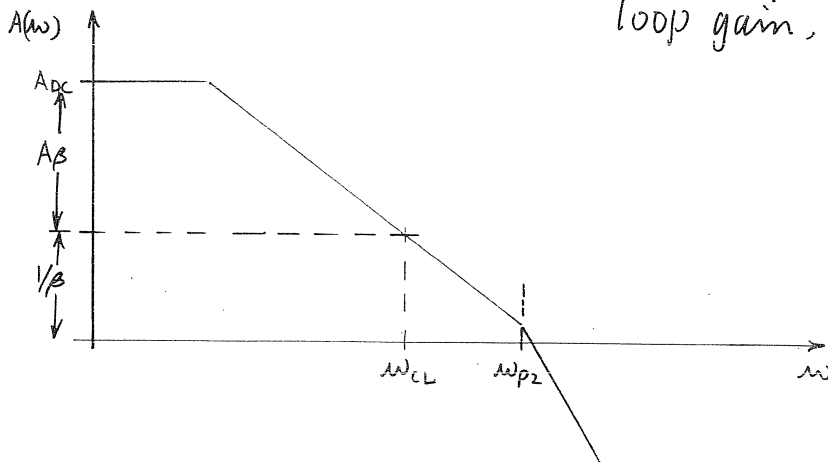
5.10) Two stage opamp:



$$g_m = 0.775 \text{ mA/V}$$

$$\omega_{p2} = 60 \text{ MHz}$$

ω_{CL} is defined as the frequency at which the loop gain, $A\beta$, is unity.



(cont.)

5.10 (cont.)

For a closed loop phase margin of 55°
eqn (5.53) states

$$\begin{aligned} \omega_t &= \tan(90^\circ - PM) \omega_{eq} \\ &= \tan(90^\circ - 55^\circ) \omega_{eq} \end{aligned}$$

$$\omega_t = 0.7 \omega_{eq}$$

For $\omega_{eq} = 2\pi \times 60 \text{ MHz}$, $\omega_t = 2\pi \times 42 \text{ MHz}$

Now including a feedback factor β into
eqn (5.46), we have

$$A(s) = \frac{\omega_t a \beta}{s(1 + s/\omega_{eq})} = \frac{\frac{g_m \beta}{C_c}}{s(1 + s/\omega_{eq})}$$

$$\text{Here } \beta = \frac{R_1}{R_1 + R_2} = \frac{1}{6}$$

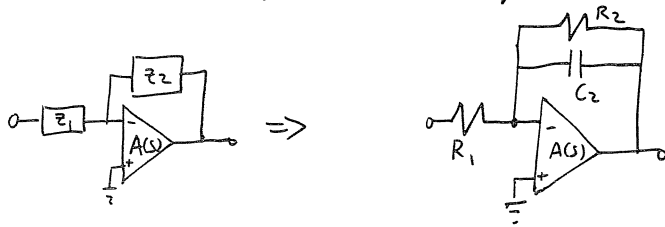
We require that $|A(j\omega_t)| = 1$ leading to

$$|A(j\omega_t)| = 1 = \frac{\frac{g_m \beta}{C_c}}{|\omega_t| \sqrt{1 + \left(\frac{\omega_t}{\omega_{eq}}\right)^2}}$$

$$\therefore C_c = \frac{g_m \beta}{\omega_t \sqrt{1 + 0.7^2}} = \frac{0.775 \times 10^{-3} \times \frac{1}{6}}{2\pi \times 42 \times 10^6 \times 1.221}$$

$$C_c = \underline{\underline{0.4 \text{ pF}}}$$

5.11) Find C_2 that provides compensation.



In Problem 5.10, we found that

$$\beta = \frac{R_1}{R_1 + R_2}.$$

This can be generalized to $\beta = \frac{z_1}{z_1 + z_2}$. We now wish to use the feedback network for compensation.

$$\text{For } z_1 = R_1, \quad z_2 = R_2 \parallel \frac{1}{sC_2} = \frac{R_2}{1 + sR_2C_2}$$

$$\beta = \frac{R_1}{R_1 + \frac{R_2}{1 + sR_2C_2}} = \frac{R_1(1 + sR_2C_2)}{sR_1R_2C_2 + R_1 + R_2} \quad \leftarrow \text{new zero added}$$

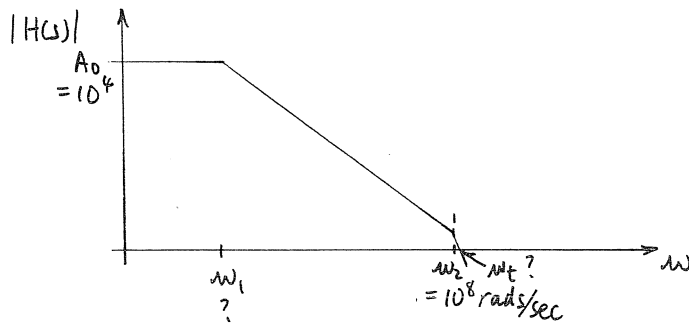
From Section 5.2, lead compensation is achieved by placing a zero at 1.2 times the unity loop gain frequency, ω_t .

$$\therefore \omega_z = \frac{1}{R_2C_2} \equiv 1.2 \times \omega_t$$

$$\therefore C_2 = \frac{1}{1.2 \times 0.7 \omega_{p2} \times R_2}$$

$$= \underline{\underline{63 \text{ fF}}}$$

5.12) Find ω_1 and ω_t .



$$\therefore \omega_z = \omega_t$$

$$\therefore H(s) = \frac{A_0(1 + s/\omega_t)}{(1 + s/\omega_1)(1 + s/\omega_2)}$$

$$\text{and } \angle H(j\omega_t) = 180^\circ - \text{PM} = -100^\circ = +45^\circ - \tan^{-1}\left(\frac{\omega_t}{\omega_1}\right) - \tan^{-1}\left(\frac{\omega_t}{\omega_2}\right)$$

$$\therefore A_0 \gg 1$$

$$\therefore \omega_t \gg \omega_1 \quad \text{by the constant gain bandwidth}$$

$$\therefore \tan^{-1}\left(\frac{\omega_t}{\omega_1}\right) \approx 90^\circ \quad \text{product}$$

$$\therefore -100^\circ \approx +45^\circ - 90^\circ - \tan^{-1}\left(\frac{\omega_t}{\omega_2}\right)$$

$$\frac{\omega_t}{\omega_2} \approx \tan(55^\circ)$$

$$\therefore \omega_t \approx 1.43 \omega_2 = 1.43 \times 10^8 \text{ rad/sec}$$

$$|H(j\omega_t)| = A_0 \sqrt{2} \times \frac{1}{\sqrt{1 + \left(\frac{\omega_t}{\omega_1}\right)^2} \times \sqrt{1 + \left(\frac{\omega_t}{\omega_2}\right)^2}} \equiv 1$$

$$\left[1 + \left(\frac{\omega_t}{\omega_1}\right)^2\right] \left[1 + \left(\frac{\omega_t}{\omega_2}\right)^2\right] = 2 A_0^2$$

$$1 + \left(\frac{\omega_t}{\omega_1}\right)^2 = \frac{2 \times 10^8}{1 + 1.43^2}$$

$$\omega_1 = \frac{1.43 \times 10^8 \text{ rad/sec}}{8.1 \times 10^3}$$

$$\omega_1 = 1.8 \times 10^4 \text{ rad/sec}$$

5.13) Given $A(s) \approx \frac{A_0(1+sT_z)}{sT_1(1+sT_2)}$, the closed loop transfer function is given by

$$A_{CL}(s) = \frac{A(s)}{1+A(s)\beta} \approx \frac{A_0(1+sT_z)/(sT_1(1+sT_2))}{1+A_0\beta(1+sT_z)/(sT_1(1+sT_2))}$$

$$= \frac{A_0(1+sT_z)}{sT_1(1+sT_2)+A_0\beta(1+sT_z)}$$

$$\therefore A_{CL}(s) = \frac{A_0(1+sT_z)}{T_1 T_2 [s^2 + s(\frac{1}{T_2} + A_0\beta \frac{T_z}{T_1 T_2}) + \frac{A_0\beta}{T_1 T_2}]}$$

Equating the coefficients of the denominator polynomial to the standard second-order pole polynomial,

$$s^2 + \frac{\omega_0}{Q}s + \omega_0^2,$$

we find that

$$\frac{\omega_0}{Q} = \frac{1}{T_2} + A_0\beta \frac{T_z}{T_1 T_2}, \quad \omega_0^2 = \frac{A_0\beta}{T_1 T_2}$$

$$\therefore \omega_0 = \sqrt{\frac{A_0\beta}{T_1 T_2}}$$

$$\therefore Q = \frac{\omega_0}{\frac{1}{T_2} + A_0\beta \frac{T_z}{T_1 T_2}}$$

$$= \frac{\omega_0 T_2}{1 + A_0\beta T_z/T_1}$$

since T_z is roughly T_1/A_0 we cannot make any approximations here.

5.14) Given $R_c = 0$, Equations (5.66) ~ (5.68) describe the denominator polynomial, $D(s)$, as

$$D(s) = 1 + sa + s^2b \quad \text{where}$$

$$a = (C_2 + C_c)R_2 + (C_1 + C_c)R_1 + g_{m7}R_1R_2C_c$$

$$b = R_1R_2(C_1C_2 + C_1C_c + C_2C_c)$$

When $R_c \neq 0$, the admittance sC_c becomes $\frac{sC_c}{1+sR_cC_c}$. Thus, we can obtain the new denominator polynomial, $D'(s)$, by simply substituting C_c with $\frac{C_c}{1+sR_cC_c}$.

$$\begin{aligned} \therefore a' &= (C_2 + \frac{C_c}{1+sR_cC_c})R_2 + (C_1 + \frac{C_c}{1+sR_cC_c})R_1 + g_{m7}R_1R_2\frac{C_c}{1+sR_cC_c} \\ &= \frac{1}{1+sR_cC_c} [(C_2(1+sR_cC_c) + C_c)R_2 + (C_1(1+sR_cC_c) + C_c)R_1 \\ &\quad + g_{m7}R_1R_2C_c] \end{aligned}$$

$$a' = \frac{1}{1+sR_cC_c} [a + s(R_2C_2R_cC_c + R_1C_1R_cC_c)]$$

$$b' = R_1R_2 [C_1C_2 + C_1\frac{C_c}{1+sR_cC_c} + C_2\frac{C_c}{1+sR_cC_c}]$$

$$b' = \frac{1}{1+sR_cC_c} [b + sR_1C_1R_2C_2R_cC_c]$$

$$\therefore D'(s) = 1 + sa' + s^2b' = \frac{1}{1+sR_cC_c} [(1+sR_cC_c) + s(a + s(R_2C_2R_cC_c + R_1C_1R_cC_c)) + s^2(b + sR_1C_1R_2C_2R_cC_c)]$$

$\underbrace{\hspace{15em}}_{\triangleq D''(s)}$

this term is moved to the numerator \rightarrow

The poles of the system are determined by the roots of $D''(s)$.

$$D''(s) = 1 + s(a + R_cC_c) + s^2(b + R_2C_2R_cC_c + R_1C_1R_cC_c) + s^3R_1C_1R_2C_2R_cC_c$$

$$\therefore R_c \ll R_1 \text{ or } R_2$$

\therefore any time constant with R_c is much less than time constants with R_1 or R_2

$$\therefore a \gg R_cC_c \text{ and } b \gg R_2C_2R_cC_c + R_1C_1R_cC_c$$

(cont.)

5.14 (cont.)

$$\therefore D''(s) \approx 1 + sa + s^2b + s^3 R_1 C_1 R_2 C_2 R_C C_C$$

\therefore system poles are spaced far apart with $\omega_{p1} \ll \omega_{p2} \ll \omega_{p3}$

$$\therefore (1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})(1 + \frac{s}{\omega_{p3}}) \approx 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{s^3}{\omega_{p1}\omega_{p2}\omega_{p3}}$$

Equating these coefficients, we have the original

$$\omega_{p1} \approx 1/a$$

Approximations,

$$\omega_{p2} \approx a/b$$

\therefore Equations (5.70) and (5.71) still hold true.

Q.E.D.

5.15) Find R_B and V_{eff} at 70°C .

From Eq. (5.108)

$$R_B = \frac{1}{g_{m13}} = \frac{1}{(\mu_n C_{ox} W/L V_{eff})} = \frac{1}{(92 \times 10^{-6} \times 10 / 1.2 \times 0.25)}$$

$$\underline{R_B = 5.2 \text{ k}\Omega}$$

To determine the effect of temperature on V_{eff} , note that

$$\mu_n \propto T^{-3/2} \text{ (see pg 250)}$$

$$\text{Also } R_B \propto \frac{1}{\mu_n V_{eff}} \propto \frac{1}{T^{-3/2} V_{eff}} \text{ and } R_B \text{ is a constant} \text{ assuming}$$

$$\therefore T_1^{-3/2} V_{eff1} = T_2^{-3/2} V_{eff2}$$

$$\text{Let } T_1 = 300\text{K} \text{ (} 27^\circ\text{C)}, T_2 = 343\text{K} \text{ (} 70^\circ\text{C)}, V_{eff1} = 0.25\text{V}$$

$$\therefore V_{eff2} = 0.25 \times \left(\frac{300}{343}\right)^{-3/2} = \underline{0.31\text{V}}$$

Results from HSPICE:

$$\text{at } 27^\circ\text{C} : V_{eff} = V_{osat} = \underline{0.26\text{V}}$$

$$\text{at } 70^\circ\text{C} : V_{eff} = V_{osat} = \underline{0.31\text{V}}$$

\therefore Results are consistent with our calculations.