A Probabilistic Pointer Analysis for Speculative Optimizations

by

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Abstract

Pointer analysis is a critical compiler analysis used to disambiguate the indirect memory references that result from the use of pointers and pointer-based data structures. A conventional pointer analysis deduces for every pair of pointers, at any program point, whether a points-to relation between them (i) *definitely* exists, (ii) *definitely does not* exist, or (iii) *maybe* exists. Many compiler optimizations rely on accurate pointer analysis, and to ensure correctness cannot optimize in the *maybe* case. In contrast, recently-proposed *speculative optimizations* can aggressively exploit the *maybe* case, especially if the likelihood that two pointers alias could be quantified. This dissertation proposes a *Probabilistic Pointer Analysis* (PPA) algorithm that statically predicts the probability of each points-to relation at every program point. Building on simple control-flow edge profiling, the analysis is both one-level context and flow sensitive—yet can still scale to large programs.

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Chapter 1

Introduction

Pointers are powerful constructs in `C` and other similar programming languages that enable programmers to implement complex data structures. However, pointer values are often ambiguous at compile time, complicating program analyses and impeding optimization by forcing the compiler to be conservative. Many pointer analyses have been proposed which attempt to minimize pointer ambiguity and enable compiler optimization in the presence of pointers [2, 6, 22, 26, 49, 62, 65, 77, 78, 80]. In general, the design of a pointer analysis algorithm is quite challenging, with many options that trade accuracy for space/time complexity. For example, the most accurate algorithms often cannot scale in time and space to accommodate large programs [26], although some progress has been made recently using *binary decision diagrams* [6, 77, 80].

The fact that memory references often remain ambiguous even after performing a thorough pointer analysis has motivated a class of compiler-based optimizations called *speculative optimizations*. A speculative optimization typically involves a code transformation that allows ambiguous memory references to be scheduled in a potentially unsafe order, and requires a recovery mechanism to ensure program correctness in the case where the memory references were indeed dependent. For example, EPIC instruction sets (eg., Intel's IA64) provide hardware

support that allows the compiler to schedule a load ahead of a potentially-dependent store, and to specify recovery code that is executed in the event that the execution is unsafe [24, 45]. Proposed speculative optimizations that allow the compiler to exploit this new hardware support include speculative dead store elimination, speculative redundancy elimination, speculative copy propagation, and speculative code scheduling [20, 51, 52].

More aggressive hardware-supported techniques, such as thread-level speculation [40, 47, 60, 69] and transactional programming [37, 38] allow the speculative parallelization of sequential programs through hardware support for tracking dependences between speculative threads, buffering speculative modifications, and recovering from failed speculation. Unfortunately, to drive the decision of when to speculate many of these techniques rely on extensive data dependence profile information which is expensive to obtain and often unavailable. Hence we are motivated to investigate compile-time techniques—to take a fresh look at pointer analysis with speculative optimizations in mind.

1.1   Probabilistic Pointer Analysis

A conventional pointer analysis deduces for every pair of pointers, at any program point, whether a points-to relation between them (i) *definitely* exists, (ii) *definitely does not* exist, or (iii) *maybe* exists. Typically, a large majority of points-to relations are categorized as *maybe*, especially if a fast-but-inaccurate approach is used. Unfortunately, many optimizations must treat the *maybe* case conservatively, and to ensure correctness cannot optimize. However, speculative optimizations can capitalize on the *maybe* case—especially if the likelihood that two pointers alias can be quantified. In order to obtain this likelihood information, in this thesis we propose a *Probabilistic Pointer Analysis* (PPA) that accurately predicts the probability of each points-to relation at every program point.

1.2   Research Goals

The focus of this research is to develop a *Probabilistic Pointer Analysis* (PPA) for which we have the following goals:

1. to accurately predict the probability of each points-to relation at every pointer dereference;

2. to scale to the SPEC 2000 integer benchmark suite [19];

3. to understand potential trade-offs between scalability and accuracy; and

4. to increase the overall understanding of program behaviour.

To satisfy these goals we have developed a PPA infrastructure, called *Linear One-Level Interprocedural Probabilistic Points-to* (LOLIPoP), based on an innovative algorithm that is both scalable and accurate: building on simple (control-flow) edge profiling, the analysis is both one-level context and flow sensitive, yet can still scale to large programs. The key to the approach is to compute points-to probabilities through the use of linear transfer functions that are efficiently encoded as sparse matrices. LOLIPoP is very flexible, allowing us to explore the scalability/accuracy trade-off space.

1.3   Organization

The remainder of this dissertation is organized as follows. In Chapter 2 the background material and related work in the fields of pointer analysis and pointer analysis for speculative optimization are described. In Chapter 3 the probabilistic pointer analysis algorithm is described. Chapter 4 describes the LOLIPoP infrastructure, including the practical implementation details and the design tradeoffs. Chapter 5 evaluates the efficiency and accuracy of the LOLIPoP infrastructure, and Chapter 6 concludes by summarizing the dissertation, naming its contributions, and listing future extensions of this work.

Chapter 2

Background

This chapter presents the background material and related work in the fields of pointer analysis and pointer analysis for speculative optimization. The organization of this chapter is divided into four main areas: Section 2.1 introduces the basic concepts and terminology involved in pointer analysis research; Section 2.2 describes some of the traditional approaches used to perform pointer analysis and outlines the strengths and drawbacks of the various approaches; Section 2.3 discusses various speculative optimizations that have been proposed to aggressively exploit overly conservative pointer analysis schemes; and finally Section 2.4 describes more recently proposed program analysis techniques that aid speculative optimizations.

2.1 Pointer Analysis Concepts

A pointer is a programming language datatype whose value refers directly to (or points-to) the address in memory of another variable. Pointers are powerful programming constructs that are heavily utilized by programmers to realize complex data structures and algorithms in `C` and other similar programming languages. They provide the programmer with a level of indirection when accessing data stored in memory. This level of indirection is very useful for two main

Table 2.1: Pointer Assignment Instructions

α = &β	Address-of Assignment
α = β	Copy Assignment
α = *β	Load Assignment
*α = β	Store Assignment

reasons: (1) it allows different sections of code to share information easily; and (2) it allows for the creation of more complex "linked" dynamic data structures such as linked lists, trees, hash tables, etc. For the purposes of this discussion, the variable that the pointer points to is called a *pointee* target. A *points-to* relation between a pointer and a pointee is created with the unary operator '&', which gives the address of a variable. For example, to create a points-to relation $\langle *\alpha, \beta \rangle$ between a pointer α and a target β, the following assignment is used α = &β. The indirection or dereference operator '*' gives the contents of an object pointed to by a pointer (i.e. the *pointee's* contents). The assignment operation '=' when used between two pointers, makes the pointer on the left side of the assignment point to the same target as the pointer on the right side. Table 2.1 describes the four basic instructions that can be used to create points-to relations. All other pointer assigning instructions can be normalized into some combination of these four types. Figure 2.1 is a `C` source code example that demonstrates how pointers can be assigned and used.

In this example found in Figure 2.1, there are three pointer variables (p, q, and r) and three pointee target variables that can be pointed at (a, b and q). Any variable, including a pointer, whose address is taken using the '&' operator is defined as a pointee target. Initially, at program point $S1$, all three pointers (p, q, and r) are *uninitialized*. A special undefined pointee target,

```
void main() {

        int a = 1, b = 2;
        int **p, *q, *r;

S1:     q = &a;
S2:     r = q;
S3:     p = &q;
S4:     *p = &b;
S5:     r = *p;

S6:     **p = 3;
S7:     *r = 4;
        ...

}
```

Figure 2.1: Sample C code that uses pointers.

denoted 'UND', is used as the target of an uninitialized pointer. Therefore, at program point $S1$, the points-to relations that exist are: $\langle *p, \text{UND} \rangle$, $\langle *q, \text{UND} \rangle$, and $\langle *r, \text{UND} \rangle$. At program point $S1$, the pointer q is assigned to point to the target a using an *address-of assignment*. Statement $S1$ has the following two side effects: (1) the points-to relation $\langle *q, \text{UND} \rangle$ is killed, and (2) the points-to relation $\langle *q, a \rangle$ is generated. At program point $S2$, the *copy assignment* instruction assigns the pointer r to point to the target of pointer q. At $S2$, The pointer q has a single target, which is a. Therefore, $S2$ kills the points-to relation $\langle *r, \text{UND} \rangle$ and creates the points-to relation $\langle *r, a \rangle$. At $S3$, there is another *address-of assignment* that assigns the double pointer p to point to the target q, which is itself is a pointer. This instruction creates the following two points-to relations $\langle *p, q \rangle$ and $\langle * * p, a \rangle$[1]. The instruction at $S4$ is a *store assignment*. Since it can be proven that at program point $S4$ the pointer p can only point to q, $S4$ kills the points-to relation

---

[1]A double pointer points-to relation such as this one is often ignored and not tracked because it can be inferred from the other points-to relations in the set.

$\langle *q, a \rangle$ and creates the points-to relation $\langle *q, b \rangle$. The instruction at $S5$ is a *load assignment* that assigns r to point to the target of $*p$, which is the target of q, which is b. Therefore, at $S6$, the following points-to relations are said to exist: $\langle *p, q \rangle$, $\langle *q, b \rangle$ and $\langle *r, b \rangle$. An analysis such as this, that tracks the possible targets that a pointer can have, is referred to as a points-to analysis. A *safe* points-to analysis reports all possible points-to relations that *may* exist at any program point.

2.1.1   Pointer Alias Analysis and Points-To Analysis

A *pointer alias analysis* determines, for every pair of pointers at every program point, whether two pointer expressions refer to the same storage location or equivalently store the same address. A pointer alias analysis algorithm will deduce for every pair of pointers one of three classifications: (i) *definitely* aliased, (ii) *definitely not* aliased, or (iii) *maybe* aliased. A *points-to analysis*, although similar, is subtly different. A *points-to analysis* attempts to determine what storage locations or pointees a pointer can point to—the result can then be used as a means of extracting or inferring pointer aliasing information. The main differences between an alias analysis and a points-to analysis are the underlying data structure used by the algorithm and the output produced. The analysis type has consequences in terms of precision and algorithm scalability—these will be discussed further in section 2.1.6. The terms *pointer alias analysis* and *points-to analysis* are often times incorrectly used interchangeably. The broader term of *pointer analysis* is typically used to encapsulate both.

2.1.2   The Importance of and Difficulties with Pointer Analysis

Pointer analysis does not improve program performance directly, but is used by other optimizations and is typically a fundamental component of any static analysis. Static program analysis aims at determining properties of the behaviour of a program without actually executing it.

It analyzes software source code or object code in an effort to gain understanding of what the software does and establish certain correctness criteria to be used for various purposes. These purposes include, but are not limited to: optimizing compilers, parallelizing compilers, support for advanced architectures, behavioral synthesis, debuggers and other verification tools. One of the main challenges of any program analysis is to safely determine what memory locations the program is attempting to access at any read or write instruction, and this problem becomes substantially more difficult when memory is accessed through a pointer. In order to gain useful insight, a pointer analysis is often required. Complicating things further, the degree of accuracy required by a pointer analysis technique is dependent on the actual program analysis being performed [44]. For example, in the sample code presented in Figure 2.1, an optimizing compiler or a parallelizing compiler would want to know whether the statements at $S6$ and $S7$ accessed the same memory locations. If the static analyzer could prove that all load and store operations were independent, then the statements could potentially be reordered or executed in parallel in order to improve performance. Many advanced architectures, behavioral/hardware synthesis tools [34, 64, 76] and dataflow processors [63, 71] attempt to reduce the pressure on a traditional cache hierarchy by prepartitioning memory accesses into different memory banks. These applications rely on a static analysis that is able to safely predetermine at compile time which memory accesses are mapped to which memory banks. The accuracy of pointer analysis is not as important for this purpose of memory partitioning [64]. As a final example a debugger uses pointer analysis to detect uninitialized pointers or memory leaks.

Currently, developing both an accurate and scalable pointer analysis algorithm is still regarded as an important yet very complex problem. Even without dynamic memory allocation, given at least four levels of pointer dereferencing (i.e. allowing pointer chains to be of length 4) the problem has been shown to be PSPACE Complete [49] and to be NP Hard given at least two levels of pointer dereferencing [48]. Given these results, it is very unlikely that there are precise, polynomial time algorithms for the C programming language. Although, there are many

different solutions that trade off some precision for scalability. These tradeoffs are discussed further in Section 2.1.6.

2.1.3   Static Memory Model

To perform any pointer analysis, an abstract representation of addressable memory called a *static memory model* must be constructed. For the remainder of this dissertation, it is assumed that a static memory model is composed of *location sets* [78][2]. A location set can represent one or more real memory locations, and can be classified as a pointer, pointee, or both. A location set only tracks its approximate size and the approximate number of pointers it represents, allowing the algorithm used to abstract away the complexities of aggregate data structures and arrays of pointers. For example, fields within a `C` struct can either be merged into one location set, or else treated as separate location sets.

2.1.4   Points-To Graph

A points-to relation between a pointer α and a pointee target β is denoted as such $\langle *\alpha, \beta \rangle$. This notation signifies that the dereference of variable α and variable β may be aliased. A *points-to graph* is typically used in a points-to analysis algorithm to efficiently encode all possible may points-to relations. A points-to graph is a directed graph whose vertices represent the various pointer location sets and pointee target location sets. An edge from node α to the target node β represents a *may*-points-to relation $\langle *\alpha, \beta \rangle$. For certain algorithms, the edge is additionally annotated as a *must* points-to edge, if the relation is proven to always persist. Figure 2.2 shows the points-to graphs for the various program points associated with the sample code in Figure 2.1.

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[2]Note that the term location set itself is not necessarily dependent on this location-set-based model.

void main() {

    int a = 1, b = 2;
    int **p, *q, *r;

S1:   q = &a;
S2:   r = q;
S3:   p = &q;
S4:   *p = &b;
S5:   r = *p;

S6:   **p = 3;
S7:   *r = 4;

    ...

}

(a) Sample program from Figure 2.1

(b) S1

(c) S2

(d) S3

(e) S4

(f) S5

(g) S6

Figure 2.2: The points-to graphs corresponding to the different program points in the sample code found in Figure 2.1.

2.1.5   Pointer Analysis Accuracy Metrics

The conventional way to compare the precision of different pointer analyses is by using either a static or dynamic metric. A static metric (also called a direct metric) asserts, without ever executing the program, that a pointer analysis algorithm X is more precise than an algorithm Y if the point-to graph generated by the algorithm X is a subset of Y, given an equal model of memory. Many researchers [22, 26, 43, 49] estimate precision by measuring the cardinality of the points-to set for each pointer dereference expression, and then calculate the average—in short,

Figure 2.3: A conservative points-to graph for the sample code found in Figure 2.1. This graph is safe for every program point and is constructed by taking the union of all possible points-to graphs (see Figure 2.2). A solid edge represents a must-point-to relation and a dotted line represents a may-point-to relation. The UND target is not included for simplicity.

the average dereference size. The disadvantage of using a static metric is that the precision result is sensitive to the static model of memory used. For example, modeling the entire heap as a single pointee target location would be advantageous for an average dereference size metric; this can result in misleading comparisons to other algorithms that model the heap more accurately.

A second method of measuring accuracy is with the use of a dynamic metric. A traditional dynamic metric evaluates the number of false points-to relations reported by the analysis; that is the number of points-to relations that never occur at runtime [54]. There are many disadvantages with a dynamic metric. The result, like the static metric, is sensitive to the static memory model used. The result is also sensitive to the input set used, which may not sufficiently cover all interesting control-flow cases. A dynamic metric also requires some form of runtime dependence profiling, which can be a computationally expensive undertaking.

A third method of measuring accuracy is to apply the analysis to a client optimization and report the speedup; this is referred to as an indirect metric. The assumption is that a more accurate analysis will result in improved performance, although it has been shown that this is

not always the case [22, 23, 44] and therefore an indirect metric may be disadvantageous as well. Hind addresses this accuracy metric problem [42] and argues that the best way to measure and compare pointer analysis precision is to concurrently use all three types: static, dynamic, and indirect.

2.1.6   Algorithm Design Choices

There are many factors and algorithm design choices that cause a pointer analysis `X` to be more accurate than a pointer analysis `Y`. Typically, when making algorithm design choices there exists a tradeoff between algorithm scalability (in time and space) and accuracy. Some of the major design considerations are [42]:

- flow-sensitivity,

- context-sensitivity,

- aggregate modeling / field-sensitivity,

- heap modeling, and

- alias representation.

A **flow-sensitive** pointer analysis takes into account the order in which statements are executed when computing points-to side effects. It is normally realized through the use of strong/weak updates applied to a data flow analysis (DFA) [1, 56] framework, which requires repeated calculations on a control flow graph until a fixed point solution is found. The analysis is considered to be highly precise; however, the analysis is generally regarded as too computationally intensive for two reasons: (1) Pointer analysis generally requires a forward and backward interprocedural DFA, which is not feasible for large programs [70]; and (2) it also requires that a points-to graph be calculated for every program point. Because of the exponential number

void fsTest(int x) {

      int a, b, c, d;
      int *p;

S1:   p = &a;
S2:   p = &b;

      if(x)
S3:   p = &c;

      if(x)
S4:   p = &d;

S5:   *p = 0;

}

(a) Sample program

(b) Flow Insensitive

(c) Flow Sensitive

(d) Path Sensitive

Figure 2.4: Sample code, and the resulting points-to graphs at program point $S5$ with various flavors of control-flow sensitivity.

of program points associated with an interprocedural analysis, achieving a scalable solution is difficult. A further and more ambitious approach is a path sensitive analysis, which requires that each non-cyclic path be analyzed individually; a means of correlating control dependences is required to prune away invalid control flow paths. Figure 2.4 shows a sample program and depicts the different points-to graphs created by the three different approaches.

A **context-sensitive** pointer analysis distinguishes between the different calling contexts of a procedure. The implication is that the points to side effects on any given path of the call graph are treated uniquely, in contrast to a context-insensitive pointer analysis which merges all the calling contexts of a procedure when summarizing the points-to side effects. We discuss the

```
int *retParam(int *x) {
    return x;
}

void csTest() {

    int a, b;
    int *p, *q;

S1:     p = retParam(&a);
S2:     q = retParam(&b);

S3:     *p = 0; *q = 0;

}
```

(a) Sample program

(b) Context Insensitive

(c) Context Sensitive

Figure 2.5: Sample code, and the resulting points-to graphs at program point $S3$ for various flavors of context sensitivity.

difficulties associated with a context sensitive analysis later in Section 2.1.7, and in Sections 2.2.2 and 2.2.3 we describe various methods that have been proposed for overcoming these difficulties. Figure 2.5 shows a sample program and depicts the two different points-to graphs created by a context insensitive and a context sensitive analysis.

A third factor affecting accuracy is **aggregate modeling** or **field-sensitivity**. Aggregate structures with multiple pointer type fields, such as arrays of pointers or C structs can be handled specially. Pointers within structs are merged and handled as a single pointer in a field-insensitive analysis. Conversely, a field-sensitive analysis handles every field and subsequently subfield as a separate and unique field. This approach additively requires a means of handling recursive data structures. Similarly, an array of pointers can be handled as a single node or each array element could be treated as a unique pointer field. Treating each array node uniquely

creates many complications because of array subscript uncertainty. A third option, proposed by Emami [26], is to represent arrays (both pointers and pointee targets) with two location sets. One of the location sets represents the first element of the array and the second element represents the remainder of the array. This added information enables a client analysis to be more aggressive when performing array subscript dependence analysis.

Heap modeling is another important, yet often neglected, design decision to be considered when performing any pointer analysis. A means of dealing with pointee targets created by dynamic allocation library calls such as `malloc` and `calloc` in C is required. The simplest and most inaccurate approach is to merge all heap allocation callsites into one location set. The most common approach is callsite allocation, which treats each individual callsite allocation program point as a unique location set target. This approach is considered to produce precise results in most cases; although, when a programmer uses a custom allocator [5, 11] it essentially degrades the analysis into a heap-merged approach. A more precise approach that addresses this custom allocator issue is to assign a unique location set for every unique context path to every allocation callsite [43, 50, 57]. Other aggressive shape analysis techniques have also been proposed. A shape analysis technique attempts to further disambiguate dynamic memory location sets by categorizing the underlying data structure created by the allocation callsite into a List, Tree, DAG, or Cyclic Graph [33].

Alias representation is yet another factor to be considered by any pointer analysis algorithm. Alias representation signifies what type of data structure is used to track aliasing information. There are two basic representation types:

- alias pairs, and

- points-to relations.

A complete alias-pairs representation stores all alias pairs explicitly, such as the one used by Landi [49]. A compact alias-pairs representation stores only basic alias pairs, and derives new

alias pairs by dereference, transitivity and commutativity. The most common representation is to store points-to relation pairs to indicate that one variable is pointing to another. Additionally, the representation can choose to distinguish between must-alias and may-alias pairs; this information is very useful for a flow-sensitive analysis that uses strong and weak updates. Figure 2.6 demonstrates a sample program created to illustrate the difference between the two representations. The flow sensitive pointer alias graph and points-to graph corresponding to program point $S5$ is shown in Figure 2.6 (b) and (c) respectively. If the compiler wanted to know, at program point $S5$, do the pointers p and q alias? The alias-pairs representation approach would output definitely *no*, whereas the point-to representation approach would answer with *maybe*. The alias analysis approach is considered to be slightly more precise but relatively much more expensive in terms of complexity.

2.1.7 Context Sensitivity Challenges

Context sensitivity is regarded as the most important contributor towards achieving an accurate pointer analysis. Conceptually, context sensitivity can be realized by giving each caller its own copy of the function being called. Two distinct approaches have been proposed:

- cloning-based analysis, and

- summary-based analysis.

The simplest way of achieving context sensitivity is through function cloning. In a cloning-based analysis every path through the call graph is cloned or conceptually inlined, and then a context-insensitive algorithm on the expanded call graph is performed. This expanded call graph is called an *invocation graph* [26]. This technique is a simple way of adding context sensitivity to any analysis. The problem with this technique is that the invocation graph, if not represented sensibly, can blow up exponentially in size.

void aliasRepTest() {

    int a, b, c;
    int *p, *q;

    if(x) {
S1:   p = &a;
S2:   q = &b;
    } else {
S3:   p = &b;
S4:   q = &c;
    }

S5:   *p = 0; *q = 1;

}

(a) Alias Representation Example

(b) Pointer Alias Graph

(c) Points-To Graph

Figure 2.6: An example illustrating the difference between using an alias graph representation or a points-to representation. Both graphs are flow sensitive and representative of the state of the pointers p and q at program point $S5$. This examples shows why an alias representation can be more accurate.

In a summary-based analysis, a summary (or a transfer function) which summarizes the effects of a procedure is obtained for each procedure. The algorithm usually has two phases: (i) a bottom-up phase in which the summaries for each procedure are collected using a reverse topological traversal of the call graph; and (ii) a top-down phase in which points-to information is propagated back down in a forward topological traversal through the call graph. This two phased approach is very advantageous for any analysis that can easily summarize the effects of a procedure. Unfortunately, for a full context-sensitive pointer analysis, this approach has not yet been shown to scale to large programs—mainly because it is difficult to concisely summarize the points-to side effects of a procedure. The difficulty lies in the inability to compute a safe and

```
void swap(int **x, int **y) {

        int *tmp;

S1:     tmp = *x;
S2:     *x = *y;
S3:     *y = tmp;


}
```

(a) swap source code

(b) swap summary function

Figure 2.7: An example that graphically demonstrates how to summarize the procedure swap found in (a).The procedure's transfer function is shown in (b). The shaded nodes represent unknown location sets (also called shadow variables) that are to be replaced when the transfer function is applied. The procedure swap is fully summarized with this linear transfer function. Obtaining a single linear transfer function is not always possible as shown in Figure 2.8.

precise one-to-one mapping (a linear transfer function) with regards to points-to graph input to output. Meaning, given any points-to set input, does there exist a single linear transfer function that can be used to summarize and compute the output. The examples found in Figure 2.7 and Figure 2.8 suggest why a full context-sensitive linear transfer function is difficult to achieve safely.

Figure 2.7(a) shows an example procedure named swap. This procedure takes in two pointer arguments (x and y) and effectively swaps their points-to targets. To summarize this procedure without knowing the points-to graph into the call of this procedure (as is the case for any summary-based approach), temporary place holder variables which we call *shadow variables* are used. A shadow variable is equivalent to Emami's invisible variable [26] or Bhowmik's dummy variable [7]. Pointer analysis is performed on the procedure and a summarizing points-to graph is built as shown in Figure 2.7(b). This points-to graph contains four shadow variables which

void alloc(int **x, int **y) {

S1:   *x = (int*)malloc(sizeof(int));

}

(a) alloc source code

(b) alloc summary function
(*x is not aliased to *y)

(c) alloc summary function
(*x definitely aliased to *y)

(d) alloc summary function
(*x might be aliased to *y)

Figure 2.8: An example that graphically demonstrates how to summarize the procedure `alloc` found in (a). The procedure's transfer functions for 3 different input cases are shown in (b), (c), and (d). For this example, (d) can be used as a safe linear transfer function for all 3 input cases, although this may introduce many spurious points-to relations.

are the shaded nodes within the points to graph. When analyzing a call to the `swap` procedure, the shadow variables within the transfer function are replaced with the input-parameter location set variables. The resulting transfer function represents the side effects obtained with a call to the `swap` procedure. This `swap` procedure is an ideal candidate procedure for a summary based approach because the transfer function is linear, meaning the side effects are independent of the input points-to graph. Figure 2.8 illustrates an example where this is not the case.

Figure 2.8(a) shows an example procedure named `alloc`. This procedure also takes in two pointer arguments (`x` and `y`), and the pointer `y` is allocated to the dynamic heap location set named `heap_s1`. Figure 2.8(b) illustrates the intuitive summarizing function which reassigns the

shadow pointer `*x` to point to the new location set `heap_s1` and the shadow pointer `*y` remains unchanged. Unfortunately, in the unlikely event that the shadow variables `*x` and `*y` are aliased (refer to the same variable) then this summarizing function is incorrect. In the event that `*x` and `*y` definitely alias, then the summarizing function found in Figure 2.8(b) should be applied, which additionally reassigns `*y` to point to `heap_s1`. In the event of a may-alias relation between `*x` and `*y`, Figure 2.8(c) depicts the transfer function that should be applied. This problem of aliasing input pointer arguments is referred to as Shadow Variable Aliasing (SVA). All summary based approaches must somehow solve this SVA problem.

The SVA problem stems from the use of call by reference parameters and global variables passing between procedures which introduce aliases, an effect where two or more l-values [1] refer to the same location set at the same program point. More precisely aliases created by multiple levels of dereferencing impede summary-based pointer analyses; i.e. pointers with k levels of dereferencing where $k > 1$ and k is the maximum level of pointer dereferencing. Ramalingam [58] shows that solving the may-alias problem for $k > 1$ level pointer is undecidable. However, if $k = 1$ the problem has been shown to be almost trivial [8, 49]. Most summary based approaches attempt to somehow map the $k = n$ problem into a $k = 1$ problem[3]. There are four known approaches to achieving this mapping.

1. Perform a one-level analysis n times [26].

2. For each incoming alias pattern, specialize each procedure summary for all possible input alias patterns [78]; this approach is called a Partial Transfer Function (PTF) approach.

3. Hoist and inline multi-level pointer assignment instructions from caller to callee until they can be resolved into a one-level pointer assignment instruction [9].

4. Summarize each procedure using a single safe [12] or unsafe [7] summary function.

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[3]n represents the maximum level of dereferencing found in the program

These approaches will be further discussed in Section 2.2.

2.2   Conventional Pointer Analysis Techniques

Pointer analysis is a well-researched problem and many algorithms have been proposed, yet no one approach has emerged as the preferred choice [44]. A universal solution to pointer analysis is prevented by the large trade-off between precision and scalability as described in Section 2.1.6. The most accurate algorithms are both context-sensitive and control-flow-sensitive (CSFS) [26,49,78]; however, it has yet to be demonstrated whether these approaches can scale to large programs. Context-insensitive control-flow-insensitive (CIFI) algorithms [2, 65] can scale almost linearly to large programs, but these approaches are regarded as overly conservative and may impede aggressive compiler optimization. Several approaches balance this trade-off well by providing an appropriate combination of precision and scalability [6, 22, 77, 80], although their relative effectiveness when applied to different aggressive compiler optimizations remains in question.

2.2.1   Context-insensitive Flow-insensitive (CIFI) Algorithms

The simplest and most common types of pointer analysis algorithms are typically both context-insensitive and flow-insensitive (CIFI). Not surprisingly, there exists many different types of CIFI algorithms. Currently, most commercially available compilers simply use an address-taken CIFI pointer analysis. This type of analysis simply asserts that every object that has its address taken may be the target of any pointer object. This is obviously a very imprecise approach, although it is quite effective at improving the programs performance when applied to a client optimization relative to using no pointer analysis [44].

The two classic approaches to realizing an efficient context-insensitive flow-insensitive (CIFI) analysis are:

- inclusion based, and

- unification based.

An inclusion based analysis treats every pointer assignment instruction as a constraint equation and iteratively solves for the points-to graph by repeatedly performing a transitive closure on the entire graph, until a fixed solution is realized. Andersen [2] initially proposed a simple and intuitive CIFI inclusion based pointer analysis with worst case complexity of $O(n^3)$; where n is the size of the program. Fahndrich, et al [27] improved on the runtime of Andersen's pointer analysis by collapsing cyclic constraints into a single node, and by selectively propagating matching constraints. With this optimized approach, they demonstrated orders of magnitude improvement in analysis runtime. Rountev and Chandra [61] also proposed a similar and more aggressive algorithm for collapsing cyclical constraints and avoiding duplicate computations.

A unification based approach is a much more scalable, yet slightly less accurate approach to performing a FICI analysis. In a unification based approach, location set nodes within the points-to graph are merged such that every node within the graph can only point to a single other node. This merge operation eliminates the transitive closure operation required by the inclusion based approach and therefore allows for linear scaling. Steensgaard [65] proposed the first unification based algorithm, which runs in almost linear time. The algorithm uses Tarjan's union-find data structure [73] to efficiently perform the analysis. The disadvantage of a unification approach is that it may introduce many spurious point-to relations as is the case in the example found in Figure 2.9. In this example, the unification based approach creates the following spurious (false) points-to relations: $\langle *r, b \rangle$, $\langle *s, a \rangle$, and $\langle *q, b \rangle$.

2.2.2   Context-sensitive Flow-sensitive (CSFS) Algorithms

The most precise pointer analysis algorithms are typically both context-sensitive and flow-sensitive (CSFS). Emami, Ghiya, and Hendren [26] introduced the invocation graph and pro-

void main() {

    int **p, *q, *r, *s;
    int a, b;

S1:   q = &a;
S2:   p = &r;
S3:   r = &a;
S4:   p = &s;
S5:   s = &b;

}

(a) Sample program

(b) address-taken

(c) inclusion based

(d) unification based

Figure 2.9: Different types of flow insensitive points-to analyses. All three points-to graphs are flow insensitive and therefore safe for all program points.

posed a cloning-based CSFS points-to analysis for `C` programs. The analysis was designed to be very accurate for disambiguating pointer references that access stack based memory locations. The analysis is field sensitive and maps each stack variables to a unique location set, including locals, parameters, and globals. Arrays are modeled as two location sets, one representing the first element of the array and the other representing the rest of the array. The heap is modeled inaccurately and allocated a single location set. Pointers through indirect calls are handled iteratively using a transitive closure. Additionally, the analysis tracks both may and must points-to relationships and use both strong and weak updates while propagating a flow sensitive points-to graph. Similar to a summary-based approach, Emami's approach caches the side-effects of each function call for reuse. The shadow variable aliasing problem was handled by repeating their one-level analysis n times, where n is the maximum level of dereferencing used. Although

this analysis dramatically improves the precision in relation to the CIFI analysis, the programs analyzed were very small and in general the analysis is not expected to scale to larger programs.

Wilson and Lam [78] proposed a summary based CSFS analysis that recognized that many of the calling contexts were quite similar in terms of alias patterns between the parameters. As such, they proposed the use of a partial transfer function (PTF) to capture the points-to information for each procedure. A PTF is a set of linear transfer functions representative of all possible aliasing scenarios that a given procedure may encounter. The PTF does this by using temporary location sets to represent the initial point-to information of parameters, and uses the PTF to derive the final point-to information of the procedure. For the benchmarks used, they showed only a 30% increase in the number of transfer functions needed. This analysis also allocated each stack variable and global a unique location set and, in addition, they mapped each heap allocation callsite to a unique location set. This pointer analysis was shown to run on benchmarks as large as five thousand lines of code.

Chaterjee, Ryder, and Landi [12] proposed a modular context-sensitive pointer analysis, using the same static memory model as Wilson and Lam [78], but fully summarizing each procedure using a single safe transfer function. By detecting strongly connected components in the call graph, and analyzing them separately they showed space improvements, but no results on benchmarks larger than 5000 lines of code. Cheng and Hwu [15] extended [12] by implementing an access path based approach, and partial context-sensitivity, distinguishing between the arguments at only selected procedures. They demonstrated scalability to hundreds of thousands of lines of code.

2.2.3   Context-sensitive Flow-insensitive (CSFI) Algorithms

Fahndrich, Rehof, and Das [28] proposed a one-level unification based CSFI pointer analysis. This analysis distinguishes between the incoming calls to a given procedure, rather than expand

each path in the call graph. The authors showed that this analysis can analyze hundreds of thousands of lines of code in minutes, and showed precision improvements over the flow-insensitive context-insensitive unification based analysis. Das also proposed a summary-based generalized one level flow (GOLF) [22] analysis which adds limited context sensitivity to his unification based CIFI one level flow idea from [21]. GOLF is field-insensitive and uses callsite allocation. It is regarded as a relatively precise algorithm that scales well beyond the SPECint95 [19] benchmarks.

Some progress has also been made recently using a *binary decision diagram* (BDD) [6,77,80] data structure to efficiently solve the pointer analysis problem accurately. A BDD based approach uses Bryant's [10] Reduced Order Binary Decision Diagram (ROBDD, also called BDD) to solve the pointer analysis problem using superposition. Similar to the dynamic programming paradigm, a BDD is able encode and solve exponential problems, on average (and not in the worst case), in linear time if repetition or duplication exists. The CSFI pointer analysis problem is formulated using a cloning based approach and the invocation graph is encoded using boolean logic into a BDD. The problem is then solved quite efficiently because much of the analysis on an invocation graph involves analyzing duplicated context paths.

2.3 Speculative Optimization

Studies suggest that a more accurate pointer analysis does not necessarily provide increased optimization opportunities [22, 23, 44] because ambiguous pointers will persist. The fact that memory references often remain ambiguous even after performing a thorough pointer analysis has motivated a class of compiler-based optimizations called *data speculative optimizations*. A data speculative optimization typically involves a code transformation that allows ambiguous memory references to be scheduled in a potentially unsafe order, and requires a recovery mechanism to ensure program correctness in the case where the memory references were indeed

dependent. Speculative optimizations benefit from pointer analysis information that quantifies the likelihood of data dependences and is therefore the target use of the probabilistic pointer analysis proposed by this thesis. Proposed forms of data speculative optimization can be categorized into two classes: (1) Instruction Level Data Speculation and (2) Thread Level Speculation (TLS). Instruction level data speculation attempts to increase the available instruction level parallelism or eliminate unnecessary computation by using potentially unsafe compiler transformations. Thread level speculation allows the compiler to speculatively parallelize sequential programs without proving that it is safe to do so.

2.3.1   Instruction Level Data Speculation

Due to the complexity of scaling the conventional superscalar out-of-order execution paradigm, the processor industry started to re-examine instruction sets which explicitly encode multiple operations per instruction. The main goal is to move the complexity of dynamic scheduling of multiple instructions from the hardware implementation to the compiler, which does the instruction scheduling statically. One main drawback to this approach is that to extract instruction level parallelism statically, information regarding aliasing memory references becomes essential. As described in Section 2.2, even if the most accurate techniques are used ambiguous pointers will persist. As a direct consequence, HP and Intel have recently introduced a new style of instruction set architecture called EPIC (Explicitly Parallel Instruction Computing), and a specific architecture called the IPF (Itanium Processor Family). EPIC is a computing paradigm that began to be researched in the 1990s. EPIC instruction sets (eg., Intel's IA64) provide hardware support that allows the compiler to schedule a load ahead of a potentially-dependent store, and to specify recovery code that is executed in the event that the execution is unsafe [24, 45].

The IPF uses an Advanced Load Address Table (ALAT) to track possibly dependent load and store operations at runtime. The ALAT is a set-associative structure accessed with load/store

addresses. The IA64 instruction set provides four instructions that interface with the ALAT to enable many different kinds of speculative optimization: a speculative load (`ld.s`), a speculation check (`chk.s`) an advanced load (`ld.a`), and an advanced load check (`chk.a`). Figure 2.10 illustrates a simple and relevant example on how data speculative optimization can be realized. In this example, if the compiler is able to determine with some amount of probabilistic certainty that pointers p and q do not alias, then it could be beneficial to eliminate the redundant load instruction and schedule any other dependent instructions earlier to increase the instruction level parallelism. To realize this speculative optimization an advanced load (`ld.a`) instruction is used in place of a normal load, then an advanced load check (`chk.a`) instruction that specifies a recovery branch if p and q where to indeed alias at runtime.

Proposed and successful speculative optimizations that allow the compiler to exploit this new hardware support include speculative dead store elimination, speculative redundancy elimination, speculative copy propagation, speculative register promotion, and speculative code scheduling [18,20,51,52,55]. Other envisioned and promising potential speculative optimizations include speculative loop invariant code motion, speculative constant propagation, speculative common sub-expression elimination and speculative induction variable elimination. To drive the decision of when it is profitable to speculate, many of these techniques rely on extensive data dependence profile information [79] which is expensive to obtain, sensitive to the input set used, possibly inaccurate, and often unavailable. The main drawback is that this type of profiling can be very expensive since every memory reference needs to be monitored and compared pair-wise to obtain accurate results. Lin, et al [51, 52] proposed a lower cost alias profiling scheme to estimate the alias probability and in addition, when alias profiling is unavailable, they used a set of effective, and arguably a conservative, set of heuristic rules to quickly approximate alias probabilities in common cases.

```
                                              ld.a w = [p];

                                              y = w;
                                              z = y * 2;
                                              ...

              w = *p;
                                              st [q] = x;

              *q = x;
                                              chk.a w, recover;

              y = *p;
                                              recover:
                                                  ld y = [p];
              z = y * 2;                             z = y * 2;
              ...                                    ...

          (a) original program              (b) speculative version
```

Figure 2.10: Example on an instruction level data speculative optimization using the EPIC instruction set. If the compiler is fairly confident that the pointers p and q do not alias, then the redundant load instruction y = *p can be eliminated speculatively and replaced with a copy instruction y = w. The copy instruction can then be hoisted above the store *q = x to increase ILP and enable other optimizations. To support this optimization, extra code for checking and recovering in the event of failed speculation is inserted (shown in bold).

2.3.2   Thread Level Speculation

Performance gained through instruction level data speculation, although promising, is ultimately limited by the available instruction level parallelism [46]. To overcome this limit, techniques to exploit more parallelism by extracting multiple threads out of a single sequential program remains an open and interesting research problem. This parallelism, called Thread-Level Parallelism (TLP), requires the compiler to prove statically that either: (a) the memory references within the threads it extracts are indeed independent; or (b) the memory references within the threads are definitely dependent and must be synchronized. Ambiguous pointers are again problematic. To compensate for these shortcomings, aggressive speculative solutions have been

Figure 2.11: TLS program execution, when the original sequential program is divided into two epochs (E1 and E2).

proposed to assist the compiler.

Hardware-supported techniques, such as thread-level speculation (TLS) [40, 47, 60, 69] and transactional programming [37, 38] allow the speculative parallelization of sequential programs through hardware support for tracking dependences between speculative threads, buffering speculative modifications, and recovering from failed speculation.

TLS allows the compiler to automatically parallelize general-purpose programs by supporting parallel execution of threads, even in the presence of statically ambiguous data dependences. The underlying hardware ensures that speculative threads do not violate any dynamic data dependence and buffers the speculative data until it is safe to be committed. When a dependence violation occurs, all the speculative data will be invalidated and the violated threads are re-executed using the correct data. Figure 2.11 demonstrates a simple TLS example.

The sequential program in Figure 2.11(a), is sliced into two threads of work (called epochs), and labeled as E1 and E2 in Figure 2.11(b) and (c). They are then executed speculatively in parallel, even though the addresses of the pointers p and q are not known until runtime. In essence, TLS allows for the general extraction of any potentially available thread-level paral-

lelism. A read-after-write (true) data dependence occurs when `p` and `q` both point to the same memory location. Since the store produces data that will be read by the dependent load, these store and load instructions need to be executed in the original sequential program order. In Figure 2.11(b), `p` and `q` do not point to the same location. Therefore, speculation is successful and both speculative threads can commit their results at the end of execution. However, as shown in Figure 2.11(c), in the unlucky event that both pointers point to the same location, a true data dependence is detected and a violation occurs. In this case, speculation fails because it leads to an out-of-order execution of the dependent load-store pair, which violates the sequential program order. The offending thread is halted and re-executed with the proper data.

There are various proposed thread-level speculation systems that aim at exploiting thread-level parallelism in sequential programs by using parallel speculative threads. Among all different types of thread-level speculation systems that have been proposed, there are three common key components: (i) breaking a sequential program into speculative threads—this task must be done efficiently to maximize thread-level parallelism and minimize the overhead; (ii) tracking data dependences—since the threads are executed speculatively in parallel, the system must be able to determine whether the speculation is successful; (iii) recovering from failed speculation—in the case when speculation has failed, the system must repair the incorrect architectural states and data, and discard the speculative work. Different TLS systems have different implementations of these three components.

In the Multiscalar architecture [31, 32, 75], small tasks are extracted by the compiler and distributed to a collection of parallel processing units under the control of a centralized hardware sequencer. The architecture originally used an Address Resolution Buffer (ARB) [30] to store speculative data and to track data dependences. The ARB was later succeeded by the Speculative Versioning Cache (SVC) [36] which used a different cache coherence scheme to improve memory access latency. The Hydra chip multiprocessor [39, 40] uses a secondary cache write buffer and additional bits that are added to each cache line tag to record speculation

states. Using these components data dependences are tracked and violation detection occurs before a thread commits. Similarly, Stampede TLS [3, 4, 66–69] leverages cache coherence to detect failed dependence violations. The compiler, with the assistance of feedback-driven profiling information, decides which part of the program to speculatively parallelize. Unlike the other two approaches, Stampede TLS does not use any special buffer for storing speculative data. Instead, both speculative and permanently committed data is stored in the cache system and an extended invalidation based cache coherence scheme is used to track the speculative state of the cache lines [66]. All of the various techniques have different tradeoffs in terms of violation penalties, dependence tracking overhead, and scalability making a general compilation framework or approach that much more difficult to realize.

Similar to the speculative optimizations that target Instruction Level Parallelism, the decision of when it is profitable to speculate when compiling for any TLS infrastructure remains a nontrivial task. All techniques rely on extensive profile information which is expensive to obtain, possibly inaccurate, and often unavailable for real world applications. Section 2.4 will discuss some of the existing work on compiler analyses intended to aid speculative optimization.

2.4 Static Analyses Targeting Speculative Optimizations

In this section, related work in the field of static program analysis for speculative optimization is described. Speculative optimizations have recently been proposed to bridge this persistent disparity between safety and optimization. As described in the previous section, deciding when to use speculative optimization remains an important and open research problem. Most proposed techniques rely on extensive data dependence profile information, which is expensive to obtain and often unavailable. Traditional pointer analysis and dependence analysis techniques are inadequate because none of these conventional techniques quantify the probability or likelihood that memory references alias. Recently, there have been some studies on speculative alias

analysis and probabilistic memory disambiguation targeting speculative optimization.

Ju et al. [17] presented a probabilistic memory disambiguation (PMD) framework that quantifies the likelihood that two array references alias by analyzing the array subscripts. The probability is calculated using a very simple heuristic based on the overlap in the equality equation of the array subscripts. Their framework uses an intuitive profitability cost model to guide data speculation within their compiler. Their experimental results showed a speedup factor of up to `1.2x`, and a `3x` slowdown if unguided data speculation is used. This approach may be sufficient for speculating when array references are encountered, but this approach is not applicable to pointers.

Chen et al. [13, 14] recently developed the first and only other CSFS probabilistic point-to analysis algorithm that computes the probability of each point-to relation. Their algorithm is based on an iterative data flow analysis framework, which is slightly modified so that probabilistic information is additionally propagated. As described in section 2.1.6, an iterative data flow framework requires that points-to information be propagated until a fixed point solution is found; which is known not to scale when used interprocedurally. It is also unclear if a fixed point solution is guaranteed to occur with their proposed probabilistic framework, which is no longer a monotonic [56] framework. Their approach optionally uses control-flow edge profiling in order to compute the various control dependence probabilities. Interprocedurally, their approach is based on Emami's algorithm [26], which is a cloning-based approach that requires an invocation graph. No strategy for caching and reusing shadow variable aliasing information was described in their research. Their experimental results show that their technique can estimate the probabilities of points-to relationships in benchmark programs with reasonably small errors, although they model the heap as a single location set and the test benchmarks used are all relatively small.

Fernandez and Espasa [29] proposed a pointer analysis algorithm that targets speculation by relaxing analysis safety. The key insight is that such unsafe analysis results are acceptable

because the speculative optimization framework can tolerate them, converting a safety concern into a performance concern. They gave some experimental data on the precision and the misspeculation rates in their speculative analysis results. Finally, Bhowmik and Franklin [7] present a similar unsafe approach that uses linear transfer functions in order to achieve scalability. Their approach simply ignores the shadow variable aliasing problem in order to realize a single transfer function for each procedure. Unfortunately, neither of these last two approaches provide the probability information necessary for computing cost/benefit trade-offs for speculative optimizations.

2.5 Summary

This chapter described the terminology and basic concepts in the fields of pointer analysis and pointer analysis for speculative optimization. Different approaches to solving the pointer analysis problem were presented and contrasted. The tradeoffs and challenges involved in designing a pointer analysis algorithm were also identified and discussed. We argued that the traditional approaches to pointer analysis are overly conservative because ambiguous pointers will always persist. Speculative optimizations, which relax safety constraints, were motivated to alleviate the inadequacies of traditional pointer analysis approaches. Finally, the chapter also surveyed related work in the new research area of pointer analysis for speculative optimization, which is closely related to work presented in this dissertation. The next chapter will introduce an innovative and scalable probabilistic pointer analysis algorithm that targets speculative optimization.

Chapter 3

A Scalable PPA Algorithm

This chapter describes our PPA algorithm in detail. We begin by showing an example program. We then give an overview of our PPA algorithm, followed by the matrix framework that it is built on. Finally, we describe the bottom-up and top-down analyses that our algorithm is composed of.

3.1   Example Program

For the remainder of this chapter, the example program in Figure 3.1 will be used to illustrate the operation of our algorithm. This example was created to quickly demonstrate the inter-procedural nature of the analysis. Simple pointer assignment instructions are blended with different types of control dependences and procedure calls to illustrate how path frequency feedback is used to extract points-to probability information. In this sample program there are three pointer variables (a, b, and tmp) and two variables that can be pointed at (x and y), each of which is allocated a unique location set. We assume that edge profiling indicates that the `if` statement at $S3$ is taken with probability 0.9, the `if` statement at $S13$ is taken with a probability of 0.01, and the `while` loop at $S5$ iterates exactly 100 times (note that our

```
        int x, y;
        int *a, *b;                                              void f() {
                                                                    int *tmp;
        void main() {
                                                      S9:      tmp = b;
S1:        a = &x;                                    S10:     b = a;
S2:        b = &y;                                    S11:     a = tmp;
                                                      S12:     g();
S3:        if(...)                                             }
S4:            f();

S5:        while(...) {                                        void g() {
S6:            g();                                   S13:     if(...)
S7:            ... = *b;                               S14:         a = &x;
S8:            *a = ...;                                        }
           }
        }
```

Figure 3.1: In this sample program, global pointers a and b are assigned in various ways. The `if` statement at $S3$ is taken with probability 0.9, the `if` statement at $S13$ is taken with a probability of 0.01, and the `while` loop at $S5$ iterates exactly 100 times.

algorithm can proceed using heuristics in the absence of profile feedback). It is important to note, that our algorithm currently assumes that all control dependences are independent and that their frequencies are mutually exclusive. Initially a and b are assigned to the addresses of x and y respectively. The function $f()$ is then potentially called, which effectively swaps the pointer values of a and b and then calls the function $g()$. The function $g()$ potentially assigns the pointer a to the address of x, depending on the outcome of the `if` at $S13$ which is taken 1% of the time.

An optimizing compiler could target the loop at $S5$ as a potential candidate for optimization. However, knowledge about what variables are being dereferenced at $S7$ and $S8$ is required to safely optimize. If both instructions always dereference the same location (i.e., $*a == *b$), the dereferences can be replaced by a single temporary variable. Conversely, if the dereference tar-

gets are always different and also loop invariant then the corresponding dereference operations can be hoisted out of the loop. If the compiler cannot prove either case to be true, which is often the result in practice because of the difficulties associated with pointer analysis, then it must be conservative and halt any attempted optimization. In this particular example, neither optimization is possible. However, it would be possible to perform either optimization speculatively, so long as the optimized code was guarded with a check and recovery mechanism [52]. To decide whether a speculative optimization is desirable, we require the probabilities for the various points-to relations at $S7$ and $S8$.

3.2 Algorithm Overview

The main objective of our probabilistic pointer analysis is to compute, at every program point s the probability that any pointer α points to any addressable memory location β. More precisely, given every possible points-to relation $\langle \alpha, *\beta \rangle$, the analysis is able to solve for the probability function $\rho(s, \langle \alpha, *\beta \rangle)$ for all program points s. The expected probability is defined by the following equation

$$\rho(s, \langle \alpha, *\beta \rangle) = \frac{E(s, \langle \alpha, *\beta \rangle)}{E(s)} \tag{3.1}$$

where $E(s)$ is the expected runtime execution frequency associated with program point s and $E(s, \langle \alpha, *\beta \rangle)$ is the expected frequency for which the points-to relation $\langle \alpha, *\beta \rangle$ holds dynamically at the program point s [13]. Intuitively, the probability is largely determined by the control-flow path and the procedure calling context to the program point s—hence an approach that is both control-flow and context sensitive[1] will produce the most probabilistically accurate results.

---

[1]A *context-sensitive* pointer analysis distinguishes between the different calling contexts of a procedure, and a *control-flow-sensitive* pointer analysis takes into account the order in which statements are executed within a procedure.

(a) Conventional points-to graph.   (b) Probabilistic points-to graph

Figure 3.2: This is a points-to graph and the corresponding probabilistic points-to graph associated with the program point after $S4$ and initially into $S5$ (P_{S5}) in the example found in figure 3.1. A dotted arrow indicates a maybe points-to relation whereas a solid arrow denotes a definite points-to relation. `UND` is a special location set used as the sink target for when a pointer's points-to target is undefined.

To perform pointer analysis, we must first construct an abstract representation of addressable memory called a *static memory model.* For our PPA algorithm the static memory model is composed of *location sets* [78].[2] A location set can represent one or more real memory locations, and can be classified as a pointer, pointer-target, or both. A location set only tracks its approximate size and the approximate number of pointers it represents, allowing us to abstract away the complexities of aggregate data structures and arrays of pointers. For example, fields within a `C` struct can either be merged into one location set, or else treated as separate location sets. Such options give us the flexibility to explore the accuracy/complexity trade-off space without modifying the underlying algorithm. We also define a special location set called `UND` as the sink target for when a pointer's points-to target is undefined.

Since a location set can be a pointer, a location set can point to another location set. Such a relation is called a *points-to* relation, and the set of all such relations is called a *points-to graph*—a directed graph whose vertices represent location sets. Specifically, a directed edge

---

[2]Note that our algorithm itself is not necessarily dependent on this location-set-based model.

from vertex α to vertex β indicates that the pointer α may point to the target β. In a flow-sensitive analysis where statement order is considered when computing the points-to graph, every point in the program may have a unique points-to graph. Our PPA algorithm computes a probabilistic points-to graph, which simply annotates each edge of a regular points-to graph with a weight representing the probability that the points-to relation will hold. Figure 3.2(a) shows an example points-to graph based on the example code given in Figure 3.1, and Figure 3.2(b) gives the corresponding annotated probabilistic points-to graph. It is important to note that the sum of all outgoing edges for a given vertex is always equal to one in order to satisfy Equation 3.1.

To perform an inter-procedural context-sensitive analysis, and to compute probability values for points-to relations, our PPA algorithm also requires as input an *interprocedural control flow graph* (ICFG) that is decorated with expected runtime frequency; we explain the construction of the ICFG in greater detail later in Chapter 4. The ICFG is a representation of the entire program that contains the control flow graphs for all procedures, connected by the overall call graph. Furthermore, all control-flow and invocation edges in the ICFG are weighted with their expected runtime frequency. These edge weights can be obtained through the use of simple edge profiling (eg., the output from `gprof`) or by static estimation based on simple heuristics.

Because our analysis is a control flow-sensitive analysis, every point s the program is said to have a probabilistic points-to graph denoted P_s. Given a second point in the program $s\prime$ such that a forward path from s to $s\prime$ exists, the probabilistic points-to graph $P_{s\prime}$ can be computed using a transfer function that represents the changes in the points-to graph that occur on the path from s to $s\prime$, as formulated by

$$P_{s\prime} = f_{s \to s\prime}(P_s). \tag{3.2}$$

Figure 3.3: Fundamental PPA Equation

3.3   Matrix-Based Analysis Framework

As opposed to conventional pointer analyses which are set-based and can use analysis frameworks composed of bit vectors or BDDs, a PPA requires the ability to track floating-point values for the probabilities. Conveniently, the probabilistic points-to graph and transfer functions can quite naturally be encoded as matrices, although the matrix formulation in itself is not fundamental to the idea of PPA. The matrix framework is a simple alternative to propagating frequencies in an iterative data flow framework [59]. We choose matrices for several reasons: (i) matrices are easy to reason about, (ii) they have many convenient and well-understood properties, and (iii) optimized implementations are readily available. Our algorithm can now build on two fundamental matrices: a *probabilistic points-to matrix P*, and a *linear transformation matrix T*. Thus we have the fundamental PPA equation

$$P_{out} = T_{in \to out} \times P_{in}. \tag{3.3}$$

One key to the scalability of our algorithm is the fact that the transformation matrix is linear, allowing us to compute the probabilistic points-to graph at any point in the program by simply performing matrix multiplication—we do not require the traditional data flow framework used by other flow-sensitive approaches [13].

3.3.1   Points-to Matrix

We encode a probabilistic points-to graph using an $N \times M$ points-to matrix where M is the number of location set vertices that can be pointed at, and N is the number of pointer location sets plus the number of target location sets—therefore the vertices that act both as pointers and pointee-targets have two matrix row entries and are hence counted twice. The following equation formally defines the points-to matrix format:

$$P_s = \begin{bmatrix} p_{1,1} & \cdots & p_{1,M} \\ p_{2,1} & \cdots & p_{2,M} \\ \vdots & \ddots & \vdots \\ p_{N,1} & \cdots & p_{N,M} \end{bmatrix}$$

(3.4)

$$p_{i,j} = \begin{cases} \rho(s, \langle i\prime, j\prime \rangle) & i \leq N - M \\ 1 & i > N - M \text{ and } i = j + (N - M) \\ 0 & \text{otherwise} \end{cases}$$

The rows 1 to $N - M$ are reserved for the pointer locations sets and the rows $N - M + 1$ to N are reserved for the target location sets. To determine the row associated with a given pointer or pointee variable, the `row_id($\alpha$)` function is used. Given a pointer variable α, the function `row_id($\alpha$)` is equal to the matrix row mapped to the pointer α and the function `row_id(&$\alpha$)` is equal to the matrix row mapped to the address of α. A pointer with n levels of dereferencing is additionally mapped to n different rows, one row for each shadow variable represented in every level the pointer can be dereferenced. For a points-to matrix, pointer-target location sets are mapped to their corresponding column number by computing `row_id(&$\alpha$)` $- (N - M)$. The inner matrix spanning rows 1 to $N - M$ fully describes the probabilistic points-to graph; the other inner matrix spanning rows $N - M + 1$ to N is the identity matrix, and is only included in

order to satisfy the fundamental PPA equation. Finally, but crucially, the matrix is maintained such that every row within the matrix sums to one—allowing us to treat a row in the matrix as a probability vector $\overrightarrow{P}$.

Example Consider the points-to graph depicted in Figure 3.2. We assume that `row_id(a) = 1`, `row_id(b) = 2`, `row_id(tmp) = 3`, `row_id(&x) = 4`, `row_id(&y) = 5`, and `row_id(UND) = 6`. We also assume that the columns correspond to x, y, and `UND` respectively. This produces the corresponding points-to matrix:

$$
P_{S5} = \quad
\begin{array}{c}
\\ a \\ b \\ tmp \\ x \\ y \\ \text{UND}
\end{array}
\begin{array}{ccc}
x & y & \text{UND} \\
\left[\begin{array}{ccc}
0.101 & 0.899 & 0 \\
0.9 & 0.1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]
\end{array}
$$

This points-to matrix indicates that at the program point after $S4$ and initially into $S5$ (P_{S5}): (1) a points to x and y with probabilities 0.101 and 0.899 respectively; (2) b points to x and y with probabilities 0.9 and 0.1 respectively; and (3) `tmp` is in an undefined state.

3.3.2 Transformation Matrix

A transfer function for a given points-to matrix is encoded using an $N \times N$ transformation matrix, where N is the number of pointer location sets plus the number of target location sets. Each row and column is mapped to a specific location set using the equivalent `row_id(`α`)` function. Transformation matrices are also maintained such that the values in every row always sum to one. Given any possible instruction or series of instructions, there exists a transformation matrix that satisfies Equation 3.3. If a statement has no effect on the probabilistic points-to graph, then

the corresponding transformation matrix is simply the identity matrix. The following sections describe how transformation matrices are computed.

3.4   Representing Assignment Instructions

In any C program there are four basic ways to assign a value to a pointer, creating a points-to relation:

1. address assignment: $a = \&b$;

2. pointer assignment: $a = b$;

3. load assignment: $a = *b$;

4. store assignment: $*a = b$.

For each of the four cases there exists a corresponding transformation matrix. Types (1) and (2) generate a safe transformation matrix, whereas types (3) and (4) are modeled using a one-level unsafe transformation. The dereferenced target that is introduced in type (3) or (4) is modeled as a shadow variable and any ensuing shadow variable aliasing is ignored (for now), which is of course unsafe. Since speculative optimizations do not necessitate safety we exploit the safety requirement when multi-level pointer assignment instructions are encountered. Safety can be added, if it is a requirement, by using some other lightweight alias analysis (this will be discussed further in Section 3.8). For each of the four cases, a transformation matrix is computed using the following equation:

$$T_{[\alpha=\beta,p]} = \begin{bmatrix} t_{1,1} & t_{1,2} & \dots & t_{1,N} \\ t_{2,1} & t_{2,2} & \dots & t_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ t_{N,1} & t_{N,2} & \dots & t_{N,N} \end{bmatrix}$$

$$(3.5)$$

$$t_{i,j} = \begin{cases} p & i = \texttt{row\_id}(\alpha) \text{ and } j = \texttt{row\_id}(\beta) \\ 1-p & i = j = \texttt{row\_id}(\alpha) \\ 1 & i = j \text{ and } i \neq \texttt{row\_id}(\alpha) \\ 0 & \text{otherwise} \end{cases}$$

In this equation, α represents the pointer location set on the left side of the assignment and β denotes the location set (pointer or target) on the right side of the assignment. The probability value p represents the binding probability for the transformation and it is equal to 1 divided by the approximate number of pointers represented by the pointer location set α as defined in Equation 3.6. A pointer location set can represent multiple pointers. Such a location is required to represent the following cases: (1) an array of pointers; (2) pointers within recursive data structures are statically modeled as a single location set; (3) pointers within C structs are merged when a field-insensitive approach is utilized; and (4) shadow variable aliasing (as described in Section 3.8). A heuristic is used to approximate how many pointers are represented by a given pointer location set if this information can not be determined statically. A probability of $p = 1$ is equivalent to a *strong update* used in a traditional flow-sensitive pointer analysis, where as a probability of $p < 1$ is representative of a *weak update*.

$$p = \frac{1}{\text{approx \# of pointers in } \alpha} \tag{3.6}$$

It is important to note that the transformation matrix used for pointer assignment instructions is simply the identity matrix, with the exception of one row that represents the left side of the assignment.

Example The transformation matrices corresponding to the pointer assignment statements $S1$ and $S10$ from Figure 3.1 are:

$$T_{S1} = T_{[a=\&x,1.0]} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{S10} = T_{[b=a,1.0]} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

To compute the points-to matrix at $S2$ we use T_{S1} and the fundamental PPA equation as follows:

$$P_{S2} = T_{S1} \cdot P_{S1}$$

$$
\begin{array}{c}
\\
a\\
b\\
tmp\\
x\\
y\\
\text{UND}
\end{array}
\begin{array}{ccc}
x & y & \text{UND}\\
\end{array}
\begin{bmatrix}
1 & 0 & 0\\
0 & 0 & 1\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 1\\
0 & 0 & 1\\
0 & 0 & 1\\
1 & 0 & 0\\
0 & 1 & 0\\
0 & 0 & 1
\end{bmatrix}
$$

The resulting points-to matrix at $S2$ shows that the points to relation $\langle a, \&x \rangle$ exists with probability 1, while all other pointers (b and tmp) are undefined.

3.5 Representing Basic Blocks

For a basic block with a series of instructions $S1 \ldots Sn$ whose individual transformation matrices correspond to $T_1 \ldots T_n$, we can construct a single transformation matrix that summarizes the entire basic block using the following:

$$
T_{bb} = T_n \cdot \ldots \cdot T_2 \cdot T_1. \tag{3.7}
$$

Therefore, given any points-to matrix at the inbound edge of basic block, the points-to matrix at the outbound edge can be computed simply by performing the appropriate matrix multiplications. Note that the construction of a transformation matrix is a backward-flow analysis: to solve for the transformation matrix that summarizes an intraprocedural path from s to $s\prime$, the analysis starts at $s\prime$ and traverses backwards until it reaches s.

Example The basic block that contains statements $S1$ and $S2$ from Figure 3.1 can be summarized as:

$$
T_{bb(S1-S2)} = T_{S2} \cdot T_{S1}
$$

$$
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Assume that we are given the points-to matrix for the start of the basic block at $S1$ in Figure 3.1; also assume that all pointers are undefined at that point. The points-to matrix at the end of the basic block (i.e., at $S3$) can be computed as follows:

$$
P_{S3} = T_{bb(S1-S2)} \cdot P_{S1} =
\begin{array}{c}
\\
a \\
b \\
tmp \\
x \\
y \\
\mathtt{UND}
\end{array}
\begin{array}{ccc}
x & y & \mathtt{UND} \\
\end{array}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

The resulting points-to matrix indicates that at $S3$ the points-to relations $\langle a, \&x \rangle$ and $\langle b, \&y \rangle$ exist with a probability of 1.

3.6   Representing Control Flow

The main objective of this matrix-based PPA framework is to summarize large regions of code using a single transformation matrix. To summarize beyond a single basic block, our transformation matrices must be able to represent control flow. Recall that the construction of a

(a) Forward edge   (b) Back-edge, outside   (c) Back-edge, inside

Figure 3.4: Control flow possibilities

transformation matrix proceeds backwards, from $s\prime$ backwards to s. When the start of a basic block is encountered during this backwards analysis, the analysis must categorize the basic block's incoming edges. For now we consider the following three non-trivial cases for each edge:

1. the edge is a forward edge (Figure 3.4(a));

2. the edge is a back-edge and $s\prime$ is *outside* the region that the back-edge spans (Figure 3.4(b));

3. the edge is a back-edge and $s\prime$ is *within* the region that the back-edge spans (Figure 3.4(c)).

The following considers each case in greater detail.

3.6.1 Forward Edges

When there exists a single incoming forward edge from another basic block the transformation matrix that results is simply the product of the current basic blocks transformation matrix and the incoming blocks transformation matrix. When there are exactly two incoming, forward edges from basic blocks γ and δ, we compute the transformation matrix as follows:

$$T_{if/else} = p \cdot T_{\gamma} + q \cdot T_{\delta} \tag{3.8}$$

T_γ and T_δ represent the transformation matrices from the program point s to the end of each basic block γ and δ respectively. The scalar probability p represents the fan-in probability from basic block γ, and q represents the fan-in probability from basic block δ. It is required that p and q sum to 1. This situation of two forward incoming edges typically arises from the use of `if/else` statements. In this case we compute the transformation matrix as follows:

$$T_{cond} = p_i \sum T_i \tag{3.9}$$

This equation is simply a generalized version of Equation 3.8 with the added constraint that $\sum p_i = 1$.

Example From Figure 3.1 the function $g()$ can be fully summarized using Equation 3.9:

$$T_{g()} = 0.01 \cdot T_{S14} + 0.99 \cdot I = \begin{bmatrix} 0.99 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The identity matrix I is weighted with a probability 0.99 since there is no `else` condition. Recall that the `if` statement at $S13$ executes 1% of the time. This matrix indicates that after the function $g()$ executes, a has 1% chance of pointing at x and 99% chance of remaining the same.

3.6.2 Back-Edge with $s\prime$ Outside it's Region

When a back-edge is encountered and $s\prime$ is *outside* the region that the back-edge spans, we can think of the desired transformation matrix as similar to that for a fully-unrolled version

of the loop—eg., the same transformation matrix multiplied with itself as many times as the trip-count for the loop. In the case where the loop trip-count is constant, we can model the back-edge through simple exponentiation of the transformation matrix. Assuming that T_x is the transformation matrix of the loop body, and C is the constant loop trip-count value, we can model this type of back-edge with the following:

$$T_{loop} = T_x{}^C \tag{3.10}$$

When the loop trip-count is not a constant, we estimate the transformation matrix by computing the distributed average of all possible unrollings for the loop. Assuming that the back-edge is annotated with a lower-bound trip-count value of L and an upper-bound value of U, the desired transformation matrix can be computed efficiently as the geometric series averaged from L to U:

$$T_{loop} = \frac{1}{U - L + 1} \sum_{L}^{U} T_x{}^i \tag{3.11}$$

Example Consider the `while` loop found at statement $S5$ in Figure 3.1. The transformation matrix for the path from $S5$ to $S8$ is:

$$T_{S5 \rightarrow S8} = (T_{S6 \rightarrow S8})^{100} = (T_{g()})^{100} = \begin{bmatrix} 0.37 & 0 & 0 & 0.63 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix indicates that after the loop fully iterates, a has a 63% chance of pointing at x and a 37% chance of remaining unchanged.

3.6.3   Back-Edge with $s\prime$ Inside it's Region

The final case occurs when the edge is a back-edge and $s\prime$ is *inside* the region that the back-edge spans, as shown in Figure 3.4(c). Since $s\prime$ is within the loop the points-to relations at $s\prime$ may change for each iteration of that loop. In this case we compute the desired transformation matrix using a geometric series such that U is the maximum trip-count value:

$$T_{loop} = \frac{1}{U} \sum_{0}^{U-1} T_x{}^i \tag{3.12}$$

Example In Figure 3.1 this scenario occurs if we require the transformation matrix for the path $S1$ to $S7$, since in this case $s\prime$ is $S7$, which is within the `while` loop at $S5$. The required transformation matrix can be computed as follows:

$$T_{S1 \rightarrow S7} = T_{g()} \cdot \tfrac{1}{100} \sum_{0}^{99} (T_{S6 \rightarrow S8})^i \cdot T_{S1 \rightarrow S5}$$

3.7   Bottom Up and Top Down Analyses

We have so far described a method for computing probabilistic points-to information across basic blocks, and hence within a procedure. To achieve an accurate program analysis we need a method for propagating a points-to matrix inter-procedurally. To ensure that our analysis can scale to large programs, we have designed a method for inter-procedural propagation such that each procedure is visited no more than a constant number of times.

We begin by computing a transformation matrix for every procedure through a reverse topological traversal of the call graph—eg., a bottom-up (BU) pass. Recursive edges in the call-graph are *weakened* and analyzed iteratively to ensure that an accurate transformation matrix is computed. The result of the bottom-up pass is a linear transformation matrix that probabilistically summarizes the behavior of each procedure.

In the second phase of the analysis, we initialize a points-to matrix by (i) computing the result of all statically defined pointer assignments, and (ii) setting all other pointers to point at the undefined location set (UND). We then propagate the points-to matrix throughout the entire program using a forward topological traversal of the call-graph (eg., a top-down (TD) pass). When a load or store instruction is reached, the probability vector for that dereference is retrieved from the appropriate row in the matrix. When a call instruction is reached we store the points-to matrix at that point for future use. Finally, we compute the initial points-to matrix into every procedure as the weighted average of all incoming points-to matrices that were previously stored.

Example The call graph for our example dictates that in the bottom-up phase of the analysis the procedure-level transformation matrices are computed in the following order: $T_{g()}$, $T_{f()}$, and then $T_{main()}$. This is intuitively necessary since $T_{f()}$ requires $T_{g()}$; and $T_{main()}$ requires both $T_{f()}$ and $T_{g()}$. The algorithm then proceeds into the top-down phase which visits the procedures in the reverse order. Initially, the following input points-to matrix into main() is used since there are no static declarations:

$$
P_{main()\_in} = \begin{array}{c} \\ a \\ b \\ tmp \\ x \\ y \\ \text{UND} \end{array}
\begin{array}{ccc} x & y & \text{UND} \\ \left[\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \end{array}
$$

The algorithm propagates and updates this matrix forward until a pointer dereference or a procedure call instruction is reached. At $S4$, the points-to matrix P_{S4} is cached so that when the procedure f() is analyzed the points-to matrix representing the initial state of f() will be

available.

$$P_{f()\_in} = P_{S4} = \begin{array}{c} \\ a \\ b \\ tmp \\ x \\ y \\ \text{UND} \end{array} \begin{array}{ccc} x & y & \text{UND} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Similarly, the points-to matrices P_{S6} and P_{S12} are also stored in order to analyze procedure g() during the top-down phase. These matrices are later merged using a weighted sum based on the fan-in frequencies from their respective callee procedures:

$$P_{g()\_in} = 100 \times P_{S6} + P_{S12}$$

3.8 Safety

At this point our algorithm does not necessarily compute a safe result in all circumstances: for certain code, the theoretical points-to transfer function may contain non-linear side effects which are not captured by our linear transformation matrix encoding. Non-linear effects occur when multiple levels of dereferencing are used to assign to pointers: (i) load assignment: $a = *b$; (ii) store assignment: $*a = b$. We optionally handle multi-level dereferencing by instantiating *shadow variable* pointer location sets, a technique that is similar to invisible variables [26]. These shadow variable location sets can potentially alias with other pointer location sets and cause unsafe behavior if ignored. In this case we want to handle these nonlinearities safely, we use a lightweight, context-insensitive and flow insensitive unification based alias analysis [65] to precompute any shadow variable aliasing that can potentially occur. We assume that aliasing

between location sets occurs with a probability that is inversely-proportionally to the size of the aliasing set. All transformations involving shadow variables that alias are amended safely to handle these nonlinearities. However, it is important to note that since we are supporting speculative optimizations, we also have the option of ignoring safety.

3.9   Summary

This chapter describes the PPA algorithm that provides the foundation for our LOLIPoP infrastructure. The points-to matrix and the transformation matrix were introduced as components used to track and store probabilistic points-to graphs. A simple example was used to show how transformation matrices can be derived and used to summarize a single instruction, a basic block, any path, and an entire procedure. A method for leveraging our matrix-based framework inter-procedurally using a bottom-up and top-down flow was also described. Additionally, a simple method that utilized a FICI pointer analysis for multiple levels of pointer dereferences was described as a means of obtaining a safe analysis. The next chapter will describe our design decisions in greater detail and present the details of our implementation of LOLIPoP.

Chapter 4

The LOLIPoP PPA Infrastructure

This chapter presents the Linear One-Level Interprocedural Probabilistic Pointer (LOLIPoP) analysis infrastructure. We begin by highlighting and discussing the features that the infrastructure offers. The design decisions made and important implementation details to realize this infrastructure are also discussed in this chapter.

4.1 Implementing LOLIPoP

Figure 4.1 shows a block diagram of the LOLIPoP infrastructure, which operates on `C` source code. The LOLIPoP infrastructure is our prototype implementation of the algorithm described in Chapter 3. The foundation of LOLIPoP is a SUIF 1.3 compiler pass [74]. The SUIF front end parses the code into the SUIF intermediate representation (SUIF-IR). Using the generated SUIF-IR, LOLIPoP executes in four sequential phases:

1. Interprocedural Control Flow Graph (ICFG) construction;

2. Static Memory Model (SMM) generation;

3. Bottom-Up (BU) call graph traversal to *collect* all the procedure-level transformation

Figure 4.1: The LOLIPoP infrastructure.

matrices;

4. Top-Down (TD) call graph traversal to *propagate* the points-to matrices.

This section will describe these four phases. For reference purposes, a simplified version of the source-code implementation of the BU phase can be found in Appendix A, which presents source level details on the methods that are invoked to compute a transformation matrix.

4.1.1 Interprocedural Control Flow Graph (ICFG)

The pass begins by building the Interprocedural Control Flow Graph (ICFG) [49] using the SUIF-IR as input. An ICFG is a collection of control flow graphs linked together with a global call graph. The ICFG is able to fully represent the complete flow of execution through a given input program. By linking all control flow graphs together in one data structure, the compiler is able to perform various interprocedural analyses. We have developed a light-weight edge-profiler which instruments C source code to track control flow edge counts, invocation edge counts, and indirect function call targets. Our ICFG is optionally annotated with any available profile information, which is obtained through the use of the edge profiler. The analysis is

optionally able to operate without this profile information through the use of simple static heuristics, which is discussed further in Section 4.1.5. The ICFG provides a convenient API so that subsequent phases can easily query for information or traverse the embedded graphs in various ways. The most important type of traversal predominantly utilized by the BU and TD phases is a topological traversal of the call graph. However, as a result of recursion, call graphs may have cyclic components. This prevents us from performing a topological traversal in the general sense because a topological sorting algorithm requires a directed *acyclic* call graph. Therefore, we must somehow remove all cycles. We accomplish this using edge *weakening*. Weakening means that we iteratively tag edges within SCCs [72] as 'weakened' and then ignore them during the topological traversal. The implication is that a weakened edge is a recursion-causing invocation edge.

4.1.2   Static Memory Model (SMM)

After building our ICFG, the next step is to build the static memory model (SMM). The SMM uses the SUIF symbol table and the ICFG to construct an abstract representation of the program's addressable memory space. LOLIPoP uses *location sets* [78] as defined in Chapter 2 to abstractly model real memory locations. For our purposes, as described in Chapter 3, a location set need only track its approximate size and the approximate number of pointers it represents. Through a linear pass of the SUIF symbol table and ICFG, location sets are extracted, merged, and represented based on various settings specified by the user. For example, the user is able to specify how the heap is to be managed: (1) as a single location set, or (2) as a unique location set that maps every call of `malloc()`, `calloc()`, `realloc()`, etc. to its own location set. All aggregate structures and arrays are optionally merged into a single location set for a field-insensitive analysis. This system also allows for a field-sensitive analysis that separates `C` `struct` fields into individual location sets. Recursive data structures are merged into one data

structure after one level of recursion. If necessary, the SMM is responsible for performing a shadow variable alias analysis pass to guarantee an overall safe analysis. LOLIPoP currently uses Steensguard's [65] unification based CIFI alias analysis to extract this alias information. The various optional user level settings that our SMM provides allow for a fair comparison with other pointer analysis techniques. They also permit us to further study the various tradeoffs between efficiency and accuracy.

4.1.3 Bottom-up (BU) Pass

Phase three of our analysis is the bottom-up (BU) pass. The bottom-up pass, as described in Section 3.7, computes a probabilistic summarizing transformation matrix for every procedure through a reverse topological traversal of the call graph. We exploit the sparse matrix library available through MATLAB's runtime library to perform all matrix operations. The matrices are cached in an indexable data structure that intelligently monitors its memory usage. If memory becomes scarce, this storage data structure swaps matrices to disk using traditional caching algorithms.

At the intraprocedural level, the key method required for the BU phase of the analysis is the method to compute a transformation matrix given any valid intraprocedural path. This method fully encapsulates the intraprocedural algorithm described in Chapter 3. The source code can be found in Section A.4 of Appendix A. This recursive method takes in any two connected basic blocks and returns the transformation matrix that summarizes the path between those two basic blocks. Quite naturally, this recursive method is used to compute the transformation matrix for an entire procedure by passing in the initial and final basic blocks associated with that procedure[1]. An important optimization added to the implementation of the algorithm previously described in Chapter 3 is the use of memoization. To avoid duplicate path computations when

---

[1]The ICFG normalizes all control flow graphs so that they have exactly one initial and one final synthetic basic block [56].

recursively analyzing the same control flow graph, transformation matrices are hashed into a global table using the path as the hashing index. Because this algorithm is recursive, this optimization is required to prevent an exponential evaluation of all paths which helps to achieve scalability.

A further and essential optimization that enables both increased performance and accuracy is UND-termination. UND-termination sets all local variables, parameters, and return value location sets to point at UND (the undefined location set) when they can no longer be referenced—i.e. when they are popped off the program stack. This optimization is completely safe since location sets local to a procedure no longer point to valid memory locations once the procedure returns. Although this subtle optimization appears to be inconsequential, its main objective is to maintain matrix sparsity when calculating other transformation matrices. Without it, matrices quickly become non-sparse which causes the amount of memory required to grow exponentially. An example program that has been refactored using UND-termination can be found in Figure 4.2.

4.1.4 Top-down (TD) Pass

The fourth and final stage of the analysis is the top-down (TD) phase. In this phase we visit every procedure in a forward topological traversal of the call graph. MATLAB's sparse matrix runtime library is again utilized to perform all the basic matrix mathematical operations. At the entry point to every procedure being analyzed we initialize an input points-to matrix. As described in Section 3.7, this matrix is the weighted sum of all points-to matrices that exist when this procedure is called. If a procedure has no callee's, such as the `main()` procedure, we initialize an input points-to matrix by (i) computing the result of all statically defined pointer assignments, and (ii) setting all other pointers to point at the undefined location set (UND).

At the intraprocedural level, the points-to matrix is also propagated in a top-down traversal of the control flow graph using an equivalent traversal algorithm as the one used at the inter-

```
                                              int **swap_x, **swap_y;
        void swap(int **x, int **y) {         void swap() {
            int *tmp;                             int *tmp;

  S1:       *tmp = *x;                   S1:       *tmp = *swap_x;
  S2:       *x = *y;                     S2:       *swap_x = *swap_y;
  S3:       *y = *tmp;                   S3:       *swap_y = *tmp;
        }
                                                  tmp = UND;
                                              }

        void foo() {                          void foo() {
            int **a, *b;                          int **a, *b;
                                                  ...
                                                  swap_x = a; swap_y = &b;
            ...                           S4:       swap();
                                                  *a = *swap_x; b = *swap_y;
  S4:       swap(a, &b);                          swap_x = UND; swap_y = UND;
            ...                                   ...
        }                                     }
```

(a) Original program   (b) After applying UND-termination

Figure 4.2: In the simple program found in (a), the function `swap` is called from `foo`. The refactored code in (b) is a version of (a) with UND termination applied. The SMM pass conceptually promotes all function parameters into global variables. All location sets are assigned to point at the undefined location set when they are no longer needed.

procedural level. The points-to matrix into every basic block is computed using a weighted sum of all the output points-to matrices from all predecessor basic blocks. Memoization is again used to avoid the duplicate calculations. Within a basic block, the points-to matrix for the program points of interest (i.e. pointer dereferences and call instructions) are computed. When a pointer dereference instruction is reached (which maps to a load or store instruction in SUIF), the probability vector for that dereference is retrieved from the appropriate row in the matrix. The probability vector is annotated to the SUIF-IR and subsequently becomes the output of our entire LOLIPoP system. Recall that our main objective is to accurately predict the probability

of each points-to relation at every pointer dereference program point. When a call instruction is reached we store the points-to matrix at that point for future use. The data structure that stores these matrices performs the interprocedural merge operation immediately to reduce the total number of matrices required for storage.

4.1.5 Heuristics

To a large extent, the probabilistic points-to information extracted at every pointer dereference point is dependent on the edge-profile information used to guide the analysis—the assumption being that the degree of accuracy used in obtaining edge profile information is directly related to the final probabilistic accuracy. The exact impact of an accurate edge-profile pass was unclear at the outset of our investigation and the actual measured impact will be discussed later in Chapter 5. Our analysis requires four different types of information from the edge profiler: (i) the fan-in probability of all control flow graph edges (ignoring back edges); (ii) the upper and lower bound trip counts for all back edges per loop invocation; (iii) the fan-in probability of all invocation call graph edges; and (iv) the probability of indirect function call targets. For experiments when edge-profile information is not available, simple heuristics are used in its place. The following three compile-time heuristics are used: (i) we assume that fan-in probability is a uniform distribution between all incoming branches; (ii) when the upper and lower bound for the trip count of a loop cannot be determined through a simple inspection of the loop bounds, we assume that the lower bound is zero and the upper bound is ten; and (iii) we assume that fan-in call graph invocation edge counts have a uniform distribution between all callee procedures. For this thesis we do not yet use a heuristic for indirect function call targets and therefore require the edge-profiler to always provide that information.

4.2 Accuracy-Efficiency Tradeoffs

During the design and implementation of LOLIPoP, many important design decisions involving accuracy/efficiency tradeoffs were encountered. Many of these tradeoffs are common to all pointer analysis algorithms, as discussed in Section 2.1.6. Our system was designed so that whenever possible, and especially when no approach was clearly favourable, the trade-off was left for the eventual client of LOLIPoP to control. This section will briefly discuss some of the important tradeoffs encountered and the approach taken.

4.2.1 Safe vs Unsafe Analysis

Because our analysis targets a speculative client optimization, safety is not a firm requirement. That said, safety can potentially aid a speculative optimization by reducing the number of memory references which must be treated as speculative. For example, reordering loads and stores using the EPIC instruction set requires the memory references to always be speculative instructions if an unsafe analysis is used. In contrast, if a safe analysis is used the compiler would be able to, in certain cases, guarantee that an optimization is safe and could avoid the overhead that results from tracking speculative loads and stores. Clearly a tradeoff exists here between accuracy and efficiency. Performing a safe analysis adds complexity and increases the run time of our algorithm for three reasons: (i) the added time required to perform the shadow variable alias analysis pass; (ii) the increased complexity in deriving and computing transformation matrices that represent instructions where an operand aliases with another location set; and (iii) the decrease in matrix sparsity as a consequence of adding and tracking many likely false points-to relations. For the reasons described here, LOLIPoP was designed to allow the user to optionally decide if a safe or unsafe analysis should be performed.

4.2.2 Field Sensitivity

Field sensitivity or aggregate modeling is a classic example that illustrates how pointer analysis accuracy can be traded off for increased algorithmic performance. Aggregate structures with multiple pointer type fields, such as `C structs` are merged and handled as a single pointer location set in a field-insensitive analysis. Conversely, a field-sensitive analysis handles every field and subsequently every subfield as a separate and unique field. Clearly, a field sensitive analysis provides more accuracy than a field insensitive analysis. With regards to performance, LOLIPoP is particularly susceptible to this field-sensitivity tradeoff because the number of location sets extracted by the Static Memory Model determines the size N of all transformation matrices. A larger value of N causes more memory to be used and requires added complexity to all matrix operations performed. LOLIPoP currently only supports field insensitivity, but that support for field-sensitivity can easily be added in the future.

4.2.3 Other Tradeoffs

LOLIPoP provides many other interesting user level runtime options that are very minor and beyond the scope of our discussion. In this section we will briefly highlight some of these tradeoffs. Appendix B outlines all of the possible runtime options currently available to the user. The user is able to model the heap as a single location set or model each heap allocation callsite site as a unique location set; unique callsite allocation is the default choice. No support is yet provided for unique context-sensitive callsite heap modeling or shape analysis. The analysis is also optionally allowed to fully trust the edge-profiler, which enables it to ignore all paths that have a probability of zero. This results in both improved performance and accuracy; unfortunately it also produces an unsafe and heavily profile biased result. Finally, the user is able to substitute the more expensive geometric series operation with an unsafe and more efficient linear interpolation operation.

4.3 Optimizations

To meet our objective to scale to the entire SPEC 2000 integer benchmark suite, we put a great deal of effort into optimizing the execution of LOLIPoP. In particular we exploit sparse matrices, we compress matrices before exponentiating, we perform aggressive memoization of matrix results at the intraprocedural level, and we use an efficient implementation for computing the geometric series for Equations 3.11 and 3.12. This section describes the main optimizations utilized to realize our scalability objective.

4.3.1 Matrix Representation

The typical data structure used for a matrix is a two dimensional array. Each entry in the array represents an element $a_{i,j}$ of the matrix and can be accessed by the two indices i and j. For our purposes, the matrices are almost always *sparse*, and implementing them using this naive approach is an inefficient use of memory. In this regard, we exploit the fact that our matrices are sparse and therefore encode them using Matlab's sparse matrix library [41, 53]. We also take special care, as discussed previously, to ensure that the matrices remain sparse to effectively take advantage of the Matlab library. A sparse matrix contains many zero entries. The basic idea when storing sparse matrices is to only store a representation of non-zero entries as opposed to storing all entries. Depending on the number and distribution of the non-zero entries, different data structures can be used and yield huge savings in memory when compared to a naive approach.

Matlab encodes sparse matrices using the Yale Sparse Matrix Format [25, 35]. It stores an initial sparse $N \times N$ matrix M in row form using three arrays: PR, IR, and JC. The variable NZ denotes the number of nonzero entries in matrix M. PR is an array of type `double` that stores all nonzero entries within M. This array is therefore at least length NZ and we ensure that it does not grow beyond $2 \times NZ$ (surprisingly, Matlab does not maintain this for us).

The array IR is also at least of length NZ. It is an integer array that stores the row offsets associated with the values in PR. JC is an integer array of length $N+1$. IR stores at $JC[i]$ the position of the first element of row i in the sparse array PR. The length of row i is determined by $JC[i+1] - JC[i]$.

4.3.2 Matrix Operations

A further optimization we use is matrix compression. The matrices we maintain have the property that all rows sum to 1.0, which implies that they contain at least N nonzero entries—therefore, we don't fully take advantage of the sparsity features provided by Matlab. Whenever storing transformation matrices, we compress them by subtracting the identity matrix. Since most transformation matrices are very close to the identity matrix, this causes them to become much more sparse. They are therefore stored using a more memory efficient means, which indirectly results in improved performance.

When performing exponentiation or the geometric series operation we utilize a different type of matrix compression. We compress the matrix into a smaller matrix by eliminating all rows and columns that will remain unchanged after the operation is performed [16]. We then perform the exponentiation or geometric series operation on the smaller matrix. The matrix is then repaired back into its former state by reinserting the rows and columns that were removed. This type of matrix compression can be performed on the Yale matrix type quite easily and matlab performs much more efficiently when exponentiating a smaller matrix.

For performing the expensive operation of exponentiation, LOLIPoP relies on the Matlab library for which it is undocumented how exponentiation is performed efficiently. Furthermore, it is hypothesized that LOLIPoP could improve upon Matlab's implementation of exponentiation by further utilizing the various properties of the transformation matrix—this is left for future work. The geometric series operation is implemented by LOLIPoP as an extension to the

exponentiation operation. More information on how to map a geometric series operation into an exponentiation operation can be found in Appendix C.

4.4 Summary

This chapter presented the LOLIPoP infrastructure. We described the four phases used by the LOLIPoP framework: (1) Interprocedural Control Flow Graph (ICFG) construction; (2) Static Memory Model (SMM) generation; (3) Bottom-Up (BU) traversal; and (4) Top-Down (TD) traversal. We highlighted the features that the infrastructure offers. We discussed the different tradeoffs involved, including field sensitivity and safe vs unsafe analysis. We also provided insight into the important implementation details and optimizations that we applied such as: memoization, UND-termination, and matrix compression. The next chapter will discuss the experimental framework used and the accuracy and performance results we obtained.

Chapter 5

Evaluating LOLIPoP

In this chapter we evaluate LOLIPoP's running-time, the accuracy of the pointer analysis results in a conventional sense, and the accuracy of the resulting probabilities.

5.1   Experimental Framework

The following describes our framework for evaluating LOLIPoP. All timing measurements were obtained using a 3GHz Pentium IV with 1GB of RAM and 1GB of swap space. We report results for all of the SPECint95 and SPECint2000 benchmarks [19] except for the following: 252.EON, which is written in C++ and therefore not handled by our compiler; 126.GCC, which is similar to 176.GCC; and 147.VORTEX, which is identical to 255.VORTEX. Table 5.1 describes both the ref and train inputs used for each benchmark.

5.2   Analysis Running-Time

LOLIPoP meets our objective of scaling to the SPECint95 and SPECint2000 benchmark suites. Table 5.2 shows the running-times for both the safe (where shadow variable alias analysis is per-

Table 5.1: Benchmark inputs used.

	Benchmark	Train Input	Ref Input	Description
SPECint2000	BZIP2	default	default	compression/decompression
	CRAFTY	default	default	chess board solver
	GAP	default	default	group theory interpreter
	GCC	default	expr.i	compiler
	GZIP	default	default	compression/decompression
	MCF	default	default	combinatorial optimization
	PARSER	default	default	natural language parsing
	PERLBMK	diffmail.pl	diffmail.pl	perl interpreter
	TWOLF	default	default	place and route for standard cells
	VORTEX	default	bendian1.raw	OO database
	VPR	default	default	place and route for FPGAs
SPECint95	COMPRESS	default	reduced to 5.6MB	compression/decompression
	GO	default	9stone21.in	game playing, AI, plays against itself
	IJPEG	default	vigo.ppm	image processing
	LI	default	default	lisp interpreter
	M88KSIM	default	default	microprocessor simulator
	PERL	jumble.pl	primes.pl	perl interpreter

Table 5.2: LOLIPoP measurements, including lines-of-code (LOC) and transformation matrix size N for each benchmark, as well as the running times for both the unsafe and safe analyses. The time taken to obtain points-to profile information at runtime is included for comparison.

	Benchmark	LOC	Matrix Size	Running Time (min:sec)		
				Unsafe	Safe	PT-Profile
SPECint2000	BZIP2	4686	251	0:0.3	0:0.3	13:34
	CRAFTY	21297	1917	0:5.5	0:5.5	14:47
	GAP	71766	25882	54:56	83:38	55:56
	GCC	22225	42109	309:40	N/A	39:58
	GZIP	8616	563	0:0.71	0:0.77	3:48
	MCF	2429	354	0:0.39	0:0.61	19:46
	PARSER	11402	2732	0:30.7	0:50.0	84:52
	PERLBMK	85221	20922	44:15	89:43	N/A
	TWOLF	20469	2611	0:16.6	0:20.6	N/A
	VORTEX	67225	11018	3:59	4:56	0:0.7
	VPR	17750	1976	0:9.3	0:10.3	197:0
SPECint95	COMPRESS	1955	97	0:0.1	0:0.1	1:55
	GO	29283	651	0:2.9	0:3	5:58
	IJPEG	31457	4491	0:23.4	0:24.9	7:12
	PERL	4686	5395	5:3	7:49	8:37
	LI	27144	3868	0:28.8	0:59.15	72:5
	M88KSIM	19933	1932	0:4.9	0:5.24	0:0.2

formed) and unsafe analyses. Each running-time includes the bottom-up and top-down phases of the overall analysis, but ignores the time taken to build the interprocedural control flow graph and static memory model—but note that this time is negligible in most cases. The runtimes range from less than a second for four benchmarks up to 5 hours for the challenging benchmark (GCC). These results are promising especially given that this is an academic prototype implementation of PPA.

For speculative optimizations, the alternative to pointer analysis is to use a points-to profiler which instruments the source code to extract points-to frequency information at runtime—for comparison purposes, we have implemented a points-to profiler. Our points-to profiler instruments the source code so that the memory addresses of all location sets can be tracked and queried at runtime. This involves two key instrumentation techniques: (1) overloading library function calls to `alloc` and `free` to track heap location set addresses; and (2) storing the addresses of every stack variable (whose address is taken) when it gets put on the program's runtime stack. Every pointer dereference is also instrumented with a wrapper function that enables the points-to profiler to determine which location set is being referenced during the dereference operation. The points-to profiler queries the set of available addresses, which takes $O(n \cdot log(n))$ time, to determine which runtime point-to relation is executing—the frequency of this points-to relation is then incremented. Points-to profiling is a computationally intense analysis and furthermore has no ability to provide safety in the resulting alias information it provides.

The results in Table 5.2 show that in most cases, our analysis approach is much faster than the profiler. In fact, for two cases (PERLBMK and TWOLF) the profiler did not terminate after two weeks of execution. It is also important to note that the profiling approach is very dependent on the input set used, both in the results it provides and on the profiler's runtime. For GCC and VORTEX reduced reference input sets were used to ensure a tractable profiler time (described in Table 5.1)—in these cases the profiler outperforms LOLIPoP because LOLIPoP must analyze

Figure 5.1: Average dereference size comparison with GOLF for the SPECInt95 benchmarks: *L* is LOLIPoP, and *G* is GOLF. LOLIPoP's results are *safe*, except for GCC which reports an unsafe result. LOLIPoP's results are more accurate than GOLF for all benchmarks.

the entire program while the profiler only analyzes the subset of code that is exercised by the reduced input set. For the more challenging benchmarks (GAP, GCC, and PERLBMK), there is a significant increase in running-time to compute safe results—i.e., to handle pointer assignments with multiple levels of dereferencing as described in Section 3.8.

5.3   Pointer Analysis Accuracy

The accuracy of a conventional pointer analysis algorithm is typically measured and compared by computing the average cardinality of the target location sets that can be dereferenced across all pointer dereference sites in the program—in short, the average dereference size. To ensure that LOLIPoP is indeed accurate, in Figure 5.1 we compare the average dereference size for LOLIPoP's safe result with those reported by Das for the GOLF algorithm [22]. We choose GOLF for comparison for two main reasons: (i) it is one of the few analyses that scales to even SPECint95 benchmarks; (ii) GOLF is safe, one-level context-sensitive, field-insensitive, and

Table 5.3: LOLIPoP Measurements.

Benchmark	Avg. Dereference Size		
	Safe	p > 0.001	Unsafe
BZIP2	1.00	1.00	1.00
COMPRESS	1.080	1.08	1.08
CRAFTY	1.830	1.40	1.83
GAP	143.84	77.61	6.21
GCC	N/A	N/A	2.64
GO	3.290	2.15	3.29
GZIP	1.41	1.31	1.45
IJPEG	6.740	2.46	1.33
LI	80.10	14.70	4.34
M88KSIM	1.82	1.66	1.84
MCF	1.51	1.51	1.51
PARSER	42.52	2.09	3.23
PERLBMK	18.48	5.45	3.10
PERL	88.98	8.40	35.35
TWOLF	1.26	1.25	1.19
VORTEX	6.06	3.61	6.13
VPR	1.18	1.10	1.09

callsite-allocating—matching those aspects of LOLIPoP. However, our goal is not to compete with GOLF since there are several reasons such a comparison would not be fair: (i) LOLIPoP is flow-sensitive while GOLF is not; (ii) LOLIPoP relies on profiling to identify the targets of function pointers; and (iii) LOLIPoP does not yet model all library calls, while GOLF does. Flow-sensitivity is the main reason that LOLIPoP does not scale as well as GOLF, although this is also the main source of LOLIPoP's significantly improved accuracy as reported in Figure 5.1. In summary, even without considering probability information LOLIPoP provides an accurate approach to performing pointer analysis.

Table 5.3 shows the average dereference sizes for all benchmarks studied, showing the safe result, the result when any points-to relation with a probability less than 0.001 is ignored, and

the unsafe result (when shadow variable aliasing is ignored)—average maximum certainty will be discussed later in Section 5.4.1. One very interesting result is that the benchmarks with a relatively large average dereference size for the safe analysis (GAP, LI, PARSER, PERLBMK, PERL) show a dramatic decrease when unlikely points-to relations are ignored (i.e., those for which $p < 0.001$). This result suggests that many points-to relations are unlikely to occur at runtime, underlining the strong potential for speculative optimizations. As expected, a similar result is observed for the unsafe version of the analysis since the safe analysis introduces many inaccuracies through the flow-insensitive, context-insensitive pass that addresses shadow variable aliasing. These inaccuracies create many low probability points-to relations that are unlikely to ever occur at runtime. For example, the safe average dereference size for GAP is relatively high at 143.84, while the unsafe size is only 6.21.

5.4 Probabilistic Accuracy

We now measure the accuracy of the probabilities computed by LOLIPoP by comparing the two probability vectors $\overrightarrow{P}_s$ and $\overrightarrow{P}_d$ at every pointer dereference point. $\overrightarrow{P}_s$, as defined in Section 3.3.1, represents the probability vector reported by LOLIPoP—the static probability vector. $\overrightarrow{P}_d$ represents the dynamic probability vector calculated by the points-to profiler. In particular, we want to quantify the accuracy of the probability vectors $\overrightarrow{P}_s$ that are statically computed at every pointer dereference. For comparison, we use the results of the profiler where each benchmark (using it's ref input) is instrumented to track—for each pointer dereference location—a frequency vector that indicates the frequency that each location set is the target. Each resulting dynamic frequency vector is then normalized into a dynamic probability vector $(\overrightarrow{P}_d)$ so that it may be compared with the corresponding probability points-to relation vector, as described in Equation 3.1. To compare the two vectors in a meaningful way, we compute the *normalized average Euclidean distance* (NAED) as defined by:

Figure 5.2: SPECint95 - Normalized Average Euclidean Distance (NAED) relative to the dynamic execution on the ref input set. D is a uniform distribution of points-to probabilities, Sr is the safe LOLIPoP result using the ref input set, and the U bars are the unsafe LOLIPoP result using the ref (Ur) and train (Ut) input sets, or instead using compile-time heuristics (Un).

$$\text{NAED} = \frac{1}{\sqrt{2}} \cdot \frac{\sum \|\overrightarrow{P}_s - \overrightarrow{P}_d\|}{(\# \text{ pointer dereferences})} \tag{5.1}$$

This metric summarizes the average error uniformly across all probability vectors at every pointer dereference on a scale that ranges from zero to one, where a zero means no discrepancy between dynamic and static vectors, and a one means there is always a contradiction at every dereference.

Figures 5.2 and 5.3 show the NAED for the SPECint95 and SPECint2000 benchmarks relative to the dynamic execution on the ref input set (results for GAP, PERLBMK, and TWOLF are omitted because their dynamic points-to profile information could not be tractably computed).
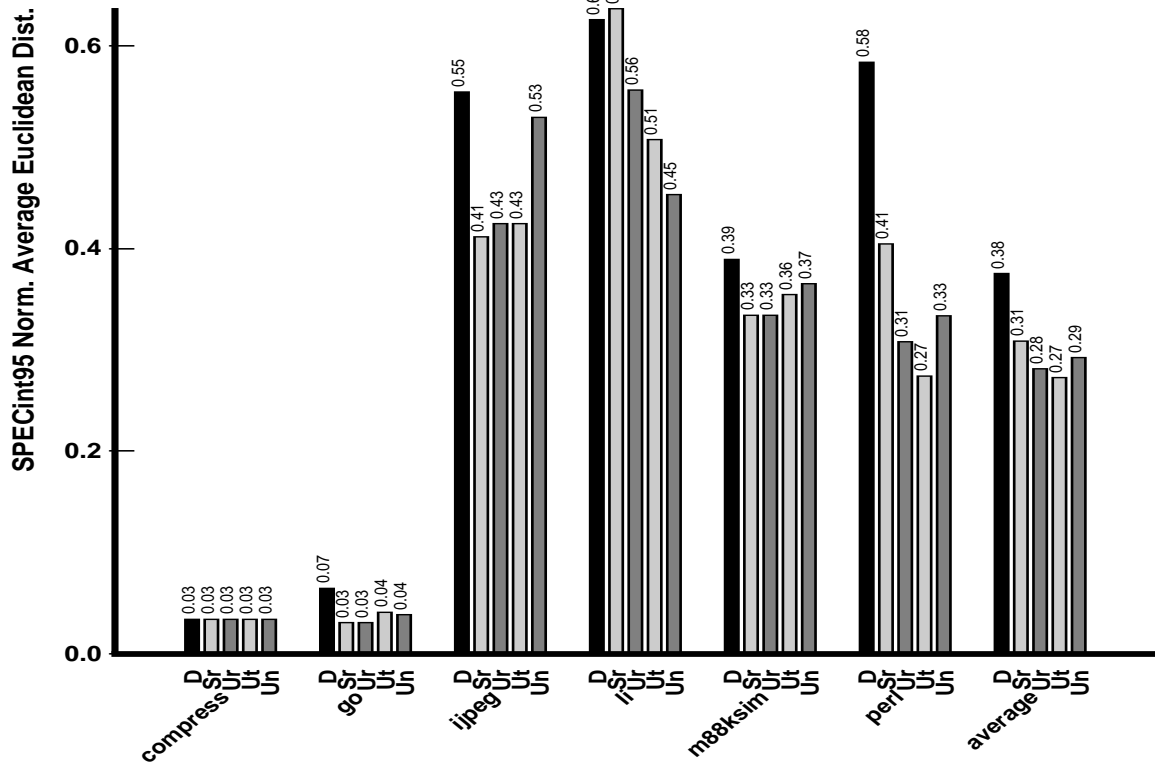
Figure 5.3: SPECint2000 - Normalized Average Euclidean Distance (NAED) relative to the dynamic execution on the ref input set. D is a uniform distribution of points-to probabilities, Sr is the safe LOLIPoP result using the ref input set, and the U bars are the unsafe LOLIPoP result using the ref (Ur) and train (Ut) input sets, or instead using compile-time heuristics (Un).

In the first experiment (D) we distribute probability uniformly to every target in the static points-to probability vector $\overrightarrow{P_s}$, making the naive assumption that all targets are equally likely. This experiment is used to quantify the value-added of probability information, and leads to an average NAED across all benchmarks of 0.32 relative to the dynamic result. It is important to notice that for BZIP2, COMPRESS, and GO even the uniform distribution (D) is quite accurate. For BZIP2 and COMPRESS this is an expected result since the average dereference size (shown in Figure 5.3) is very close to 1.0.

The second experiment (Sr) plots the NAED for the safe analysis using edge-profile in-

formation from the ref input set. Comparing the static and dynamic results using the same input set allows us to defer the question of how representative the profiling input set is. With LOLIPoP we improve the NAED to an average of 0.25 across all benchmarks, although that average can be misleading. For about half of the benchmarks probability information does not make a large difference (bzip2, compress, go, m88ksim, mcf, vpr), while for the remaining benchmarks probability information significantly improves the NAED. For LI, the probability information slightly increases the NAED. The LI benchmark contains a tremendous amount of shadow variable aliasing—we know this because of the large gap between the safe and unsafe average dereference sizes shown in Figure 5.3. The spurious points-to relations introduced by the 'safe' analysis appear to corrupt the useful probability information. A similar result would be expected for GAP. Using a more accurate shadow variable analysis pass would help to reduce this effect. Also, applying field-sensitivity would also help because it would drastically reduce the amount of shadow variable aliasing.

The next experiment (Ur) shows the NAED for the unsafe analysis, also using edge-profile information from the ref input set. Comparing with the safe experiment (Sr), surprisingly we see that on average the unsafe result is more accurate (with an NAED of 0.24): this result implies that safety adds many false points-to relations, and can actually be undesirable in the context of speculative optimizations. The exception to this is GZIP, where the NAED deteriorates when transitioning from safe to unsafe. This implies that GZIP frequently utilizes and relies on many levels of pointer dereferencing.

The final two experiments plot the NAED for the unsafe analysis when using the train input set (Ut), and when using compile-time heuristics instead of edge-profile information (Un). Surprisingly, the average NAED when using the train input set (Ut) is slightly more accurate than with the ref input set (Ur): this indicates that there are aliases which occur rarely during the ref execution, which when profiled and fed back into LOLIPoP, become a source of inaccuracy compared to the lesser coverage of the train input set. Finally, we see that the unsafe approaches

are more accurate with edge-profiling information than when compile-time heuristics are used (Un), although even with the heuristic approach LOLIPoP is more accurate than the uniform distribution (D).

5.4.1   Average Maximum Certainty

To further evaluate the potential utility of points-to probability information provided by LOLIPoP, we use a measure *average maximum certainty*:

$$\text{Avg. Max. Certainty} = \frac{\sum (\text{Max Probability Value})}{(\# \text{ pointer dereferences})} \tag{5.2}$$

This equation takes the maximum probability value associated with all points-to relations at every pointer dereference and averages these values across all pointer dereference sites. Data speculative optimizations benefit from increased certainty that a given points-to relation exists: since the probabilities across a probability vector sum to one, if there is one large probability it implies that the probabilities for the remaining location sets are small. In other words, the closer the average maximum certainty value is to one, the more potential there is for successful speculative optimization. The average maximum certainty for each SPEC benchmark is given in Table 5.4, and in general these values are quite high: the average value across all benchmarks is 0.899. This indicates that on average, at any pointer dereference, there is likely only one dominant points-to relation. Therefore a client analysis using LOLIPoP will be very certain of which points-to relation will exist at a given pointer dereference. The actual average max certainty values appear to be directly correlated to the average dereference size. If the average dereference size is quite high for a given benchmark then it's max certainty value is relatively smaller. Intuitively, this is an expected result.

Table 5.4: LOLIPoP Average Maximum Certainty Measurements.

Benchmark	Avg. Maximum Certainty
BZIP2	1.00
COMPRESS	0.96
CRAFTY	0.95
GAP	0.78
GCC	0.96
GO	0.94
GZIP	0.90
IJPEG	0.90
LI	0.76
M88KSIM	0.83
MCF	0.92
PARSER	0.97
PERLBMK	0.79
PERL	0.87
TWOLF	0.90
VORTEX	0.91
VPR	0.95
Average	**0.899**

5.5   Summary

This chapter discussed the experimental framework we used in evaluating LOLIPoP. The running times for both the safe and unsafe version of our PPA algorithm when applied to the SPECint2000 and SPECint95 benchmarks were presented. We showed that these results were on average much faster than the running times of our points-to profiler. We also presented the accuracy of our algorithm using the traditional average dereference size metric. We compared these results to the results obtained by GOLF [22]. This showed that even without considering probability information, that LOLIPoP provides an accurate approach to performing pointer analysis. We also showed that many of the probabilistic points-to relations observed had very small probability values, underlining the strong potential for speculative optimizations. We also showed that on average, the compiler is fairly certain about the location sets being referenced, which also motivates the use of speculative optimization. Finally, we used the NAED metric to present the accuracy of the reported probabilities relative to the runtime frequencies observed.

Chapter 6

Conclusions

As speculative optimization becomes a more widespread approach for optimizing and parallelizing code in the presence of ambiguous pointers, we are motivated to be able to accurately predict the likelihood of points-to relations without relying on expensive dependence profiling. We have presented LOLIPoP, a probabilistic pointer analysis algorithm that is one-level context-sensitive and flow-sensitive, yet can still scale to large programs including the SPECint2000 benchmark suite. The key to our approach is to compute points-to probabilities through the use of linear transfer functions that are efficiently encoded as sparse matrices.

We have used LOLIPoP to draw several interesting conclusions. First, we found that even without considering probability information, that LOLIPoP provides an accurate approach to performing pointer analysis. Second, we demonstrated that many points-to relations are unlikely to occur at runtime, underlining the strong potential for speculative optimizations. Third, we found that the unsafe version of our analysis is more probabilistically accurate, implying that safety adds many false points-to relations, and can actually be undesirable in the context of speculative optimizations. Finally, we showed that LOLIPoP still produces reasonably accurate probabilities when using compile-time heuristics instead of edge-profile information.

6.1 Contributions

This thesis makes the following contributions.

1. A novel algorithm for probabilistic pointer analysis based on sparse transformation matrices.

2. An accurate, one level context-sensitive and flow-sensitive pointer analysis that scales to the SPEC integer benchmarks.

3. A method for computing points-to probabilities that does not rely on dependence-profiling, and can optionally use control flow edge-profiling.

4. An infrastructure (LOLIPoP) for performing this analysis (as well as a points-to profiler) for use in future research.

6.2 Future Work

In the future, we plan to (i) further explore the trade-offs between scalability and accuracy; (ii) improve the flexibility of our static memory model to allow for field-sensitivity and improved models for recursive data structures; (iii) improve the efficiency of the algorithm by optimizing the matrix framework, capitalizing on the specific properties of the underlying transformation matrices; and (iv) apply LOLIPoP to a client speculative optimization to evaluate its overall effectiveness.

Appendix A

LOLIPoP Source Code

This appendix provides a high level pseudo code description of the Bottom-Up phase of our LOLIPoP infrastructure. The code is presented in `c++`. Many implementation details are omitted for simplicity and clarity. Figure A.1 illustrates the LOLIPoP infrastructure at a high level as it was described in Chapter 4. The pass begins by building an Interprocedural Control Flow Graph (ICFG) that is annotated with any available profile information. The static memory model (SMM) is then built by extracting location sets from SUIF's symbol tables. We then perform the bottom-up (BU) and top-down (TD) phases of the analysis as described in Section 3.7.

Figure A.1: The LOLIPoP infrastructure.

The following source code represents our implementation of the high level block diagram shown in Figure A.1.

```
void
LOLIPoP( Suif_IR *ir )
{

    /* Construct the interprocedural control flow graph */
    interprocedural_cfg *ICFG = build_ICFG( ir );

    if( edge_profile_enabled )
        ICFG->ep_set(< edge_profile_file >);

    /* Construct the static memory model */
    static_mem_model *SMM = build_SMM( ir , ICFG );

    /* Bottom Up Phase to collect all procedure summarizing tf matrices */
    tmtrx_depot *TMD = bottom_up_tf_collect(ICFG, SMM);

    /* Top Down Phase to propagate point−to information */
    top_down_pt_propagate(ICFG, SMM, TMD);

    /* Suif_IR is now annotated with all probabilistic points−to information */
}
```

The remainder of this appendix will present the important source code for the Bottom Up analysis phase of LOLIPoP. This algorithm was fully described in Chapter 3.

A.1 Suif Instruction to Transformation Matrix

The following method summarizes a single SUIF instruction into a transformation matrix. Only the unsafe version of this method is presented here.

```
transfer_fn_mtrx *
get_tf_mtrx ( Suif_instruction *si )
{
    if (!ICFG->has_points_to_graph_side_effects ( si ))
        return Identity_Mtrx ;

    transfer_fn_mtrx *tfm = Sparse_Identity_Mtrx ;

    /* Get matrix row-id for the location set on the left side of the asgn. */
    int ls_loc_id = StaticMemoryModel->get_ls_loc_id ( si );

    /* Get matrix row-id for the location set on the right side of the asgn. */
    int rs_loc_id = StaticMemoryModel->get_rs_loc_id ( si );


    /* Construct the matrix */

    double p = 1 / StaticMemoryModel->number_of_ptrs ( ls_loc_id );

    if ( ls_locid != rs_loc_id )
    {
        tfm->set ( ls_loc_id , ls_loc_id ) = 1 - p;
        tfm->set ( ls_loc_id , rs_loc_id ) = p;
    }


    /* Handle function call instructions */
    if ( si->is_function_call )
    {
        Suif_function_call *fc = si->get_function_call ()
        tfm = tfm * get_tf_mtrx ( fc );
    }

    return tfm ;
}
```

A.2 Function Call to Transformation Matrix

The following method summarizes a single SUIF function call instruction into a transformation matrix.

```
transfer_fn_mtrx *
get_tf_mtrx(Suif_function_call *fc)
{

    /** TMD->get_tf_mtrx() queries the tf matrix storage data structure.
     *    - If the target of the function call is a library function, then
     *       get_tf_mtrx() return it's Transformation matrix if it is
     *       available. Otherwise it returns the identity matrix.
     *    - If the matrix is yet to be stored in the tf matrix storage
     *       data structure (occurs because of recursion), then the identity
     *       matrix is returned and the current control flow graph is marked
     *       so that the analysis will later re-analyze this control flow graph.
     */

    transfer_fn_mtrx *tfm;

    if(fc->isDirectFunctionCall) /* does not use function pointer */
    {

        tfm = Sparse_Identity_Mtrx;

        /* Get the ID associated with the target cfg being called */
        int fn_callee_id = ICFG->get_function_callee_id(fc);

        /* Create matrix ensuring to map and unmap arguments to parameters */

        tfm = get_tf_mtrx_map_args_to_params(fc, fn_callee_id) * tfm;

        tfm = TMD->get_tf_mtrx(fn_callee_id) * tfm;

        tfm = get_tf_mtrx_map_params_to_args(fc, fn_callee_id) * tfm;
    }
    else if (fc->isIndirectFucntionCall) /* uses function pointer */
    {

        tfm = Sparse_Zero_Mtrx;

        for(int i = 0; i < ICFG->get_number_of_callees(fc); i++)
        {
            /* Get one of the IDs associated with a target cfg being called */
            int fn_callee_id = ICFG->get_function_callee_id(fc, i);
            double callee_prob = ICFG->get_function_callee_prob(fc, i);
```

```
        transfer_fn_mtrx tmp_tfm = Sparse_Identity_Mtrx;

        tmp_tfm =
            get_tf_mtrx_map_args_to_params(fc, fn_callee_id) * tmp_tfm;

        tmp_tfm = TMD->get_tf_mtrx(fn_callee_id) * tmp_tfm;

        tmp_tfm =
            get_tf_mtrx_map_params_to_args(fn_callee_id, fn_caller_id) * tmp_tfm;

         tfm += tmp_tmf * callee_prob;
      }
  }

  if(ICFG->isRecursive(fc))
  {
      /* If recursive, make this a weak update (RECURSIVE_BINDING_PROB = 0.6) */
      tfm = tfm * RECURSIVE_BINDING_PROB;
  }

  return tfm;
}
```

A.3   Basic Block to Transformation Matrix

The following method summarizes an entire basic block into a transformation matrix.

```
transfer_fn_mtrx *get_tf_mtrx(Basic_block *bb)
{
   if(!bb->has_points_to_graph_side_effects())
      return Sparse_Identity_Mtrx;

   /* memoization table is actually checked here, but this code is omitted */

   transfer_fn_mtrx *tfm = Sparse_Identity_Mtrx;
   for(int i = 0; i < bb->number_of_instrs(); i++)
   {
      Suif_instruction *si = bb->get_instr(i);

      tfm = get_tf_mtrx(si) * tfm;
   }
   return tfm;
}
```

A.4 Control Flow Path to Transformation Matrix

This method returns the transformation matrix associated with any control flow path within a single function. Many minor details to handle specific corner case conditions such as cycles caused by `goto` statements are omitted for clarity. The function takes in two main parameters – `bb_start` and `bb_end`, which represent the initial and final basic blocks represented by the intraprocedural control-flow path of interest. Our ICFG normalizes all control flow graphs such that they have a single synthetic and empty entry and exit. This trivially allows us to compute the transformation matrix for an entire procedure using this method.

```
transfer_fn_mtrx *get_tf_mtrx(Basic_block *bb_start, Basic_block *bb_end)
{

    /* Check the memoization table to see it this path has already
       been computed */

    if(memoize_tbl->exists(bb_start, bb_end))
    {
        return memoize_tbl->get(bb_start, bb_end);
    }

    /**
     * Recursive Base Case:
     * If the initial basic_block in the path is the same as
     * the final basic block in the path then return the matrix
     * representing this basic block
     */
    if(bb_start->equals(bb_end))
    {
        transfer_fn_mtrx *tfm = get_tf_mtrx(bb_start);

        if(bb_start->is_loop_header())
        {
            Loop lp = bb_start->get_loop();
            transfer_fn_mtrx *
              tfm_loop = get_loop_tf_mtrx(loop, is_final_bb_end_in_path);

            tfm = tfm * tfm_loop;
        }

        memoize_tbl->memoize(tfm, bb_start, bb_end);
        return tfm;
```

```
}


/* Get the direct dominator of bb_end */
Basic_block bb_end_dd = ICFG->get_direct_dominator(bb_end);


/**
 * Recursive call to get the tf matrix represented by the path
 * from bb_start to bb_end's direct dominator (=> tfm_start_to_end_dd)
 */
transfer_fn_mtrx *tfm_start_to_end_dd = get_tf_mtrx(bb_start, bb_end_dd);


/**
 * We now must compute the tf matrix represented by the path
 * from bb_end's direct dominator to bb_end (=> tfm_end_dd_to_end)
 */

transfer_fn_mtrx *tfm_end_dd_to_end = Sparse_Zero_Mtrx;

for(int i = 0; i < bb_end->num_or_predecessors; i++)
{
     transfer_fn_mtrx *
        tfm_tmp = get_tf_mtrx(bb_end_dd, bb_end->get_predecessor(i));


     /* scale the matrix based on the fan-in edge probability */
     tfm_tmp *=
        ICFG->get_fan_in_edge_probability(bb_end->get_predecessor(i), bb_end);

     tfm_end_dd_to_end += tfm_tmp;
}


transfer_fn_mtrx *tfm_end = get_tf_mtrx(bb_end);

if(bb_end->is_loop_header())
{
   Loop lp = bb_end->get_loop();
   transfer_fn_mtrx *
      tfm_loop = get_loop_tf_mtrx(loop, is_final_bb_end_in_path);

    tfm_end = tfm_end * tfm_loop;
 }

tm_end_dd_to_end = tfm_end * tm_end_dd_to_end;

/* Compute the final transformation matrix */
transfer_fn_mtrx *tfm = tm_end_dd_to_end * tm_start_to_end_dd
```

```
    /* Memoize this result */
    memoize_tbl->memoize(tfm, bb_start, bb_end);

    return tfm;
}
```

Appendix B

LOLIPoP from the Command Line

LOLIPoP is a command line utility invoked with the command `ppa`. The `ppa` utility takes in a suif-linked SUIF-IR formatted file (*.spl) as an input and generates an annotated SUIF-IR file (*.spx). In this appendix we list the various runtime options available to the user. Here we show what prints out when a user invokes the `ppa --help` command.

```
Usage: ppa -.{OUTPUT_SUFFIX} [OPTIONS] [FILES]
  For every pointer dereference, this analysis probabilistically identifies
  which static location sets are being referenced.
Example:
  scc -.spd file1.c file2.c file3.c
  linksuif file1.spd file1.spl file2.spd file2.spl file3.spd file3.spl
  ppa -.spx file1.spl file2.spl file3.spl
OPTIONS:
  [--help],                     defines the various feature enabling flags
                                provided by this application
  [-ppa_debug],                 outputs debugging & warning information
  [-output_file <string>],      specify PPA pointer dereference results
```

```
                                file
 [-stats_file <string>],        specify PPA performance & accuracy statistics

                                results file
 [-matlab_file_out <string>],   outputs a matlab transformation matrix

                                debugging script
 [{-pgm_analyze <string>}*],    debug a specific function using a pgm matrix

                                output
 [-benchname <string>],         specify the name of the benchmark being

                                evaluated
 [{-ep_file <string>}*],        specify control flow edge profiling data

                                see ep --help for more info about generating

                                such a file
 [-icall_file <string>],        specify a file to be used for tracking

                                the targets of indirect function calls

                                [default=ppa_icall.xml]
 [-quiet],                      minimizes the amount of print statements

                                & warnings
 [-conserve_memusage],          performs the analysis with minimal heap

                                memory usage [disables aggressive_memusage

                                flag]
 [-aggressive_memusage],        performs the analysis aggressively with

                                respect to heap memory usage [disables

                                conserve_memusage flag]
 [-icfg_dot_outfile <string>],  outputs the ICFG in .dot format
 [-trust_profile_info],         a probability of zero is assigned to profiled

                                edges that are not taken during the profiling
```

```
                             phase
  [-merge_heap_callsites],   merges all heap callsites into one location
                             set
  [-merge_temp_strings],     merges all temp string variables into one
                             location set
  [-field_sensitive],        enables field sensitivity for C structs
  [-pt_contexts <int>],      specify the number of points-To contexts
                             per function to be stored [default=1]
  [-use_inaccurate_gstf],    enables a faster (unsound) geometric series
                             transform
  [-phase_execute <int>],    indicates upto which phase the analysis
                             should execute: 1 - build ICFG; 2 - build
                             Static Memory Model; 3 - Bottom Up Analysis;
                             4 - Top Down Analysis; [default=4; executes
                             all 4 phases]
  [-evaluate_unreachable],   forces the analysis of functions that are
                             statically deemed unreachable
  [-safe_sva],               perform shadow variable alias (sva) analysis
                             (makes the analysis multi-level safe)
  [-blobs_supported <int>],  when performing sva, specify the maximum
                             number of blobs that can be used [default=5]
  [-blob_size_threshold <int>], when performing sva, specify the minimum
                             blob size [default=15]
  [{-custom_alloc <string>}*] specify custom allocator functions that
                             return unique heap memory
```

Appendix C

Matrix Exponentiation and Geometric Series Transformations

This appendix explains how the matrix geometric series operation can be directly mapped to solving the exponentiation operation. A given loop has a transformation matrix equal to A^n, where A is the transformation represented by the body of the loop and n is the number of iterations. For example, consider the transformations that are applied to a simple `for` loop:

```
for(i = 0; i < k; i++)
{
    ...
}
```

CASE 1   Assuming the loop has an inner transformation matrix defined by the square $n \times n$ matrix A and the loop iterates a constant k times. Therefore the overall transformation matrix defined by this `for` loop is equal to A^k. A^k can be efficiently computed using diagonalization or eigenvalue decomposition.

Definition   A square matrix A is *diagonizable* if there is an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix; the matrix P is said to *diagonalize A*.

$D = P^{-1}AP$

$A = PDP^{-1}$

This can be used in computing powers of a matrix:

$A^2 = PDP^{-1}PDP^{-1}$

$A^2 = PDIDP^{-1}$

$A^2 = PD^2P^{-1}$

In General, for any positive integer k:

$A^k = PD^kP^{-1}$

Using this property, taking a power of a diagonal matrix is trivial.

$$D^k = \begin{pmatrix} d_1^k & 0 & \dots & 0 \\ 0 & d_2^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & d_n^k \end{pmatrix}$$

CASE 2   Assuming the loop has an inner transformation matrix defined by the square $n \times n$ matrix A and the loop iterates a variable amount of times ranging from 0 to k. Therefore the overall transformation matrix defined the by this `for` loop is equal to $\frac{1}{k+1}\sum_{i=0}^{k} A^i$. This geometric series expression can also be efficiently computed using diagonalization.

$\frac{1}{k+1}\sum_{i=0}^{k} A^i = \frac{1}{k+1}[A^0 + A^1 + A^2 + \dots + A^k]$

$\frac{1}{k+1}\sum_{i=0}^{k} A^i = \frac{1}{k+1}[PD^0P^{-1} + PD^1P^{-1} + PD^2P^{-1} + \dots + PD^kP^{-1}]$

$\frac{1}{k+1}\sum_{i=0}^{k} A^i = \frac{1}{k+1}P[D^0 + D^1 + D^2 + \dots + D^k]P^{-1}$

$$\frac{1}{k+1} \sum_{i=0}^{k} A^i = \frac{1}{k+1} P[\sum_{i=0}^{k} D^i] P^{-1}$$

$$\sum_{i=0}^{k} D^i = \begin{pmatrix} \sum_{i=0}^{k} d_1^i & 0 & \cdots & 0 \\ 0 & \sum_{i=0}^{k} d_2^i & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{i=0}^{k} d_n^i \end{pmatrix}$$

$$\sum_{i=0}^{k} D^i = \begin{pmatrix} \frac{1-d_1^k}{1-d_1} & 0 & \cdots & 0 \\ 0 & \frac{1-d_2^k}{1-d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1-d_n^k}{1-d_n} \end{pmatrix}$$

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