A 3rd-Order Integrated Passive Switched-Capacitor Filter Obtained with A Continuous-Time Design Approach

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Abstract—A 3rd-order passive switched-capacitor low-pass filter is presented together with experimental results. The current input - voltage output filter structure realizes complex-conjugate poles although it is composed of switches and capacitors. The results are verified with measurements performed on the filter prototype integrated in a 0.13 µm CMOS technology. The prototype has a cut-off frequency of 470 kHz, 150 µW power consumption from 1.2 V power supply, 92 dB SFDR, and an active area of 0.06 mm². The switch-capacitor filter was obtained using a continuous-time model that is also described here and is useful for design, analysis, and simulation of oversampled switched-capacitor circuits. The model is applicable to a variety of topologies including multi-phase passive switched-capacitor filters, switched-capacitor integrators, as well as switched-capacitor DC/DC converters.

Index Terms—Discrete-time systems, continuous-time design, continuous-time modeling, low-pass filter, passive switched capacitor circuits, switched capacitor circuits.

I. INTRODUCTION

As integrated circuit technology advances towards shorter length transistors, analog designers need to reconsider conventional implementations. Advanced transistors require lower power supply voltages and make the design of high-gain high-bandwidth amplifiers more difficult. However, switch and capacitor based circuits work well in modern technologies due to lower parasitic components and fast clocking circuits. In terms of filter design, these trends favor passive switched-capacitor (PSC) architectures, where the charge transfer between capacitors do not require the presence of an active element [1, 2]. The elimination of the active element leads to high linearity and low noise filter designs with accurate corner frequencies that are set by capacitor ratios. In addition, PSC circuits are easily configurable either by using a capacitor bank or changing the sampling frequency.

In the recent literature, PSC topologies were used as an anti-aliasing integration sampler [3, 4], a channel-selection filter for receivers [5-11], a decimation filter before an ADC [12], and a charge-based DAC for transmitters [13]. The anti-aliasing integration sampler can only realize first-order filtering [3, 4]. However, this sampler shows that the continuous input current integration on the filter capacitor prior to sampling results in a first-order build-in anti-aliasing filtering, which is a property that is also inherited by subsequent higher-order filters [4]. Afterwards, higher-order filtering was achieved using a multi-phase PSC structure for channel-selection filters but had only real poles which resulted in poor filter selectivity [5]. Higher-order real-poled PSC filters were utilized to improve the filter selectivity partially [6]. By adding active feedback to the multi-phase PSC filter, complex conjugate poles were realized, but at the expense of degraded noise, linearity, and power performance [14]. A simplified continuous-time (CT) model enabled easy analysis and design of multi-phase PSC filters with grounded capacitors [9]. This modeling approach replaces switches and sampling capacitors with ideal voltage buffers and equivalent resistors leading to great simplifications in the analysis and design. As a result, a 3rd-order low-pass PSC filter with complex conjugate poles was implemented passively using only switches and capacitors [10]. This passive feedback implementation restores the sharper filtering with no additional noise, linearity, and power penalty due to active devices.

This paper extends the work in [9, 10] and presents a continuous-time model and design approach for oversampled switched-capacitor topologies by focusing on PSC structures. In contrast with [9], the model proposed here is applicable to the circuits regardless whether the capacitors are grounded or both plates are switching. The accuracy of the model is explored, and limitations are derived. The design methodology of a 3rd-order filter prototype is investigated in detail while showing measurement results that closely agree with the prediction. The paper is organized as follows: Section II describes the continuous-time modeling approach. Section III gives modeling examples for non-grounded capacitor circuits. Section IV covers the limitations of the continuous-time model. Section V describes the filter design using the continuous-time design approach and gives the implementation details, and Section VI provides measurements verifying the continuous-time model.

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address multi-phase switched-capacitor structures, where a simple equivalent resistor approach does not work. Moreover, this model is capable of addressing non-reciprocal topologies by employing branches with ideal voltage buffers, which can create unilateral paths [9]. (For completeness, the derivation of buffer + equivalent resistor model for voltage sources with input frequencies much smaller than the sampling frequency is covered in Appendix B.)

The modeling approach with buffers and equivalent resistors can address multi-phase switched-capacitor structures; however, it relies on a grounded sampling capacitor. There are a variety of switched-capacitor implementations that employ sampling capacitors with both plates switching in between different nodes. One common example is the parasitic insensitive switched-capacitor integrator shown in Fig. 2a. Conventionally, the switching parts are modeled using a &R as shown in Fig. 2b [15, pp. 417]. This approach can capture the correct positive voltage gain value. However, -R indicates that for positive V1 values, the current is flowing into V1 from the virtual ground, which is not the case. Moreover, if this continuous-time approach is used to analyze the effect of finite amplifier gain or offset voltage, the analysis may result in false or inaccurate values together with a fictitious right-half plane pole at +1/(1+ARC) rad/s, where A is the finite amplifier gain defined by \( v_{out}/(v_1-v_2) \). Apart from the parasitic insensitive switched-capacitor integrator, DC/DC converters with flying capacitors and even PSC filters can include sampling capacitors with non-grounded switching plates.

### A. Floating Switched-Capacitor Continuous-Time Model

A two-phase switched-capacitor topology where both plates are non-grounded is shown in Fig. 3a. The average current flows into V2 in one clock period can be written as \( (V_1-V_2)/f_S C_S \). This current can be modeled using a voltage controlled voltage source (VCVS) whose value is equal to V1, connected in series with an equivalent resistance of R, whose value is 1/f_S C_S. Another branch of VCVS and R is necessary to model the average current flowing into V1 node to complete the model as shown in Fig. 3b. Although it is not covered, this proposed model can be easily adapted to multi-phase switched-capacitor structures by merely increasing the number of VCVS + R branches. A similar model for switched-capacitor structures

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**Fig. 1.** Basic two-phase switched-capacitor topology modeled with a) traditional equivalent resistance, and b) two buffer + equivalent resistance branches.

**Fig. 2.** a) Parasitic insensitive switched-capacitor integrator, b) its continuous-time model.

**Fig. 3.** a) Two-phase switched-capacitor topology with non-grounded sampling capacitor and b) proposed continuous-time model.
with two phases was also previously investigated [17]. (The
derivation of the model for voltage sources with input
frequencies much smaller than the sampling frequency is
similar to the proofs given in Appendices A and B.)

III. Modelling Examples

In this section, two examples of switched-capacitor
topologies that cannot be modelled in continuous-time with a
traditional approach will be analyzed with the proposed model
to highlight the effectiveness of the model.

A. Parasitic Insensitive Switched-Capacitor Integrator

The proposed continuous-time model can be used to analyze
the parasitic-insensitive switched-capacitor integrator. Fig. 4
shows the switching parts of the integrator, where \( V_1 \)
corresponds to the input voltage, and \( V_2 \) corresponds to the
virtual ground of the amplifier shown in Fig. 2a. In \( \Phi_1, C_1 \) is
connected in between \( V_1 \) and ground nodes, this sampled
voltage value, \( (V_1-0) \), becomes the value of the VCVS in the
second branch. Whereas during \( \Phi_2, C_1 \) is connected in between
ground and \( V_2 \) nodes, thus the VCVS value in the first branch
becomes \(-V_2\). Since there exist two ground nodes, it is possible
to rearrange this continuous-time model as shown in Fig. 4
bottom side, where VCVSs can be shown by ideal inverting
voltage buffers to have a more intuitive schematic. Fig. 5 shows
the parasitic insensitive integrator where the switching parts are
replaced.

It should be noted that this continuous-time integrator model
can capture the correct sign of the voltage transfer function by
the use of inverting voltage buffers rather than having \(-R\).
Furthermore, this model captures the correct input current
direction. This new model can also be used to analyze the
effects of finite amplifier gain and offset voltages without
resulting in a fictitious right half plane pole.

B. Flying Capacitor DC-DC Converter

The switched-capacitor DC/DC converter structure shown in
Fig. 6a can be analyzed using the differential continuous-time
model to determine the Thévenin equivalent of the switching
parts. This structure involves a flying capacitor, \( C \), and large
capacitors, \( C_m \)'s, for storing the DC value of the output voltage.
The frequency of interest is much smaller than the sampling
frequency [18]. During \( \Phi_1, C \) is connected in between \( V_{DD} \) and
\( V_{out} \) nodes, sampling a voltage of \( V_{DD-V_{out}} \). Whereas, during
\( \Phi_2, C \) is connected in between \( V_{out} \) and the ground node
sampling a voltage of \( V_{DD-0} \) across itself. The top left schematic
in Fig. 6b shows analyses of this circuit using the differential
switched-capacitor continuous-time model, where \( R=1/f_s C \). It
is possible to simplify further the model as shown in Fig. 6b
and obtain the Thévenin equivalent circuit shown in Fig. 6c. In
order to find \( V_{out} \) from \( 2V_{out} \), the open circuit voltage, \( V_{DD} \), and
the output resistance, \( R/2 \), are divided by two. The conventional
approach is to use a DC transformer model as shown in Fig. 6d
[18]. The conversion ratio, \( m:n \), and the output resistance,
\( R_{out} \), can be calculated to be exactly the same as Fig. 6c.
However, this approach is not intuitive and the derivation is
cumbersome.
IV. LIMITATIONS OF THE MODEL

The branch-based continuous-time model inherits the assumptions of the traditional two-phase switched-capacitor resistor equivalence: The driving node voltage should dictate the voltage sampled by the sampling capacitor, \( C_s \). However, limited settling time and charge sharing paths may disturb this behavior. Moreover, as previously discussed, the continuous-time model is only valid for input voltage sources with frequencies much smaller than the sampling frequency. In this section, these limitations will be studied to determine the accuracy of the model when the assumptions are not met perfectly.

A. Settling Time

A resistance in series with the sampling capacitor creates a non-zero time constant, which leads to settling error. This series resistance can be due to the switch on resistance or the source resistance. For a conventional switched-capacitor topology shown in Fig. 1, when switches have on resistances of \( R_{SW} \), the equivalent resistance seen can be rewritten as follows:

\[
R = \frac{1}{f_s C_s} \cdot \frac{1 + e^{-\frac{1}{2} t_s R_m C_s}}{1 - e^{-\frac{1}{2} t_s R_m C_s}}
\]

where \( f_s \) is the sampling frequency. As \( R_{SW} \) goes to zero, \( R \) becomes \( 1/f_s C_s \), as expected. For example, if the settling time constant \( t_s R_m C_s \) is set to 15% of the sampling period, \( 1/f_s \), the error in the equivalent resistance becomes 7.40 %, while if the time constant is set to 5% of the sampling period, the accuracy becomes 0.01%.

One way to relax this problem without requiring a higher clock frequency is to increase the sampling frequency of the switched-capacitor by introducing time-interleaved stages [19], as done in the presented prototype described in Section V.

B. Charge Sharing

In the case \( C_s \) is connected to a capacitive driving impedance, there is an issue of charge sharing between two capacitors that leads to an error in the equivalent resistor capacitance. For example, Fig. 7a shows a 1\(^{st}\)-order PSC filter with current input and voltage output, where \( C_S \) is switching between an integrating capacitor, \( C_i \), and a ground node. \( C_i \) capacitor can be treated as a voltage source for \( C_i \) values much larger than \( C_S \), and thus the switching parts can be replaced by an equivalent resistor with a value of \( 1/f_s C_S \) (Fig. 7b). However, when \( C_S \) becomes comparable to the driving impedance, \( C_i \), the charge stored in \( C_S \) is not an accurate representation of the initial voltage on \( C_i \). Moreover, the pole created (proportional to \( C_i/C_S \)) moves toward the higher frequencies, where the model accuracy is limited.

It is possible to investigate the continuous-time model accuracy for this 1\(^{st}\) -order PSC filter. Fig. 8 shows the frequency transfer functions simulated using periodic steady state and periodic AC analysis for a sampling frequency of 160 MHz, \( C_S=200 \text{fF} \), and with a large (40) and a small (4) \( C_i/C_S \) ratio. For \( C_i/C_S=40 \), the error in the 3dB cut-off frequency is 1%, whereas for \( C_i/C_S=4 \), the error becomes 12%. Fig. 9 shows the percentage error in the simulated 3 dB cut-off frequency for changing \( f_{\text{dB}}/f_s \) ratios. The error increases, as \( f_{\text{dB}} \) becomes closer to \( f_s \).

C. Accuracy of the Model in PSC Filters Response

The continuous-time model is valid for bandlimited sources with frequencies much smaller than the sampling frequency. However, it is shown that PSC filter frequency responses obtained using the continuous-time model closely follows the discrete-time analysis up to Nyquist frequency, \( f_s/2 \), as long as the pole frequencies are much smaller than the sampling frequency [9, 10]. This is because when the model accuracy starts to decrease at higher frequencies, the dominant impedance is determined by the large integrating capacitors present in the system. As an example, it is possible to analyze the 1\(^{st}\)-order low-pass PSC filter shown Fig. 7. As \( C_i>>C_S \) and \( f_{\text{dB}}<<f_s \), the model can predict the pass-band gain and the pole frequency with high accuracy. For the input frequencies closer to \( f_s/2 \), i.e., for frequencies higher than the pole frequency, \( C_i \) impedance starts to be the dominant impedance determining the overall frequency response. Thus, although at those frequencies the continuous-time model of the switching parts is not accurate, the continuous-time model of the filter results in the correct transfer function.
The strength of the continuous-time approach is that it offers a new point of view to develop novel topologies. In this section, starting from a cascade of real poles, complex conjugate poles are realized using the continuous-time approach. The continuous-time design approach highlights that switched-capacitors do not only act as resistance but with the help of time-variance, it is possible to create unilateral loops. This observation is important because the unilaterality makes it possible to realize complex conjugate poles. A 3rd-order low-pass PSC filter with complex conjugate poles was designed and fabricated to validate the continuous-time design approach. Design trade-offs and implementation details will be discussed through the section.

A. Design of the PSC Filter

It is known that complex conjugate poles can be generated by closing feedback around a cascade of stages with real poles. In Fig. 10a, the feedback is realized by connecting the output of the cascade to the input through an inverting buffer and an equivalent resistance, R. This structure is also the Nth-order realization of the 2nd-order Butterworth biquad filter reported previously, which has a limited quality factor (Q) of 1/\sqrt{2} [9]. Thanks to the continuous-time approach, it is easy to analyze how the number of branches will affect the pole locations and change the associated quality factors. To determine the pole locations, the transfer function of the filter shown in Fig. 10a can be written as follows:

\[
\frac{v_{out}(s)}{i_{in}} = \frac{R}{(1+sRC_{11})(1+sRC_{12})(1+sRC_{13})...+1} \tag{2}
\]

Fig. 10b shows the locations of the poles generated by a cascade of 2, 3, and 4 elements with the same valued C1’s for each stage. For a loop gain equal to one, which is the maximum achievable with a PSC network, the maximum quality factor is obtained starting from a cascade of coincident real poles, which demands identical C1’s. The generated poles have the same quality factors as the poles of a Butterworth filter but different frequencies (Fig.10b). The shift in the pole frequencies results in in-band peaking in the filter transfer function that increases with the order of the filter.

A 3rd-order filter is chosen to be designed as it can approximate the Butterworth filter behavior with less than 1dB in-band peaking. Fig. 11a shows the 3rd-order filter continuous-time model. And, an example of a 3rd-order implementation is shown in Fig. 11b. In this filter, C3 shares charge with C11, C12, and C13 in \Phi_1, \Phi_2, and \Phi_3, respectively, creating a unilateral signal flow. During the phase change from \Phi_3 to \Phi_1, C3 is flipped to create the negative feedback. Note that Fig. 11a also shows the related phase changes corresponding to each branch.

The pole locations derived from the model can be verified by evaluating the z-domain transfer function of the filter shown in Fig. 11a as follows (assuming equal C1’s for simplicity):

\[
\frac{v_{out}(z)}{Q_{in}(z)} = \frac{1}{Q_{in}(z)} \frac{z^2(1-a^2)}{C_3 (z-a^2) + z^2(1-a^2)} \tag{3}
\]

where \alpha is C/C_1 + C_3 and \Q_{in} is the amount of charge fed into the filter in one sampling period, T_S. Although the network shown in Fig. 11a is periodic with 2T_S, T_S is defined for three phases because the output is sampled in every three phases. The quality factor of the created poles can be found by mapping z-domain poles to s-domain using the bilinear approximation. As C/C_3 ratio increases, Q of the complex conjugate pair approaches 1, which is the Q value predicted by the continuous-time model used for the synthesis of the filter.
Simulation of the proposed filter and the 3rd-order Butterworth response (with the same DC gain and 1 dB droop frequency) are compared in Fig. 11b. Although the pole location deviates from the ones of a Butterworth, the 3rd-order filter maintains similar in-band flatness and selectivity. The figure inset shows that the response of the proposed filter deviates from a Butterworth one less than 1 dB close to the filter band edge. Fig. 11b also compares the proposed solution with the 7th-order all real pole reported by Tohidian et al. in [5] by assuming a maximum in-band drooping of 1 dB. Although the proposed filter has a lower order, a higher selectivity is achieved over a decade close to the filter pass-band due to the presence of the complex conjugate poles.

B. Implementation of the PSC Filter

A fully differential 3rd-order PSC filter was designed and fabricated in 0.13 μm TSMC CMOS process. Fig. 12 shows the PSC filter implementation. The charge inversion of C3 is realized by exploiting the differential structure and cross-coupling of positive and negative nodes. The filter uses six phases: Three non-inverting phases (Φ1, Φ2, and Φ3) when top C3 is connected to the top C1’s sequentially, while the bottom C3 is connected to the C1’s on the bottom differential side. During the following three inverting phases (ΦN, Φ2N, and Φ3N) the C3’s are connected to the C1’s, on the opposite sides.

In order to increase the sampling rate, three time-interleaved blocks, which consists of C3’s and switches, are employed. Thus, the resulting sampling period becomes equal to the period of a single phase (Fig. 12). The z-domain transfer function of the time-interleaved filter can be written as:

\[
\frac{V_{out}(z)}{V_{in}(z)} = \frac{2}{C_3} \frac{z(1-a)^2}{(z-a^2)(1-a^2)}
\]

where \(a = C_1/(C_1+C_3)\), coefficient 2 is due to the differential structure, and \(Q_m\) is the amount of charge fed into the filter in one sampling period. The z-domain transfer function reported in (4) seems considerably different than the previously derived transfer function in (3). The reason is that the sampling period, \(T_s\), in (4) is defined for a single phase due to the time-interleaved structure, whereas in (3), \(T_s\) is defined for three phases. However, once mapped back to s-domain it can be seen that (3) and (4) result in similar filtering characteristics for the same \(T_s\). The quality factor of the complex conjugate pole pair can be calculated as 1.08 by mapping z-domain poles to s-domain using Bilinear approximation (assuming \(C_1/C_3=10\) and all C1’s are equal to each other).

For a sampling frequency of 160 MHz, the filter 3 dB bandwidth was designed to be 470 kHz. C1’s are chosen to be equal for maximum Q. All capacitors, C1’s and C3’s, are used as MIM capacitors with values 12.6 pF and 225 fF, respectively, together with the added parasitic capacitances. Each sampling capacitor, C3, is directly connected to six switches (Fig. 12), and 20% of the C3’s are made of switch parasitic capacitances that are added on top of MIM capacitances. Although each integrating capacitor, C1, is also directly connected to six switches, the effect of switch parasitic capacitances on C1’s is negligible due to larger capacitance sizes. Smaller switches can help lower switch parasitic capacitances; however, there is a minimum limit on switch sizes due to the settling. The switches in this prototype are implemented using transmission gates and sized for 1.5 kΩ maximum on resistances for the operation range. A ring counter is implemented to produce six non-overlapping clock phases using an external clock signal at 160 MHz. A g_m-cell was not included in the design in order not to dominate the noise and linearity responses of the PSC filter. Thus, the noise model can also be verified. In the prototype, the PSC filter is followed by an on-chip open drain output buffer to drive the probe used for the measurements. In order not to affect the noise measurements, the output buffer was designed to have lower noise spectral density compared to the filter.

C. Component Mismatch Effect on the Quality Factor

Component mismatch in between C1’s or C3’s can affect the filter transfer function by changing the filter cut-off frequency and the quality factor of the poles. It is possible to analyze these effects using the continuous-time model. As an example, a single-ended 2nd-order passive switched-capacitor filter can be investigated using the denominator of the filter transfer function given in (2) for \(N=2\) as follows:

\[
D(s) = (1+sRC_1)(1+sRC_2) + 1
\]

By rearranging the terms in (5) and equating it to the well-known 2nd-order biquad filter formula \((s^2 + \omega_0^2/Q + \omega_0 s)\), it is possible to find the corresponding 3 dB cut-off frequency, \(\omega_0\), and quality factor, \(Q\), of the poles as follows [9]:

\[
a_0 = \sqrt{\frac{\omega_0^2}{R_1C_1C_2}}\quad\text{and}\quad Q = \frac{\sqrt{\omega_0^2C_1C_2}}{C_1 + C_2}
\]

From (6) it can be calculated that for example for a 10% mismatch in between C1 (±5%) and C2 (±5%), Q decreases by 0.13 %, which corresponds to a change from 0.707 to 0.706, i.e., the phase of the complex-conjugate poles changes from 45°.
to 44.93°. C₅ mismatch in between the time-interleave stages affects the R value. Thus, it does not affect the quality factor (see (6)).

VI. MEASUREMENT RESULTS

For measurements, a voltage source is fed into the filter through large external resistors (Rₑₓ) in series, similar to what has been done in [20]. The voltage source together with Rₑₓ models the Thévenin equivalent of a gm-cell with a finite output resistance (Fig. 12). The total external resistance is 200kΩ (where the filter input resistance is 27.8kΩ, Rₑₓ=1/ᵢᶠC₅). The filter consumes 125μA from a 1.2V power supply, which is the power consumed by the phase clock generator. The active area of the chip is 0.06 mm² dominated by the integrating capacitors (Fig. 12).

Fig. 13 shows the normalized transfer functions of the prototype measurements, post-layout schematic simulations, and the continuous-time model simulation. A good agreement has been obtained between measurements, simulation, and theory. For comparison, 7ᵗʰ-order all real pole PSC filter simulation response is also added to the figure. The 7ᵗʰ order filter reported in [5] was designed to have the same DC gain and 1 dB droop frequency. It can be observed that the 7ᵗʰ-order filter transfer function has a much smoother roll-off around the cut-off frequency compared to other filter transfer functions, which causes more than 10 dB attenuation loss around the band-edge. Towards 10 MHz, the measured filter response shows a flattening caused only by leakage on the PCB used for testing. External resistors lead to around 18 dB attenuation on the PCB, which worsen the filtering profile by elevating the leakage level. Fig. 14a shows the normalized measured transfer function of the filter for changing sampling frequency, fₛ. It can be seen that the filter passband gain decreases for increasing fₛ values, because the equivalent switched-capacitor resistance, R (=1/ᵢᶠC₅), decreases, which is proportional to the low-frequency gain of the filter as derived in (2). Whereas, the filter cut-off frequency, which is proportional to the 1/RC₁ (=fₛC₅/C₁), increases for increasing fₛ values. Fig. 14b shows the first (Vₒᵤ), the second (Vₒ₂=Vₒ₂), and the third (Vₒ₃) order filtering nodes normalized measured transfer functions together with the continuous-time model simulation results, which match with very good agreement.

Output noise spectral density measurement results are shown in Fig. 15a with a resolution bandwidth of 1 Hz. The noise is measured at the output of the buffer and later referred to the output of the filter, Vₒₜ. At low frequencies, noise is equal to the noise of the Rₑₓ (i.e., 4kT/fₛC₅). The solid black line shows the simulation result of the PSC filter and the dashed line shows the continuous-time model simulation result. The noise spectral density is reported up to 1 MHz due to the comparable noise floor of the spectrum analyzer. However, this measurement can show the in-band noise spectral density as well as the out-of-band decay close to the filter band edge. It can be seen that measured noise spectral density closely follows the simulations and theory. The measured input referred noise is 15.8μV, integrated between 10 kHz and 470 kHz. Out-of-band IIP3 is extrapolated using two blockers at 3.4 MHz and 6.7 MHz creating an intermodulation product at 100 kHz. Fig. 15b shows the input signal power and the third-order intermodulation distortion measurement results referred to the chip input (Vᵢⁿ in Fig. 12). The out-of-band IIP3 extrapolated is 55.1 dBm leading to 92 dB SFDR.

Table I summarizes the measurement results and compares them with the 7ᵗʰ-order PSC filter [5] and also with the recently published 4ᵗʰ order PSC filter that uses active feedback structure.
TABLE I.
SUMMARY RESULTS AND COMPARISON TABLE

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*Integrated over *10 kHz – 470 kHz, **50 kHz – 9 MHz bandwidth.

*** Averaged from 100 kHz to 4.4 MHz

14. The SFDR obtained with this work is more than 20 dB better with a much lower power consumption partially thanks to the presence of complex conjugate poles which allowed to reduce the filter order without compromising the filter selectivity close to the cut-off frequency. However, it should be noted that both other works employ a transconductance at the input, and [14] also uses another transconductance as active feedback, which degrades filter’s linearity and increases power consumption. Although the transconductance on the feedback path is not needed in this work thanks to the passive feedback topology, the input $g_{m}$-cell is only avoided due to testing purposes. Thus, Table I should be reviewed considering these differences.

VII. CONCLUSION

A continuous-time model for oversampled switched-capacitor circuits with non-grounded sampling capacitors was introduced with examples. The limitations of the modelling approach were discussed. A 3<sup>rd</sup>-order passive switched capacitor filter prototype with complex-conjugate poles was obtained using the continuous-time approach. Measurements performed on the filter prototype result in state-of-art performance meanwhile verifying the continuous-time modeling and design approach. With this prototype, for the first time, switched-capacitor complex conjugate poles have been integrated on silicon without the need of any active circuitry. This represents a remarkable result that makes it possible to obtain sharp filtering profiles using PSC filters while showing that the design approach can lead to the invention of novel structures.

APPENDIX A: DERIVING EQUIVALENT SWITCHED-CAPACITOR RESISTANCE

Assume that $V_1$ and $V_2$ nodes in Fig. 1a are connected to two voltage sources with maximum frequencies well below the sampling frequency, $f_s$, and switches are ideal with zero on resistances. Then, the current supplied by $V_1$ can be written as follows:

$$i_1(t) = i_{\text{pulse}}(t) + i_{\text{cap}}(t)$$

where $i_{\text{pulse}}(t)$ is the current pulse occurs at the instant the switch is closed at $\Phi_1$, and $i_{\text{cap}}(t)$ is the current supplied to the capacitor during $\Phi_1$, as the $V_1$ voltage slowly varies. These two components can be written as below:

$$i_{\text{pulse}}(t) = C \left[ v_1(t) - v_2(t) \right] \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$i_{\text{cap}}(t) = C \frac{dv_1(t)}{dt} \sum_{n=-\infty}^{\infty} a_n e^{j2\pi fn t}$$

with $a_n = \begin{cases} 2 \pi \frac{(-1)^{n+1}}{n}, & n \text{ is odd} \\ 0, & n \text{ is even and } n \neq 0 \\ 0.5, & n = 0 \end{cases}$

(8)

where $i_{\text{pulse}}(t)$ is a series of current pulses whose area is equal to $C \Delta V (= C v_1(nT_s)-C v_2(nT_s))$, and $i_{\text{cap}}(t)$ is the capacitor current equation multiplied by the rectangular pulse train, which is represented by its Fourier series. Note that rectangular pulse train has the same phase of $\Phi_1$ with an amplitude of 1.

In order to define an impedance to model the topology as a linear time-invariant (LTI) system, the resulting current in response to the $V_1$ and $V_2$ voltages should be examined. From (7) and (8), it is possible to write the Fourier transform of the produced current as:

$$I_i(\omega) = \frac{C}{T_s} \sum_{k=-\infty}^{\infty} \left[ V_1(j\omega - jk2\pi f_s) - V_2(j\omega - jk2\pi f_s) \right]$$

$$+ j\omega C \sum_{k=-\infty}^{\infty} a_k V_1(j\omega - jk2\pi f_s)$$

(9)

where $a_k$ are the Fourier series coefficients and equal to $a_n$ for $n=k$. It can be observed from (9) that the current produced has a fundamental harmonic and also higher harmonics at the multiples of $f_s$. To model the circuit behavior for low frequencies, only the fundamental harmonic would be of interest [21], which can be written as below:

$$I_{I_{\text{fund}}}(\omega) = f_s C \sum_{k=-\infty}^{\infty} a_k \left[ V_1(j\omega) - V_2(j\omega) \right] + j\omega C \frac{V_1(j\omega)}{2}$$

(10)

It can be seen that $I_{I_{\text{fund}}}$ results in a 1/$f_s$C resistance in between $V_1$ and $V_2$ nodes and a C/2 valued capacitance in between the $V_1$ node and ground. Although C/2 is needed for the exact representation at the fundamental frequency, for
simplicity, it is ignored in this paper, as all the cases that are covered, sampling capacitors are connected to low impedance nodes, where the effect of C/2 becomes negligible. Moreover, it should be noted that 2/\omega C impedance is much greater than 1/\omega C for the frequencies of interest (\pi \omega < \omega_0).

The impedance seen from the V_2 node can be derived similarly by writing the current equation sunk into the V_2, i(t).

In this case, the Fourier Transforms of the current produced can be written as follows:

\[ I_{2fund}(\omega) = e^{j\omega t} \frac{C}{T_S} \sum_{n=-\infty}^{\infty} [V_1(j\omega - nk\pi f_c) - V_2(j\omega - nk\pi f_c)] \]

\[ I_{cap}(\omega) = j\omega C \sum_{n=-\infty}^{\infty} b_n V_2(j\omega - nk\pi f_c) \]

with \( b_n = \begin{cases} \frac{2}{\pi n} (-1)^n, & n \text{ is odd} \\ 0, & n \text{ is even and } n \neq 0 \\ 0.5, & n = 0 \end{cases} \) (11)

whose fundamental component becomes:

\[ I_{2fund}(\omega) = e^{j\omega t} \frac{C}{2} \left[ V_2(j\omega) - V_1(j\omega) \right] + j\omega C \frac{V_2(j\omega)}{2} \] (12)

The exponential term in \( I_{2fund} \) expression is due to the sampling phase shift, and it can be ignored for the frequencies of interest (\pi \omega < \omega_0). Once the exponential term is ignored, it is seen that the first part of \( I_{2fund} \) results in the same 1/\omega C resistance in between V_1 and V_2 nodes, whereas the second part of \( I_{2fund} \) results in a C/2 valued capacitance in between V_2 node to ground. This capacitance can be ignored as well because the impedance of 2/\omega C is much greater than 1/\omega C for the frequencies of interest (\pi \omega < \omega_0). Thus, the voltage and current relationship in between V_1 and V_2 voltage sources can be modeled by an equivalent resistance of 1/\omega C in between those two nodes.

**APPENDIX B: DERIVING MULTI-PHASE EQUIVALENT MODEL**

Assume that V_1, V_2, and V_3 in Fig. 16a are all connected to ideal voltage sources with input frequencies much smaller than the sampling frequency, \( f_S \), and switches are ideal with zero on resistance. Then, the current supplied by V_1 can be written as follows:

\[ i(t) = i_{node}(t) + i_{cap}(t) \]

\[ i_{node}(t) = C \left[ v_1(t) - v_3(t) \right] \sum_{n=-\infty}^{\infty} \delta(t - nT) \]

\[ i_{cap}(t) = C \frac{dv_1(t)}{dt} \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 t} \]

with \( c_n = \begin{cases} 2 \frac{\pi n}{\omega} \sin \left( \frac{\pi n}{3} \right), & n \neq 0 \\ 1, & n = 0 \end{cases} \) (13)

It can be shown that these current equations lead to a resistance in between V_1 and V_3 nodes with a value of 1/\omega C for the frequencies of interest. Moreover, (13) also leads to a C/3 valued capacitance in between the V_1 node and ground (derivation and assumptions are similar to the ones discussed in Appendix A).

In order to complete the model and to replace the switching parts, all current components (i_1, i_2, and i_3) should be considered. It can be shown that, for the frequencies of interest, there is a general current equation valid for i_1, i_2, and i_3 that can be written as follows in the Fourier domain:

\[ I_k(\omega) = f_k C \left[ I_n(\omega) - V_{n-k}(\omega) \right] \] (14)

where \( I_k \) represents the Fourier transform of the current component sourced by V_k voltage source (where k=1,2, and 3), and \( V_{n-k} \) represents the voltage source that C is connected to in the prior phase before connecting to V_k. It is important to note that although there exist three voltage sources, the current component related to each node only depends on two voltage sources that C is connected (one at the relevant phase and the other one is at the phase prior to the relevant phase).

Fig. 16b shows the schematic representation of (14) employing ideal voltage control voltage sources (VCVS) in series with the 1/\omega C valued equivalent resistances. This schematic can be modified to have a more intuitive view as shown in Fig. 16c, where VCVS’s are replaced with ideal voltage buffers.

**REFERENCES**


Together with academic experience, he also has spent a number of years in the semiconductor industry and was a co-founder of a successful IP company called Snowbush Microelectronics.

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