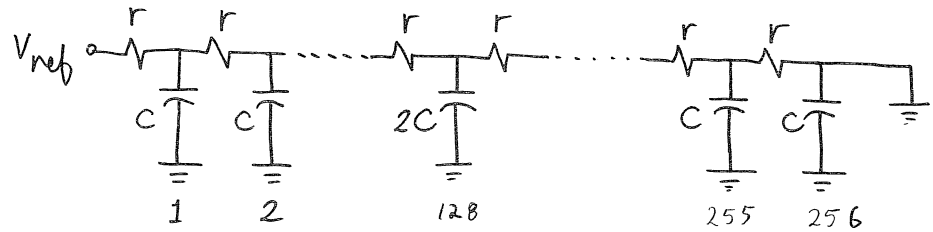
Chapter 12 – Problems

12.1) The total number of switches is $\sum_{i=1}^{N} 2^i = 2(2^N - 1)$

12.2)



In the equivalent circuit shown, $r = \frac{400}{256}\,\Omega$, $C = 0.1\,pF$.

2C represents the capacitance of the only switch that is ON.

This switch is considered to be in the middle of the string for the worst case time constant.

$$\tau = \sum_{i=1}^{255} \left[(ir) \| (256-i)r\right] C + C(128r // 128r)$$

$$= rC \sum_{i=1}^{255} \frac{i(256-i)}{256} + 64rC$$

$$= rC \sum_{i=1}^{255} i - \frac{rC}{256} \sum_{i=1}^{255} i^2 + 64rC$$

$$= rC\frac{(255)(256)}{2} - \frac{rC}{256}\,\frac{(255)(256)(510+1)}{6} + 64rC = 10987\,rC$$

$$= 10987 \times \frac{400}{256} \times 0.1\,p = 1.7\ ns$$

The settling time to 0.1% is $7\tau = 12.\ ns$

12.3) The total number of switches is $2^{N/2} \cdot 2^{N/2} + 2^{N/2} = \underline{2^N + 2^{N/2}}$

12.4) For the output opamp, the offset that can be tolerated is:

$$\frac{1}{2}\,\frac{0.1}{100} \times V_{ref} = \frac{1}{2}\,\frac{1}{1000} \times 5V = \underline{2.5\ mV}$$

For the two opamp in the middle, the offset must be less than

$$\frac{1}{2} \times 2.5\ mV \times 64 = \underline{80\ mV}$$

Note that the offset introduced from the middle opamps will be divided by 64 ($=2^6$). The "$\frac{1}{2}$" factor accounts for two opamps in the middle.

12.5) The ratio between the largest and the smallest resistor is

$$\frac{2^{10} R}{2R} = 2^9 = \underline{512}$$

the current ratio is the same as above.

12.6) The matching accuracy required for the b_2 resistor, b_3 resistor, and b_4 resistor is 2 times, 4 times, and 8 times the matching accuracy of b_1 resistor, respectively.

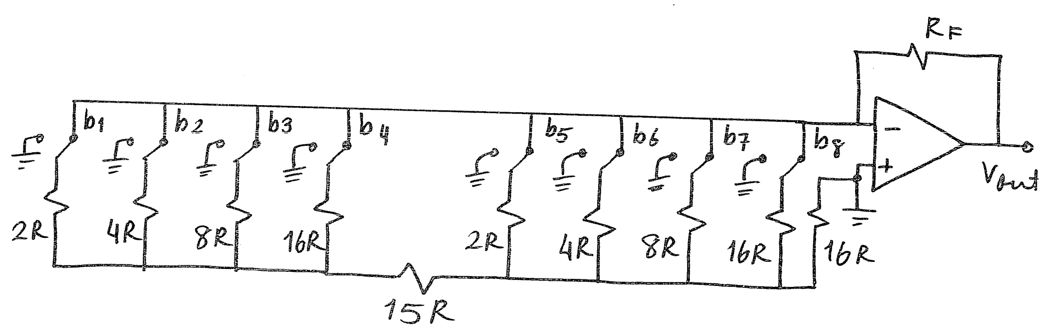
12.7) The worst case DNL happens at the transition from nominal 7LSB to 8LSB, assuming −0.5 % error for C, 2C, and 4C, and +0.5% error for 8C.

$$DNL = 8LSB\,(1.05) - 7LSB(0.95) - LSB = 0.75\ LSB$$

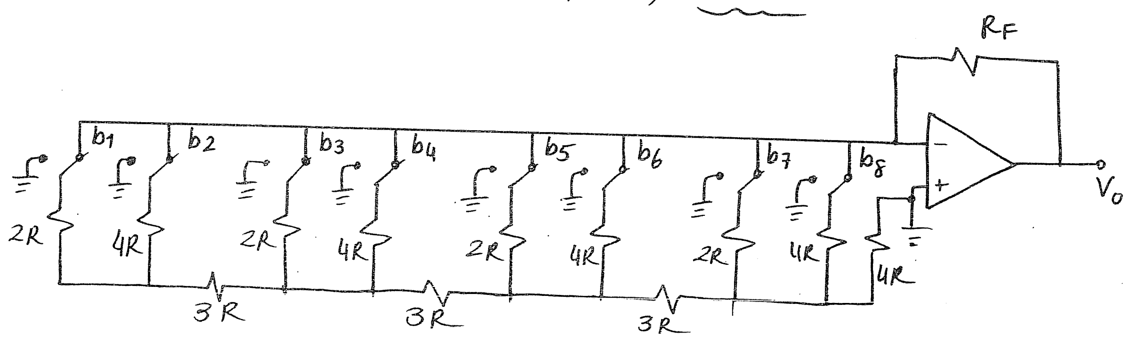
12.8) The largest resistance, in this case, is $2^{N/2}R$ while the smallest resistance is 2R! Therefore, the resistance ratio is $2^{(N/2-1)}$. Noting the resistance ratio for a binary-scaled A/D is 2^{N-1}, we have:

Resistance ratio improvement $= 2^{N/2}$ times

12.9)

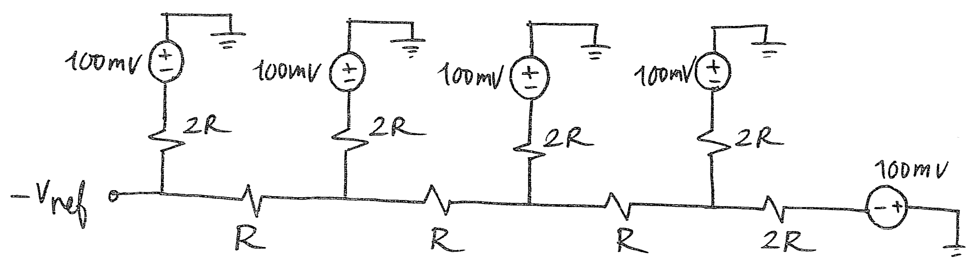


Resistance Ratio = 16R/(2R) = 8

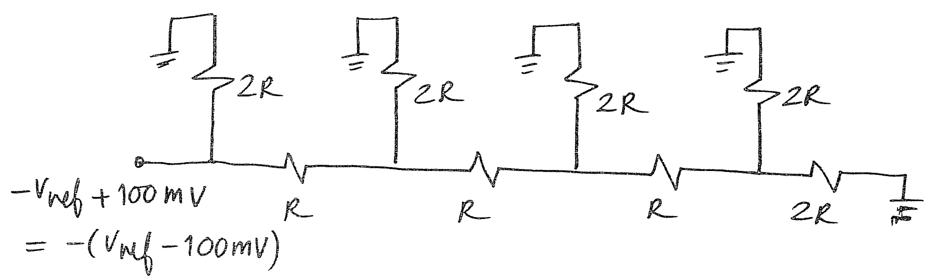


Resistance Ratio = 4R/(2R) = 2

12.10) The equivalent circuit for current calculation in 2R-branches is shown below:



The 100-mV voltage sources model the voltage drop of the switches. One end of 2R-resistor is connected to −100 mV. Therefore, they can all be connected together and the 100 mV voltage-source can be moved and added to the V_{ref} source. This will not change the current calculations of 2R-resistors.



∴ In effect, V_{ref} has decreased by 100 mV and the circuit still operates as if there is no voltage drop across the switches.

12.11) If $R_A = 2.01\, R_B$, the output error is:

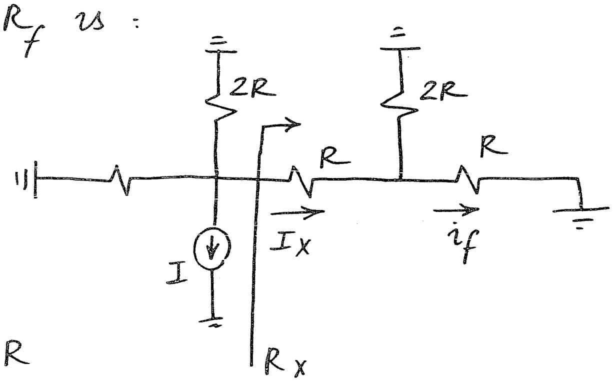
$$\left(\frac{2}{2.01} - 1\right) \times 16\ \text{LSB} \simeq \underline{0.08\ \text{LSB}}$$

If $R_C = 2.01\, R$, the output error is:

$$\left(\frac{2}{2.01} - 1\right) \times 1\ \text{LSB} = \underline{0.005\ \text{LSB}}$$

It's obvious that precision for R_A is more important than the precision for R_G.

12.12) Assuming only $b_1 = 1$, the current drawn through R_f is I.
" " $b_2 = 1$, " " " " " " $I/2$.
" " $b_3 = 1$, the equivalent circuit to find the current through R_f is:



$$R_X = R + \frac{2R}{3} = \frac{5}{3} R$$

$$I_X = I \frac{R}{R + \frac{5}{3}R} = \frac{3}{8} I$$

$$i_f = \frac{2R}{2R+R} I_X = \underline{\frac{1}{4} I}$$

With similar analysis, it can be shown that the current through b_4 & R_f is $\frac{I}{8}$. Therefore,

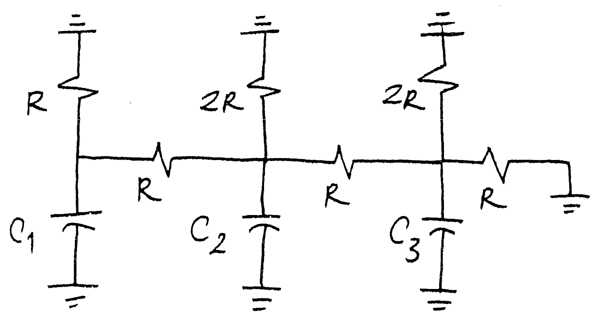
$$\underline{V_o = 2R_f I \left(2^{-1} b_1 + 2^{-2} b_2 + 2^{-3} b_3 + 2^{-4} b_4\right)}$$

(cont.)

12.12) (cont.) The equivalent circuit for the open circuit time-constant analysis is shown:

where

$R = 10\,K\Omega$, $C = 0.5\,pF$



$$\tau = C_3 \frac{R}{2} + C_2 \frac{5R}{8} + C_1 \frac{21R}{32} = 1.78125\,RC = 8.9\text{ nsec}$$

$$\Rightarrow \quad \underline{\omega_{3dB} \simeq \frac{1}{\tau} = 2\pi \times 17.9\text{ MHz}}.$$

12.13) Assuming $R_f = 2\,K\Omega \Rightarrow 8\,LSB = 2V \Rightarrow 1\,LSB = 0.25\,V$

For "0000" input, $V_o = (0 + 0.15)\,LSB = \underline{0.0375\ V}$

For "1000" input, $V_o = (8 + 0.15 + \frac{0.2}{2})\,LSB = \underline{2.0625\ V}$

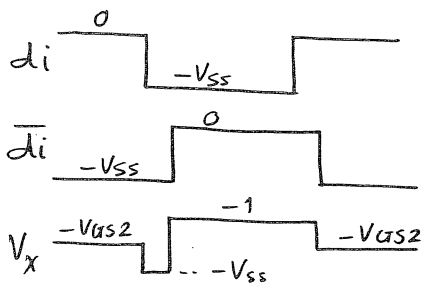
For "1111" input, $V_o = (15 + 0.15 + 0.2)\,LSB = \underline{3.8375\ V}$

12.14) $\frac{0.5}{2^{(4-1)}} = \frac{0.5}{8} = 62.5\ mV$

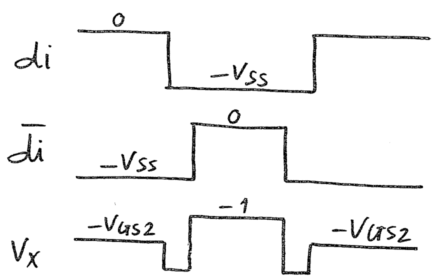
12.15) Let's denote the voltage of the node connecting Q_1, Q_2, & Q_3 by V_X. The following table shows V_X as a function of di & $\overline{di}$.

di	$\overline{di}$	V_X
0	0	-1
0	$-V_{SS}$	$-V_{GS2}$
$-V_{SS}$	0	-1
$-V_{SS}$	$-V_{SS}$	$-V_{SS}$

The V_X waveforms:

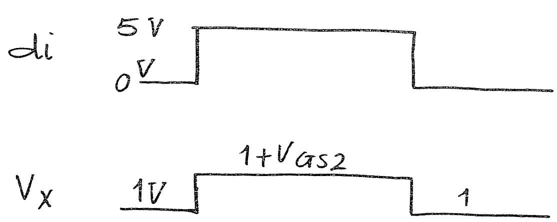


overlapping di & $\overline{di}$



non-overlapping di & $\overline{di}$

12.16) Let's denote the voltage of the node connecting Q_1, Q_2, and Q_3 by V_X.



12.17) Using (1.67) with $I_D = 0.1\, I_{ref} = 5\,\mu A$, we have

$$5\mu = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_t)^2 = (46\mu) \frac{W}{L} (3-1)^2$$

$$\Rightarrow \frac{W}{L} = 0.027$$

$$\Delta I_D = g_m \, \Delta V_{GS}$$

where $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t) = 96\mu \times 0.027 \times 2 = 5 \frac{\mu A}{V}$

& $\Delta V_{GS} = 1\,mV$

$$\Rightarrow \Delta I_D = 5 \frac{\mu A}{V} \times 1\,mV = 5\,nA$$

12.18) $50\,\mu = (46\,\mu) \frac{W}{L} (3-1)^2 \Rightarrow \frac{W}{L} = 0.27$

$$g_m = \frac{2 I_D}{V_{GS} - V_t} = \frac{100\,\mu}{2V} = 50 \frac{\mu A}{V}$$

$$\Rightarrow \Delta I_D = g_m \Delta V_{GS} = 50 \frac{\mu A}{V} \times 1\,mV = 50\,nA$$