

# Timing Recovery

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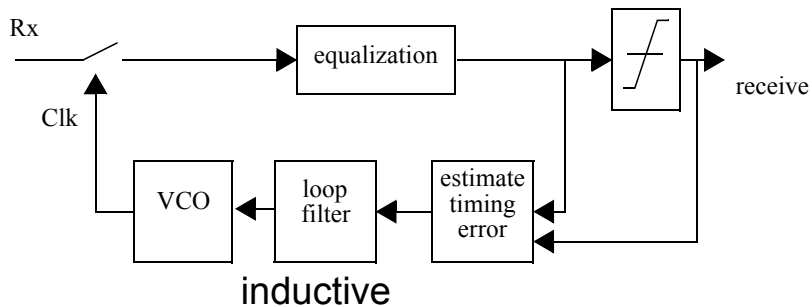
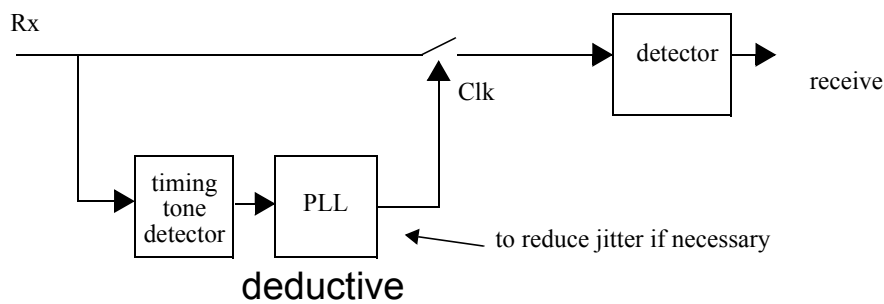


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## Timing Recovery (two types)



- Timing more difficult with less excess bandwidth.



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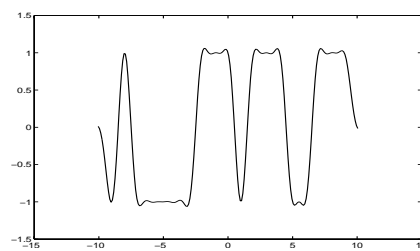
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## Deductive Timing Recovery

- Non-linear spectral line method most popular (linear spectral line method used if  $f_s$  tone present).
- Apply a non-linearity to receive signal and bandpass filter to recover  $f_s$  tone (usually with PLL).
- Works because receive signal is cyclostationary (i.e. its moments vary in time and are periodic).
- Common non-linearities used are squaring and absolute circuit (rectifier) (for low excess BW)
- **Ensemble average** of non-linear circuit output is periodic in T
- Thus, a  $f_s$  component exists (scrambled data)

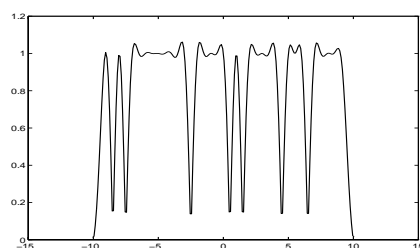


## Example (100% excess BW)

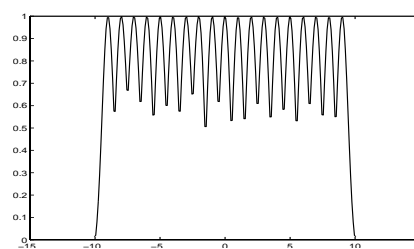


receive signal

average NOT in time but  
over transmit sequences  
(100 sequences in this case)



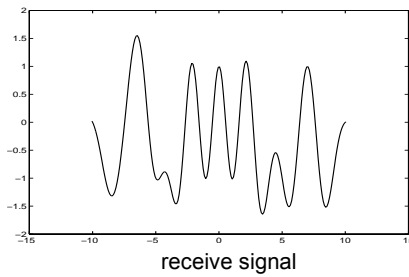
abs(receive signal)



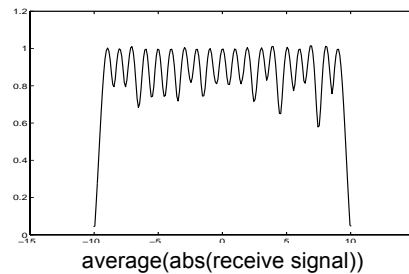
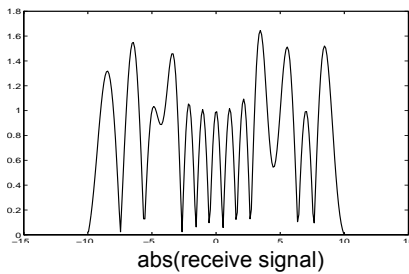
average(abs(receive signal))



## Example (20% excess BW)

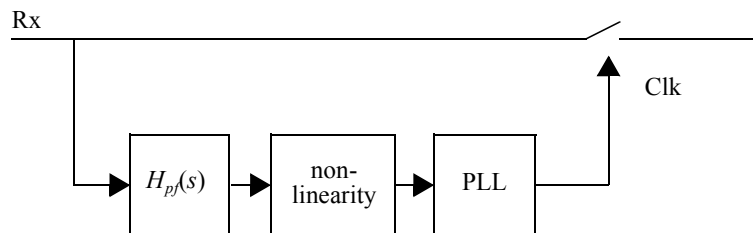
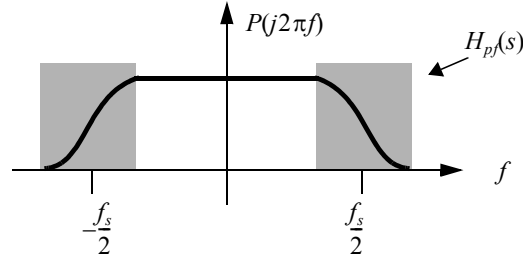


average NOT in time but  
over transmit sequences  
(100 sequences in this case)



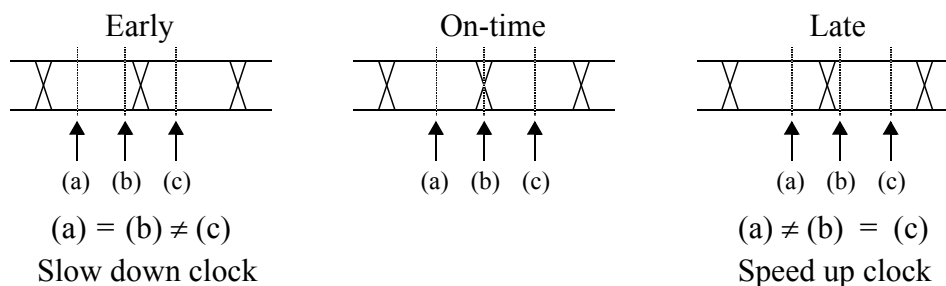
## Deductive Timing

- Can pre-filter receive signal to only non-flat portion to reduce jitter — eliminate portion that does not contribute to timing tone.



## Inductive Timing — Early Late

- Can sample at 2X and determine if clock is early or late when a transition occurs.



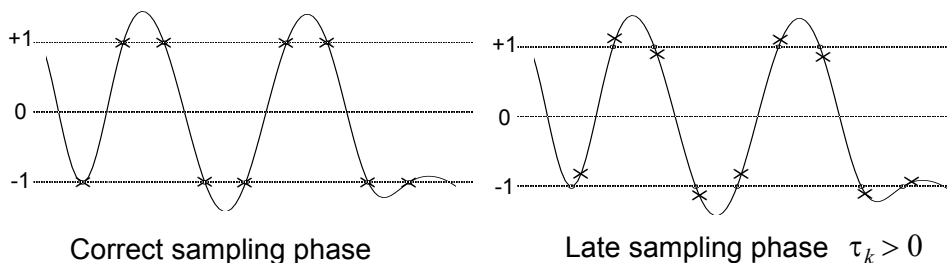
- If (a) = (b) = (c), do nothing
- However, (b) sample does not indicate how far away from zero crossing — can add dither to (b) to aid estimate.



## Inductive Timing (MMSE)

- Commonly realized as minimum mean-square error (i.e. MMSE timing)
- Also called LMS timing.
- Assume sample times are  $kT + \tau_k$

$$A_k = \pm 1$$



## Inductive Timing (MMSE)

- MMSE adjusts  $\tau_k$  to minimize

$$E[E_k^2(\tau_k)] = E[(Q_k(\tau_k) - A_k)^2] \quad (1)$$

where  $E[\bullet]$  denotes expectation,  $Q_k(\tau_k)$  is the sampled signal (it is a function of  $\tau_k$ ) and  $A_k$  is the ideal symbol.

- Stochastic gradient (as in LMS algorithm) leads to

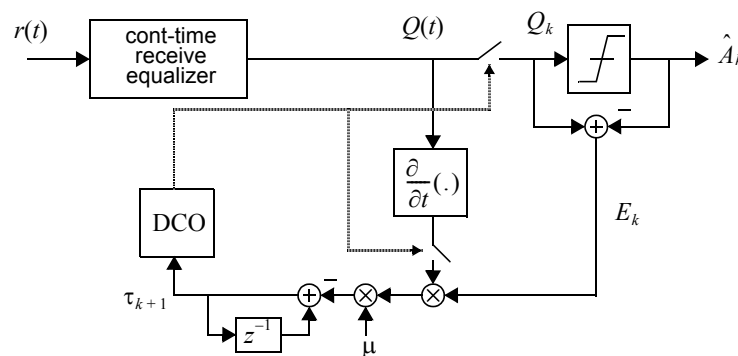
$$\tau_{k+1} = \tau_k - \mu \left( E_k(\tau_k) \times \frac{\partial Q_k(\tau_k)}{\partial \tau_k} \right) \quad (2)$$



## Inductive Timing (MMSE)

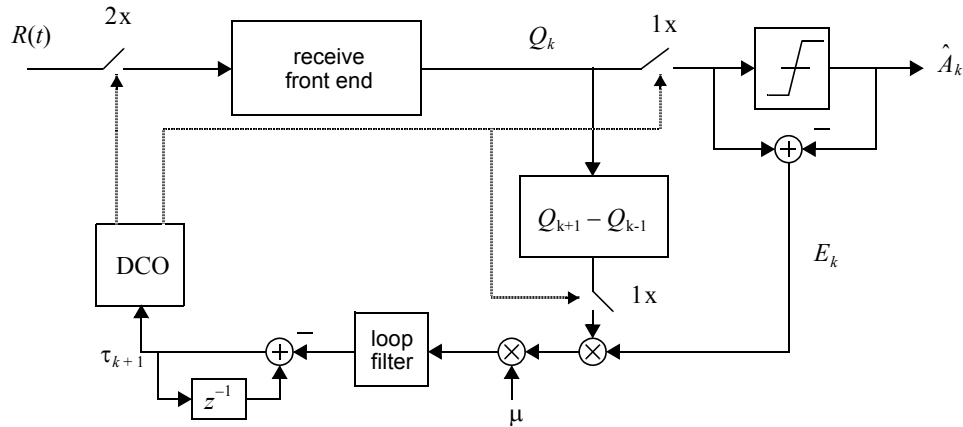
- Can replace derivative wrt  $\tau_k$  by derivative wrt time since sampled at  $t = kT + \tau_k$

$$\frac{\partial Q_k(\tau_k)}{\partial \tau_k} = \frac{\partial Q(t)}{\partial t} \Big|_{t = kT + \tau_k} \quad (3)$$



## Inductive Timing (MMSE)

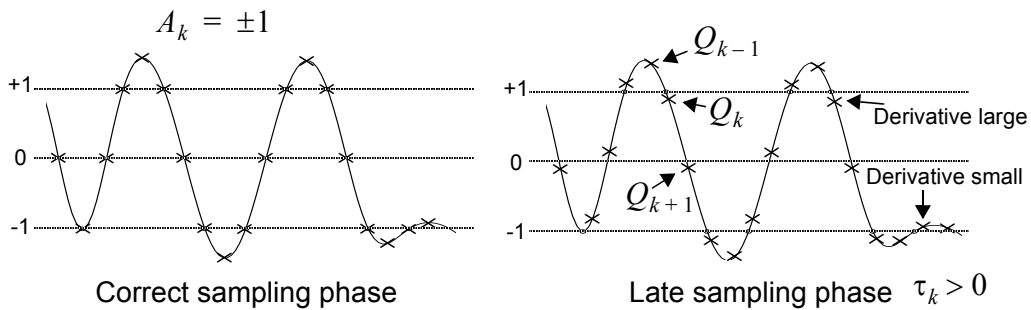
- Can sample at 2X symbol-rate and perform derivative in discrete-time.



## 2X Timing Example

- Sample at twice symbol-rate

$$\tau_{k+1} = \tau_k - \mu(Q_k - \hat{A}_k) \times (Q_{k+1} - Q_{k-1}) \quad (4)$$

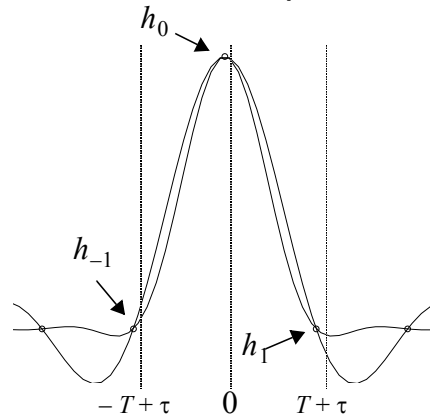


- At  $Q_k$ , slope is neg,  $E_k$  is neg, so  $\tau_k$  is decreased.
- Use absolute values then 50% duty cycle not needed



## Inductive Timing — Baud-Rate

- If all sampling done at symbol-rate, MMSE timing can still be used — base it on impulse response.



- Early-late — adjust so  $h_1 - h_{-1} = 0$
- Zero-crossing — adjust so  $h_1 = 0$



## Inductive Timing — Baud-Rate

- To obtain impulse response estimates, cross correlate received signals with received symbols.
- Recall

$$Q(t) = \sum_m A_m h(t - mT) + n(t) \quad (5)$$

- Sampled at time  $kT + \tau$ , we have

$$\begin{aligned} Q_k &\equiv Q(kT + \tau) \\ &= \dots + A_{k-1} h(kT + \tau - (k-1)T) + A_k h(kT + \tau - kT) + \dots \\ &= \dots + A_{k-1} h_1(\tau) + A_k h_0(\tau) + A_{k+1} h_{-1}(\tau) + \dots \end{aligned} \quad (6)$$

where  $h_k(\tau) \equiv h(kT + \tau)$



## Inductive Timing — Baud-Rate

- To estimate  $h_1(\tau)$ , use  $Q_k \times A_{k-1}$
- **All other terms go to zero since  $A_{k-1}$  is uncorrelated with  $A_j$  when  $k \neq j$**
- To estimate  $h_{-1}(\tau)$ , we need to use a delayed version of  $Q_k$

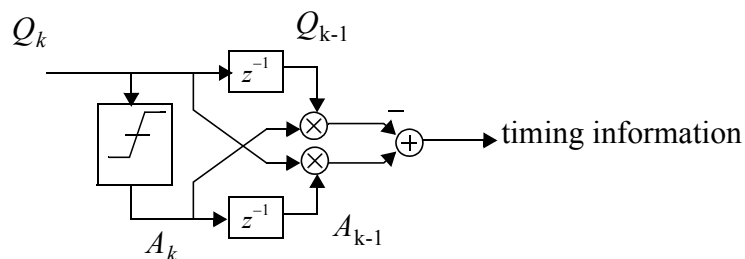
$$Q_{k-1} = \dots + A_{k-1}h_0(\tau) + A_k h_{-1}(\tau) + A_{k+1}h_{-2}(\tau) + \dots \quad (7)$$

- To estimate  $h_{-1}(\tau)$ , use  $Q_{k-1} \times A_k$



## Inductive Timing — Baud-Rate

- To build early-late scheme,

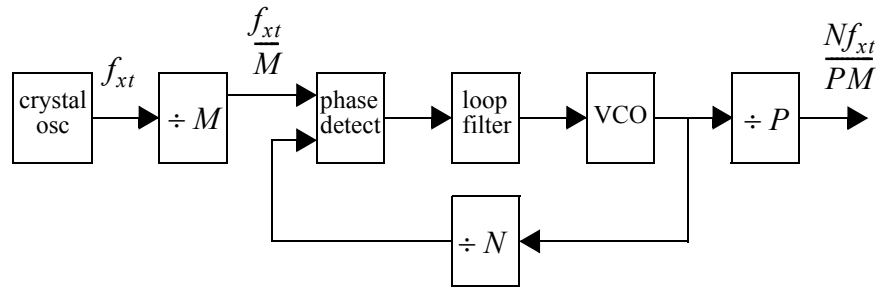


- Early-late is insensitive to amplitude distortion.
- Zero-crossing is better where phase distortion dominates
- $h_0$  factor should be known otherwise adaptation gain will vary (can divide it out in algorithm).



## A Fractional-N Frequency Synthesizer

- Often need a low jitter clock that can have arbitrary frequency.
- A voltage-controlled crystal oscillator is expensive.
- Use oversampling within a PLL



$$N = \{k-1, k, k+1\}$$

A digital controlled oscillator



## Elastic Buffer

- Used to deal with low frequency input clock jitter
- Allows attenuation of clock jitter to next stage

### Example

- Input clock rate — 1MHz but varies from 0.9MHz to 1.1MHz in sinusoidal fashion at 1kHz
- Output clock rate — fixed at 1MHz
- Input clock high — 16 extra bits stored in buffer
- Input clock low — 16 bits removed from buffer
- Keep elastic buffer half-full on-average through feedback

