Theoretical Predictions of EM-induced Degradation in Test-Structures and On-Chip Power Grids with Analytical and Numerical Analysis

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Abstract—Paper discusses the state of the art physics-based analytical modeling and numerical analysis techniques developed for the prediction and description of electromigration (EM) induced conductance degradation of individual interconnect metal lines and on-chip power grids. Mechanical stress evolution caused by an electric current driven redistribution of vacancies and plating atoms, which populate the metal grain boundaries (GB) and interfaces, initiates the growth of preexisted crystal imperfections such as micro cavities and interfacial/intergranular delaminating. It is described as a major cause of the failure. A role in the failure development played by the interfacial and GB atomic diffusions and their variation is covered. A close relation between the interfacial-adhesion energy and so-called “critical stress” is clarified. A physics-based statistical formulation of EM phenomenon is discussed.

Different kind of analytical/numerical techniques employed for analysis of EM degradation in different cases characterized by the scales varying from the size of an individual line to the multibillion segment power grids are discussed. Conditions for employment of 1D EM approximation (Korhonen’s equation) are validated by direct comparison with results of 3D FEA simulations. Implementation of the novel compact model- and FDA-based approaches for analyzing EM-induced IR-drop degradation in power nets is demonstrated.

Index Terms—Electromigration, reliability, diffusion, on-chip interconnect, power grid, test-structures.

I. INTRODUCTION

Ever increasing interconnect current density due to continuous reduction in critical dimensions of the advanced technology nodes makes EM assessment as one of the most important design sign-off checks. An employment of FinFET devices in chip design adds additional concerns about the increase in chip electric current density and acceleration of the age-induced failures rates. Excessive Joule heat generation caused by elevated current densities makes the development of these failures even faster. Nevertheless, modern semiconductor chips, while working in the range of elevated current densities and temperatures, should satisfy the scope of strict reliability requirements. One of them is a requirement of 10 year chip immortality, which is the same as for the elder generation chips. To satisfy this requirement the chips should be designed in accordance with a special set of design rules and practices. A well-known conservatism of these rules results in the problems with chip performance. In order to understand the source of this conservatism we should examine the methodology employed for these design rules generation. As we know the major EM problem is a degradation of the circuit electrical characteristics. It can result in both the short and open circuits. Voids generated in the metal can be responsible for the resistance increase, and, in the worst case, for the open circuit. Hillock formation, which is a metal extrusion into an inter-metal isolation, can form shorts between the neighboring wires. A standard EM testing is performed on Cu lines connected to a lower or upper metal levels by vias. The electric current flows from wide metal supply lines through the vias into the test line. The changes in the voltage and resistance taking place with the duration of time are detected. Increase in resistance of the individual line above the threshold value is considered as a failure. Details describing this methodology characterized by a variety of measurement techniques were published in many papers, see for example [1]. Despite the deference in implementations of the testing procedure, the determination of failure is the same: a multilink structure of the conducting wires is failed when any single wire is failed (the series failure mode). While being valid for estimation of the time to failure (MTTF), after a corresponding averaging procedure is performed on a set of the identical wires, this failure mode does not describe well the failure of a large electrical circuit where nodes form a grid-like structure. A redundancy for the current conductance existing in such circuits is responsible for the sustain operability even when some fraction of individual wires were failed. It’s known that the electrical connectivity between opposite sides of an electrical net characterized by a mixture of sequentially or parallel connected current conducting segments will not be broken till a critical fraction of the segments, so-called percolation threshold, will be cutoff. This threshold fraction depends on the net dimension, structure, and size [2]. The on-chip interconnect power-ground
(p/g) grids of the modern semiconductor chips are similar to such nets. The failure criterion for the p/g grid is of course not a complete cutting off the electrical conductance but a development of the voltage drop exceeding the threshold drop specified by a designer. This threshold voltage drop means a critical reduction in the supply voltage (Vdd/Vss) happening somewhere in the layout at the particular gate, which destroys the proper cell functionality.

Hence, the accepted failure criteria depend on a scale of the analyzed problem migrating from the simple threshold increase in the resistance of an individual line (wire) to the threshold increase in the voltage drop in the case of large circuits. It should be stressed out that while the failure criteria are different, the physical mechanism causing the failures is the same. It is an increase in the resistance of individual wires caused by EM induced voiding. Thus, in order to accomplish the EM assessment we should have a capability to calculate the EM induced resistance change of each and every single wire of the p/g grid at any instance in time. Since the dynamics of EM-induced resistance change is caused by a void nucleation and growth, the physics-based modeling technique providing a circuit-wide distribution of the void nucleation times and void growth kinetics should be available. Majority of the EM assessment models described in literature are empirical, derived on the basis of the fit with available experimental data [3]. They are not allowing to take into account many important physical details that affect the metal degradation occurring when electrical stressing is applied. These are the grain size and texture determining the GB atom/vacancy diffusivity, residual stress, etc. effecting the void nucleation and growth.

It is obvious that problems characterized by different scales can afford quite different level of details, which can be included in the predictive modeling. For example, the role played by a particular grain size distribution and a texture in the EM induced stress evolution can be addressed by the finite-element analysis (FEA) performed on the individual interconnect test-structure, [4], and cannot be addressed (with the same level of accuracy) by the one-dimensional (1D) simulation of the stress evolution in the interconnect trees or p/g grids.

A primary target of this paper is demonstrating some of the recently developed models, discussing the ranges of applicability of different modeling approaches, and the ways of implementation of different kind of numerical and analytical techniques for the EM assessment.

The reminder of the paper is organized as follows. In Section II we present some background material regarding EM, a general model of the evolution of vacancy and plated atom concentrations and stress under the action of electric stressing. Employment of the phase-field method for description of void evolution is discussed. Sections III and IV deal with 1D approximation of EM phenomenon and its comparison with 2D/3D models. Section V demonstrates the employment of 1D approach for EM assessment of large p/g grids. Section VI concludes the paper.

II. PHENOMENOLOGICAL MODEL OF EM

A. EM-Induced Atomic Perturbation

As it is agreed, the EM phenomenon is the directional migration of lattice atoms and defects under the action of the electric field and current. This forced redistribution of atoms can generate non-uniformity in chemical composition, and can introduce non-uniform elastic/plastic deformation and stress. Directional migration of atoms in solids can happen under the action of oriented forces only. These forces, which are proportional to the gradient of the potential energy \( F(\vec{r}) = -\nabla U(\vec{r}) \), push atoms to migrate along this gradient with the velocity \( \vec{v} = \mu F(\vec{r}) \). The coefficient proportionality \( \mu \), representing the atom mobility, relates to the atomic diffusivity \( D_a \) through the Einstein relation, [5]: 
\[
\mu = \frac{D_a}{k_b T}
\]
where, \( k_b \) and \( T \) are the Boltzmann constant and absolute temperature. The state of equilibrium is achieved when a balance between the tendency of atoms to migrate toward the lower potential energy \( U(\vec{r}) \) and their tendency to spread out due to the thermal diffusion is established, or, in other word, when the total atomic flux is vanished. Describing the atomic flux \( \Gamma(\vec{r}) \) as a number of atoms crossing the unit surface normal to the direction of the atomic flow per unit time, we can write:
\[
\Gamma(\vec{r}) = \mu F(\vec{r}) N_A(\vec{r}) = -N_A(\vec{r}) \frac{D_a}{k_b T} \nabla U(\vec{r})
\]  
Here, \( N_A(\vec{r}) \) is the coordinate dependent atomic concentration.

Potential energy of a lattice atom depends on a number of physical factors such as the temperature, chemical composition, mechanical stress, electrical field, etc. Variation in \( U(\vec{r}) \) caused by variations of these parameters can initiate the matter redistribution. For example, when an electric field is applied to the metal body (wire, film, etc.), the atomic flux caused by the combined interaction between metal ions and electric field and conduction electrons flows toward the anode, and is described as
\[
\Gamma_{EM}(\vec{r}) = -N_A(\vec{r}) \frac{D_a}{k_b T} e Z^* \rho(\vec{r})
\]  
Here, \( Z^* \) is the, so-called, effective valence, which depends on the metal properties, [6], \( \rho(\vec{r}) \) is the metal resistivity and \( j(\vec{r}) \) is the current density.

When the electric current passes through the metal wire it generates the volumetric strain caused by the atom density redistribution. In the case of a metal line confined by a rigid surrounding this volumetric strain generates nonuniform distribution of the hydrostatic stress along the wire. As a result, an additional atom migration is caused by the action of forces generated by an inhomogeneity of elastic energy accumulated in the lattice. The change in the free energy of the regular lattice atom caused by the presence of the hydrostatic stress \( \sigma_{Hyd} \) is described as, [7]:
\[
\Delta U(\vec{r}) = U_{Hyd}(\vec{r}) = -\Omega \sigma_{Hyd}(\vec{r})
\]
Here, $\Omega$ is a volume occupied by a lattice atom, and the hydrostatic stress $\sigma_{\text{Hyd}}$ is the mean of the sum of normal stress components. It can be understood as a work done by this stress when an extra volume of $\Omega$ is created inside a confined wire. Thus, the atomic flux caused by an inhomogeneous distribution of the hydrostatic stress can be presented as

$$\Gamma_\sigma(\vec{r}) = N_\sigma(\vec{r}) \frac{D_\sigma \Omega}{k_B T} \nabla \sigma_{\text{Hyd}}(\vec{r}) \quad (4)$$

An additional atomic flux generated by an inhomogeneous distribution of the atom concentration is

$$\Gamma_\rho(\vec{r}) = -D_\rho \nabla N_\sigma$$

(5)

It should be mentioned that the Soret diffusion is not considered in this paper.

Now we need to understand how a redistribution of the atomic density can result in an electrical degradation of the metal. A redistribution of the atomic density is accompanied by generation of the volumetric deformation. When a metal wire is embedded into a rigid confinement, which is the case of the interconnect metallization, the dilatation generates the stress. Over time, the lasting unidirectional electrical load increases these stresses, as well as the stress gradient along the metal line, Fig. 1. This stress can reach a critical level, resulting in a void nucleation at the cathode and/or hillock formation at the anode ends of line, which affect interconnect electrical characteristics. EM-induced generation of hillocks and corresponding development of the electrical shortages is not considered in this paper due to its infrequent nature. It should be noted that the Soret diffusion is not considered in this paper.

An estimation of the $\sigma_{\text{crit}}$ on the basis of the classical model of the homogeneous nucleation can be found in [10, 11]. Assuming that a void can be nucleated only when a pre-existing flaw exists, it was concluded that the flaw with the initial size $r_f$ will start growing if the surrounding stress exceeds the value of $\sigma_{\text{crit}} = 2\gamma/r_f$, where $\gamma$ is the surface energy per unit area.

Thus, assuming a variation in sizes of the void precursors we can come out with a range of critical stresses, which means that a growth of a particular precursor will start when the stress generated by electric current will reach the corresponding level. Thus, the so-called void nucleation is getting a clear statistical nature: void initiation depends on locations and sizes of the pre-existing defects. This can explain a poor EM performance traditionally observed at the early stages of new technology development when the metal interfaces are not optimized yet. It also corresponds well to the observations made in [12] of the correlation between the EM resistance and adhesion properties of the metal interfaces.

### B. Evolution of the Vacancy Concentration caused by EM

In order to create a formalism describing the stress evolution we need, first, to extract the atom concentration evolution, and, second, to generate a link between the changes of atom concentration and stress. The first task can be easily accomplished by solving an appropriate continuity equation coupling the rate of evolution of the local atomic concentration $N_\sigma(\vec{r}, t)$ with the atomic flux divergence occurring at this particular location $\vec{r}$

$$\frac{\partial N_\sigma(\vec{r}, t)}{\partial t} + \vec{\nabla} (\Gamma_\rho(\vec{r}, t)) = 0 \quad (6)$$

where $\Gamma_\rho(\vec{r}, t)$ is the total atomic flux described by a combination of fluxes given by (1)–(5) at the location $\vec{r}$, at the instance in time $t$. It is known, [6], that typical interconnect metals such as Al and Cu are characterized by the vacancy mechanism of diffusion. It means that an atomic diffusivity depends on the vacancy concentration, which, in turn, is a function of the temperature and hydrostatic stress. Since the flux of atoms equals to the vacancy flux but is oriented in an opposite direction, a modeling of the vacancy diffusion can be used instead of atomic diffusion. A vacancy diffusivity is represented by the standard Arrhenius form

$$D = D_0 \exp \left( -\frac{E_{\text{TD}} - \Omega \sigma_{\text{Hyd}}}{kT} \right) \quad (7)$$

Here, $E_{\text{vd}}$ is the activation energy of the vacancy diffusion, which is a sum of the energy of vacancy formation and the energy required for diffusing atom to be moved from its initial site to the saddle point (activated complex), $E_{\text{vd}} = E_v + E_\sigma$, [13]. $\Omega^* = 0.9\Omega$ is a combination of the vacancy formation volume, which is the crystal volume change upon formation of a vacancy in its standard state, and the migration volume, which is the additional volume change when the defect reaches the saddle point in its migration path, [14]. EM induced fluxes of vacancies are described by expressions (2)-(5) where the atomic concentration and diffusivity are replaced by the vacancy concentration $N(\vec{r})$ and diffusivity $D(\vec{r})$, and $\Omega$ is replaced by the $(1 - f)\Omega$, where $f = \Omega_v/\Omega$, which is the ratio of the vacancy and atomic volumes. A polycrystalline structure of interconnect metals introduces additional complications to
the description of the matter transfer. Indeed, metal interfaces (IF) and GBs are characterized by much faster atomic diffusivities than grain interiors. We are describing this well-known effect by introducing the lower energies for vacancy formation and diffusion at the IFs and GBs in comparison with grain bulks.

Stressing the solid with different types of loads (electrical, mechanical, thermal) disturbs the state of thermo-chemical equilibrium of the lattice defects. A system evolves to a new state of equilibrium corresponding to new conditions. It is known that an infinite crystalline solid in the state of zero stress condition is characterized by a thermally equilibrium concentration of vacancies: 

$$N_0^{eq} = \Omega^{-1} \exp\left\{-E_f/k_BT_0\right\},$$

where $E_f$ is the energy of vacancy generation, $T_0$ is the temperature corresponding to the state of zero stress condition, and $\Omega^{-1}$ is the density of lattice sites. The presence of stresses yields the new equilibrium vacancy concentration equals to:

$$N^m = \exp\left\{-\frac{E_f - f\Omega \sigma_{sd}}{k_BT}\right\} = N_0(T) \exp\left\{\frac{f\Omega \sigma_{sd}}{k_BT}\right\}$$  \hspace{1cm} (8)

Here and everywhere below the vacancy concentration is expressed as a fraction of the lattice sites density $\Omega^{-1}$ not occupied by atoms. A presence of the multiplier $f$ in (8) can be explained by the work that should be done against the hydrostatic stress for the vacancy formation. It includes the work done against pressure when a lattice atom transferred to the GB or IF deforms its thicknesses: $\Omega \sigma_{sd}$, and the work performed during the volume relaxation around the newly formed vacancy: $-(1-f)\Omega \sigma_{sd}$.

Mechanism of the transition to the new state of equilibrium distribution of vacancies consists of two independent steps: vacancy generation-annihilation and vacancy diffusion. Formation of an extra vacancy at the lattice site is accompanied by removing a lattice atom occupying this site to any place in the solid where the regular lattice structure is destroyed: IFs, GBs, dislocation edges, etc. Atoms transferred to IFs or GBs can build (plate) a continuation of the solid body will be deformed on account of the dilatation effect. Because of absence of external forces and the uniform temperature, the GB or IF deforms its thicknesses: $\Omega \sigma_{sd}$, and the work induced strain we have

$$\Delta \Omega = \Delta \Omega - \Delta \Omega = -\left(1 - \frac{f}{\Omega}\right) \Delta N.$$  \hspace{1cm} (11)

Here, $N$ is the concentration of vacancies in GB/IF, $N^m$ is the GB/IF equilibrium vacancy concentration (8), and $\tau$ is the appropriate time constant. In equation (9), a sink/source term $G$ is introduced in order to describe the vacancy/plated atom pair generation/annihilation kinetics, which takes place at GBs/IFs. Because of the big difference in mobility of atoms and vacancies we expect plating atoms to stay where they were generated inside interfaces and GBs, while vacancies should migrate under the action of various driving forces. It should be mentioned that the phenomenon of the additional volume generation/disappearance due the pair formation/annihilation has been accounted by Kirchheim [15], and Sarichev et al. [16] by introducing an additional volume evolution equation. In our approach it is described by the introduced plated atom kinetics. The same sink/source term $G$ should exist in the continuity equation for evolution of vacancy concentration at the GB/IF:

$$\frac{\partial N}{\partial t} + \nabla \cdot (D_{gb/IF} \nabla N) - \frac{D_{gb/IF} N}{k_BT} (1-f) \Omega \nabla \sigma_{sd} - \frac{D_{gb/IF} N}{k_BT} \varepsilon Z * \rho) + G = 0$$  \hspace{1cm} (12)

Here, $D_{gb/IF}$ is vacancy diffusivity in the GB/IF. Evolution of the vacancy concentration inside grain bulks is described by the equation similar to (11) with omitted generation/annihilation term:

$$\frac{\partial N}{\partial t} + \nabla \cdot (D \nabla N) - \frac{DN}{k_BT} (1-f) \Omega \nabla \sigma_{sd} - \frac{DN}{k_BT} \varepsilon Z * \rho) = 0$$  \hspace{1cm} (12)

Here, $D$ is the vacancy diffusivity in the grain interior.

The sink/source term $G$ in (13) is the same as in equation (11) describing the evolution of the vacancy concentration in GB/interface, which is in line with the assumption of simultaneous generation/recombination of the vacancy and plated atom, i.e. vacancy - plated atom pair.

C. Linking with the Stress

In this section, we will demonstrate a way how to calculate the stress distribution based on known distributions of vacancies and plated atoms i.e. the distribution of the inelastic volumetric strain. This formalism is similar to the methodology of thermal stress calculation developed by Landau and Lifshitz, [17]. If the concentrations of vacancies and plated atoms are changed in any way then, even in the case of absence of external forces and the uniform temperature, the solid body will be deformed on account of the dilatation effect.

A change in the vacancy concentration occurs as a result of the generation or annihilation of vacancy-plated atom pairs and the diffusion of vacancies. Both these steps result in a local volume change. Because of a difference between the vacancy volume $\Omega_v$ and the atomic volume $\Omega$, a vacancy generation and annihilation results a dilatational volume strain $\varepsilon_v = (\nabla \Omega - \Omega) \Delta N = -(1-f) \Delta N$. Here, $\Delta N = N - N^{eq}$ is a change in vacancy concentration. Thus, for the vacancy-induced strain we have $\varepsilon_v = (\nabla \Omega - \Omega) \Delta N = -(1-f) \Delta N/3$. Dilatation caused by the atom plating we describe as generation or
removal of an extra volume \( \Omega \) in the GBs and/or interfaces. Thus, the total volume strain can be described as:

\[
\theta = -(1 - f) \Delta N + M. \tag{13}
\]

It should be mentioned, that \( \Delta N \) and \( \Delta M \) in (13) are not necessarily equal each other. A standard derivation provides us with the following expression for the stress components

\[
\sigma_i = \lambda e + 2Ge, \quad (14)
\]

where the Lame coefficient \( \lambda \) and the shear modulus \( G \) have the form: \( G = E/(1 + \nu) \) and \( G = E/(1 + 2\nu) \). Here, \( E \) and \( \nu \) are the Young’s modulus and the Poisson’s factor. Substituting \( \sigma_i \) from (14) into the partial differential equation (PDE) for force balance, \([18]\), and assuming absence of the body forces, we can find:

\[
(\lambda + G) \frac{\partial e}{\partial x_i} + G\Delta u = \frac{E}{3(1 - 2\nu)} \left( \frac{\partial M}{\partial x_i} - (1 - f) \frac{\partial N}{\partial x_i} \right) = 0 \tag{15}
\]

where the terms

\[
X_i = -\frac{E}{3(1 - 2\nu)} \left( \frac{\partial M}{\partial x_i} - (1 - f) \frac{\partial N}{\partial x_i} \right) \tag{16}
\]

\[
X_j = E(1 - f) \frac{\partial N}{\partial x_j} \tag{17}
\]

take place of the body force components in GB/IF and grain interiors correspondingly. Conditions of the equilibrium at the surface of the solid body with the vacancies and plated atoms yield the following surface force balance PDE

\[
\frac{E}{3(1 - 2\nu)} (M - (1 - f) \Delta N) \nu = \lambda e l + G \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \frac{\partial u}{\partial z} \right) + \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} m + \frac{\partial u}{\partial z} \right) \tag{18}
\]

where the terms

\[
Y_i = \frac{E}{3(1 - 2\nu)} (M - (1 - f) \Delta N) \nu_i \tag{19}
\]

take place of the surface force components.

Thus, the displacements \( u_i \) generated by vacancies and plated atoms distributed in the GBs, and IFs are equal to the displacements produced by the body forces (16) and (17) for the body forces in grain interiors, and by the tension (19) distributed over the solid body surface. From the equation (14) we can see that the normal stress components consist of two parts, a “traditional” elastic part, which is derived by using elastic strain/stress relations, and the inelastic part describing the dilatational-induced pressure of the amount of:

\[
\Delta P_{\text{IF/GB}} = -\frac{E}{3(1 - 2\nu)} (M - (1 - f) \Delta N), \tag{20}
\]

\[
\Delta P = \frac{E(1 - f)}{3(1 - 2\nu)} \Delta N
\]

for the GBs/IFs, and grain interiors, which are proportional to the concentrations of vacancies and plated atoms.

Thus, the EM-induced stress evolution inside an arbitrary taken interconnect segment can be obtained by a solution of the system of coupled second order PDE. These are the continuity equations (9)-(12) describing the evolution of the concentrations of vacancies and plated atoms, the force balance equations (15) for all three components of the displacement, and the Laplace’s equation for the current density distribution everywhere inside the segment. In general case this system can be solved numerically by employing the FEA technique. Using appropriate boundary conditions (BC) for all involved modules such as zero flux conditions for the metal-barrier interfaces in the continuity equations, zero displacements for the interfaces with the rigid confinement in the force balance equation, and the isolation, voltage or current sources BC for the Laplace’s equation allows us to get physically viable solutions for many cases. Fig. 1 demonstrates the hydrostatic stress distribution along an upstream electron flow test structure. Figures 2 and 3 show the distributions of vacancies and hydrostatic stress in the polycrystalline test structure with grains characterized by different crystallography orientations relative the line direction, and the correlation between the interfacial shear stress distribution, caused by the anisotropy in mechanical properties of polycrystalline copper line, and the voiding locations.
D. Void Growth and Postvoiding Stress Evolution

The described above formalism predicts the stress evolution occurring in the interconnect segment loaded with electric current until the voiding is started.

Origination of a free surface inside the metal line due to void nucleation can be described by means of the phase-field approach [19]. A phase field \( \phi \) (also called an order parameter) describes the void and metal regions as two different phases in a way that all material properties such as metal conductivity, Young’s modulus, diffusivity of vacancies, etc. are vanished inside the void. This is achieved by the following phase-field definition

\[
\phi = 1 \quad \text{in metal}, \\
\phi = -1 \quad \text{in void},
\]

which provides the following representation of the materials and transport properties everywhere inside the metal

\[
\eta = \frac{1 + \phi}{2}; \quad \gamma = \gamma \eta, \quad E_\phi = E \eta, \quad D_\phi = D \eta
\]

The model provides a smooth transition between the two phases by corresponding smooth behavior of \( \phi \) in a narrow region \( \delta \) representing the void-metal interface. Evolution of the order parameter \( \phi \) is described by the non-linear equation:

\[
\frac{\partial \phi}{\partial t} + \xi \nabla^2 \phi + (\phi^2 - 1) \frac{\partial \phi}{\partial \lambda} + \partial \phi = f(\sigma)
\]

where the parameter \( \xi \) defines the interface thickness \( \delta = \sqrt{2 \xi} \) and \( V \) is the rate of surface motion. Right-hand term in (23) allows to introduce void region in the initially void-less metal, when the EM-induced stress achieves \( \sigma_{\text{void}} \):

\[
f(\sigma_{\text{void}}) = \begin{cases} 
0, & \text{if } \sigma_{\text{void}} < \sigma_{\text{crit}} \\
1, & \text{if } \sigma_{\text{void}} \geq \sigma_{\text{crit}}
\end{cases}
\]

The duration of nucleation process is controlled by the parameter \( f_0 \); the term (24) vanishes when \( \phi \) achieves new equilibrium state \( \phi = -1 \). The velocity of the void surface motion is determined by the vacancy inflow flux from the metal bulk directed normal to the surface, and by the divergence of the flux of vacancies migrating along the void surface:

\[
V = \Omega (-\nabla J_1 + J_s)
\]

Here, \( J_s = -\frac{D}{k_B T} \left( \nabla \sigma + \frac{\Omega}{\xi} \nabla \mu \right) \) and \( J_1 = -\frac{D}{k_B T} \nabla \mu \), where \( D \) is the surface diffusivity. The chemical potential \( \mu = \gamma \nabla^2 \phi + \frac{\partial W}{\partial \phi} \), is determined by the surface energy \( \gamma \) and by variation of strain energy \( W(\epsilon, \phi) \) along the surface, [19].

Since the inflow of vacancies into the growing void is a major factor effecting the void shape evolution, the grain microstructure providing the distribution of GB around the void plays crucial role in void shaping. Fig. 4 demonstrates the void evolution taking place when GBs are uniformly distributed around the growing void – (a), and when a single GB crossing the week top interface provides a venue for the fast vacancy migration – (b).

As an additional demonstration of the simulation capabilities, Fig. 5 demonstrates the simulated drift of the line edge caused by a DC current applied to the Blech strip test structure of EM.

\[
\sigma_{\text{Hyd}} = -B \theta
\]

Here, \( B \) is the effective bulk modulus, and \( \theta \) is the volumetric strain. Variation in \( \theta \) is originated by redistribution of the lattice atoms along the line caused by the electron wind and by the equilibration of the vacancy concentration with the stress. The introduced effective bulk modulus \( B \) should effectively describe an interaction between volumetrically deformed line and confinement; see for example [22].

A major challenge for such approaches is an accurate calculation of the volumetric strain \( \theta \) and stress generated by interaction with confinement. As it was demonstrated above, the inelastic volume strain generated in an unit volume by vacancy migration is

\[
\theta_{\text{inc}} = ((\Omega - \Omega)/\Omega) \Delta N_v = -(1-f)\Delta N_v
\]
and by vacancy/plated atom generation/annihilation is 

\[ \theta_v = \Omega^{-1} \left( \Omega M + (\Omega - \Omega) \Delta N_e \right) = M - (1 - f) \Delta N_e, \]

where \( M = -\Delta N_e \). Here, \( \Delta N_e \) and \( \Delta N_m \) are the vacancy concentration changes caused by generation/annihilation and migration. Thus, a total vacancy change is \( \Delta N = \Delta N_e + \Delta N_m \), and a total inelastic volume strain is described by (13). It should be mentioned that if not for a confinement, this inelastic volume change would certainly occur. Since the rigid confinement doesn’t allow the metal volume changing, the generated elastic change would certainly occur. Since the rigid confinement doesn’t allow the metal volume changing, the generated elastic strain demonstrates a sign, which is opposite to the sign of inelastic strain.

Introducing the effective diffusion coefficient and effective generation/annihilation rate, the both are averaged on the entire cross section of the line, [9, 10], the considered above system of PDEs can be reduced to 1D case

\[ \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left[ - D_n \frac{\partial N}{\partial x} + \sigma \frac{\partial \rho}{\partial x} \right] = -G \]

(27)

Accepting Korhonen’s assumption of the very fast equilibration of the vacancy concentration with the stress, i.e. assuming the validity of: \( \Delta N = N^{\Omega} = N^{\Omega,} \), we obtain

\[ \frac{\partial \Delta N}{\partial t} = \frac{\partial}{\partial t} \left( \frac{\beta N^{\Omega}}{kT} \frac{\partial \sigma}{\partial t} \right) \]

(28)

Combining together the equations (26)-(28), we come to the following equation

\[ \frac{\partial N}{\partial t} + G = \frac{\partial \Delta N}{\partial t} - \frac{\partial M}{\partial t} = \left( \frac{1}{B} + \frac{\beta N^{\Omega}}{kT} \right) \frac{\partial \sigma}{\partial t} \]

(29)

Assuming a validity of the approximation \( \beta N^{\Omega} / kT \ll 1 \), which is same that was used in [9], and recalling that the atomic diffusivity is determined as \( D_n = (1 - f)DN \), the equations (29) and (28) yield the Korhonen’s equation

\[ \frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left[ D_n \frac{\partial \sigma}{\partial x} + \sigma \frac{\partial \rho}{\partial x} \right] \]

(30)

Hence, making exactly same assumptions, which were made in [9] for development of the equation (30), we were able to reduce our full multiphysics EM model to the same 1D representation. Below we will demonstrate some results obtained with this 1D approximation.

A solution of (30) for the case of the wire of length \( L \) with diffusion blocking ends loaded with DC current density \( j \) provides the well-known kinetics of stress evolution, [9]

\[ \sigma(x,t) = \sigma_f - \frac{Gx + 4GL}{2} - 4GL \sum_{k=0}^{\infty} \frac{\cos((2k + 1) \pi x/L)}{(2k + 1)^2} \times \exp\left\{ -\Gamma(2k + 1) \frac{\pi^2}{L^2} \right\} \]

× \exp\left\{ -\Gamma(2k + 1) \frac{\pi^2}{L^2} \right\} \]

(31)

Here, \( \chi^2 = D_n B \Omega / kT \), \( G = eZ \rho j / \Omega \), \( \sigma_f \) is the initial stress.

Fig. 7 shows the evolution of distribution of the hydrostatic stress along the line, which yields a steady state linear stress distribution \( \sigma_{\Omega}(x, \infty) = \sigma_f - x \rho \Omega j / \Omega \), at \( t \to \infty \):

\[ t_nuc = \frac{L^2 \rho j T}{D_n B \Omega} - \left[ \frac{eZ \rho j L / 2\Omega}{\sigma_f - eZ \rho j L / 2\Omega - \sigma_{\Omega}} \right] \]

(32)

Equation (32) clearly demonstrates that in the case of \( \sigma_f > \sigma_f + \sigma_{\Omega} \), where \( \sigma_{\Omega} = eZ \rho j L / 2\Omega \), which is the Blech condition of immortality, [24]: \( j \times L < 2\Omega (\sigma_{\Omega} - \sigma_f) / eZ \rho \), \( t_nuc \) diverges indicating the line immortality. When \( \sigma_f > \sigma_{\Omega} \), the equation (32) yields the negative void nucleation time, which is the condition for stress voiding. Indeed, if the residual stress exceeds the critical stress needed for void nucleation, then a void will be nucleated without any electrical stressing. Representing \( t_nuc \), given by (32), in the traditional form of the Black equation, \( t_nuc = A j^{-n} \exp\left\{ -E_a / \sigma_f \right\} \), and extracting the current density exponent \( n \) and apparent activation energy \( E_a \), confirms an existence of the discussed in literature dependencies of \( n \) and \( E_a \) on \( \sigma_f \) and \( j \) by the fit between the

\[ \text{Fig. 6. Evolution of the hydrostatic stress (a) along the metal line loaded with DC current, and at the cathode end of line, (b) } j = 5 \times 10^9 \text{A/m}^2, \text{ } T = 400 \text{K.} \]

As it was shown in [23], keeping just the slowest decaying term of the infinite series (31), provides an approximate void nucleation time \( t_nuc \) as an instant in time when \( \sigma \) reaches \( \sigma_{\eqref{eq:critical}} \) at the line cathode end

It should be mentioned, that everywhere above \( D_n \) is the effective atomic diffusivity, which is determined mainly by the
interfacial and GB diffusivities, [10]. Dependence of $D_{\sigma}$ on the grain size distribution demonstrates an additional to $\sigma_{cr0}$ parameter reflecting the statistical nature of the EM phenomenon.

It is obvious that a change in line resistivity will be initiated at $t_{nuc}$. Growing void increases the line resistivity and, at the end, when it occupies the entire cross section of the wire, pushes the electric current to flow through the highly resistive metal liners. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal. A kinetics of the void volume evolution in 1D approximation was derived in [27]. Void nucleation occurs near the cathode edge of the line when the increasing with time tensile stress creates a condition for a stable growth of the preexisted process-induced flaws located at the metal/passivation interface. It means, the metal in the vicinity of the growing void is under essential tension, about $\sigma_{cr0}$. Large stress gradient build in between the void surface, characterized by zero normal stress, and metal bulk pushes atoms to move from the void surface into the metal.

Equation (30) can be used also for predicting the stress evolution in interconnect lines loaded with different kind of time-dependent electric stressing $j(t)$, such as AC and pulsed currents, [28]. General solution takes the form:

$$\sigma(x, t) = \int_0^t \left( \sigma(x, \tau) / B \right) d\tau$$

As an example, the Fig. 9 shows the current switch-off induced relaxation kinetics of stresses accumulated at the cathode end of line during 4x10^7 s of stressing by DC current with the density of 1x10^9 A/m² at $T = 373$ K and 400 K.

IV. A COMPARISON OF 1D AND 2D/3D PREDICTIONS

We have demonstrated that the proposed 1D EM approximation can generate physically correct solutions for a number of important problems. Now we will examine a direct comparison between the predictions made with the simple 1D considerations and results obtained by 2D simulations.
immortal, which is highly optimistic due to the Blech effect \cite{24}, then the tree would appear to be branches happen to be short so that they are deemed immortal branches of an interconnect tree. As a result, if the individual connected and atomic flux can flow freely between the power grids, many branches within the same layer are material flow between branches. In today's mesh structured extrapolation, \cite{26}. Second, Black's model ignores the conditions, and this leads to significant errors in lifetime accelerated testing conditions are not valid at actual operating time) to determine the failure time for the whole grid. This approach is highly inaccurate, for at least three reasons. First, the fitting parameters obtained for Black’s model under accelerated testing conditions are not valid at actual operating conditions, and this leads to significant errors in lifetime extrapolation, \cite{26}. Second, Black’s model ignores the material flow between branches. In today’s mesh structured power grids, many branches within the same layer are connected and atomic flux can flow freely between the branches of an interconnect tree. As a result, if the individual branches happen to be short so that they are deemed immortal due to the Blech effect \cite{24}, then the tree would appear to be immortal, which is highly optimistic and can be entirely misleading for design. Finally, the third problem lies with the series model assumption. A series model is the case where a power grid is deemed to have failed as soon as the first of its branches has failed, typically due to an open circuit. However, modern power grids use a mesh structure. As such, there are many paths for the current to flow from the C4 bumps to the underlying logic, a characteristic we have referred above to as redundancy. Mesh power grids are in fact closer to (but not quite) a parallel system. As such, it is highly pessimistic to assume that a single branch failure will always cause the whole grid to fail. Thus, there is a need for a new EM checking approach that accurately models EM degradation using physics-based models, combined with a mesh model to account for redundancy, while being fast enough to be practically useful.

This novel approach has been developed recently, \cite{29, 30}. While both teams have implemented similar physical models, the advanced numerical technique developed in \cite{30} appears to be promising for EM assessment in power grids of large VLSI circuits. The developed in \cite{30} approach consists of the decomposing the power grid into a number of interconnect trees, solving the set of PDEs \cite{30} for all branches of each tree characterized by different current densities and geometries (length and width), and linking the solutions to each other through the proper BC at the segment junctions, which represent the continuity of stress and atomic fluxes. The latter secures an accurate calculation of stress evolution inside multi-segment tree. Fig. 13 shows the kinetics of stress evolution at all tree junctions and the evolution of the stress distribution across the 3-terminal tree shown in Fig. 14. An improved computational speed is achieved using an efficient filtering scheme and a fast predictor-based approach. This has been proved to have minimal impact on accuracy.

The extended Korhonen model starts out as a system of PDE coupled by the boundary laws, which are then scaled and discretized to reduce the model to a system of ordinary differential equations (ODE). The method then moves on to numerically solve the ODE system at successive time-points to track the stress evolution and find the corresponding time of void nucleation(s).

The random nature of EM degradation is accounted for using a Monte Carlo method. Successive samples of grid time-to-failure are found, until the estimate of the overall MTF has converged to satisfy user provided thresholds.

Computation speed is enhanced by using a filtering scheme that estimates upfront the set of trees that are most likely to impact the MTF assessment of the grid, with minimal impact on accuracy. The process also includes a predictive scheme that allows for faster MTF estimation by extrapolating the solution (stress curve) obtained from a few initial time-points.

The developed approach was tested on a number of IBM power grid benchmarks, \cite{31}, on a quad-core 3.4GHz Linux machine with 32GB of RAM. The MTFs estimated using the physics-based approach were on average 3x longer than those based on a (calibrated) Black’s model, supporting the claim that Black’s model is not accurate enough for modern power grids and confirming the need for physical models. Having achieved a run-time of less than three hours for the largest grid

\[ \text{Fig. 11. Post voiding stress evolution with the time instances indicating the time elapsed since the void was nucleated, (a). Simulated void evolution, (b). Evolution of void volume obtained with 1D (c), and FEA (d) simulations.} \]

\[ \text{Fig. 12. Simulated growth of the line corner void caused by scavenging the vacancy flux and agglomerating with the small voids drifting along the top interface in two cases: the diffusion along the top interface prevails } D_h >> D_{cb}, (a); \text{ the diffusion along the GB prevails } D_h << D_{cb}, (b), [4]. \]

All these examples demonstrate a good capability of 1D EM model to capture the EM induced degradation in one dimension-like metal segments. It means it can be employed for the EM assessment in large electrical circuits such as on-chip power grids.

V. \textbf{POWER GRID EM CHECKING USING PHYSICS-BASED MODELS}

Standard practice employed in the industry for the EM assessment is to break up a grid into isolated metal branches, assess the reliability of each branch separately using Black’s model and then use the series model (earliest branch failure time) to determine the failure time for the whole grid. This approach is highly inaccurate, for at least three reasons. First, the fitting parameters obtained for Black’s model under accelerated testing conditions are not valid at actual operating conditions, and this leads to significant errors in lifetime extrapolation, \cite{26}. Second, Black’s model ignores the material flow between branches. In today’s mesh structured power grids, many branches within the same layer are connected and atomic flux can flow freely between the branches of an interconnect tree. As a result, if the individual branches happen to be short so that they are deemed immortal due to the Blech effect \cite{24}, then the tree would appear to be immortal, which is highly optimistic and can be entirely misleading for design. Finally, the third problem lies with the series model assumption. A series model is the case where a power grid is deemed to have failed as soon as the first of its branches has failed, typically due to an open circuit. However, modern power grids use a mesh structure. As such, there are many paths for the current to flow from the C4 bumps to the underlying logic, a characteristic we have referred above to as redundancy. Mesh power grids are in fact closer to (but not quite) a parallel system. As such, it is highly pessimistic to assume that a single branch failure will always cause the whole grid to fail. Thus, there is a need for a new EM checking approach that accurately models EM degradation using physics-based models, combined with a mesh model to account for redundancy, while being fast enough to be practically useful.

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for large VLSI circuits. This approach has been demonstrated as suitable for analysis of EM degradation in different cases characterized by micro caverns and interfacial/inter-granular delaminating. Initiation of preexisted crystal imperfections such as driven redistribution of vacancies and plating atoms, which mechanical stress evolution caused by an electric current in power grids. We demonstrated a general model of the degradation of individual interconnect metal lines and on-chip modeling and numerical analysis approaches currently evolution with time, (b). Here $L_1=L_2=50$ mm, and $j_1=-j_2=6 \times 10^9$ A/m$^2$. [30].

**REFERENCES**