Pre-FEC and Post-FEC BER as Criteria for Optimizing Wireline Transceivers

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***Abstract*—Forward-error-correction (FEC) codes have become an integral part of high-speed wireline links. Signal-to-noise ratio, minimum mean-squared error, and pre-FEC BER are common performance metrics used to design and optimize link parameters, such as the tap coefficients in feed-forward and decision-feedback equalizers. This paper shows that the equalizer parameters found by conventional methods do not necessarily minimize post-FEC BER due to the unaccounted-for negative impact of DFE error propagation on FEC performance. However, the introduction of 1/(1+D) pre-coding eliminates long error bursts so that both pre-FEC and post-FEC BER are minimized with the same equalizer coefficients. These observations may have implications on the architecture and optimization of wireline transceivers.**

***Keywords*—forward error correction (FEC), bit error rate (BER), wireline, receiver, optimization, feed-forward equalization (FFE), decision-feedback equalization (DFE), pre-coding**

I. Introduction

In high-speed wireline applications, the prevailing trend is towards architectures relying heavily on digital signal processing (DSP), which benefits from CMOS technology scaling. Forward-error-correction (FEC) codes have also become an integral part of the DSP, lowering the post-FEC BER by several orders of magnitude compared to the raw pre-FEC BER. The feed-forward equalizer (FFE) and decision-feedback equalizer (DFE) are common DSP blocks at the receiver, each offering benefits and shortcomings. For example, a linear finite-impulse-response (FIR) FFE can reduce both the pre-cursor inter-symbol interference (ISI) and post-cursor ISI but may lead to noise amplification. On the other hand, the DFE does not suffer from noise amplification but can only remove post-cursor ISIs. Moreover, alleviating the critical timing path in the DFE feedback loop requires parallelization, which generally limits the DFE to only 1-2 taps in 100Gb/s+ wireline applications [1-2]. FFE and DFE tap coefficients are typically optimized to maximize signal-to-noise ratio (SNR) or to minimize the mean-squared error (MMSE) or pre-FEC BER [3-5]. However, the parameters found by these conventional methods do not necessarily correspond to the minimum post-FEC BER operating point, which is what we ultimately care about.

Error propagation in decision-feedback equalization can significantly impact BER [6-7]. A DFE removes channel ISI by registering past equalized symbols in the feedback path and using them to estimate and cancel ISI from the current symbol. However, if any past symbol registered in the DFE is wrong, the receiver's ISI estimate is biased and may increase the probability of additional symbol errors. Errors may thus propagate around the DFE feedback loop and result in FEC code failures. In current long reach wireline SerDes applications, such as 100GBase-KP4, Gray-coded 4-PAM signaling and Reed-Solomon (RS) FEC are standard [8-9]. In linear FEC codes on GF(2^m), the encoder groups every m bits into one FEC symbol, and the decoder can correct up to t erroneous FEC symbols in an n-symbol codeword. All m bit errors in each erred FEC symbol are corrected so long as the total number of FEC symbol errors does not exceed t. Hence, higher-order RS codes can correct longer error bursts and have therefore been specified, in part, to accommodate DFE error propagation. Error bursts can become much longer when DFE tap weights are large and/or alternating in sign [10]. Such burst errors can reduce the coding gain offered by popular FEC codes [6][11]. Even RS codes often used in wireline links and generally considered effective at correcting bursts are still significantly hampered by DFE error propagation.

This paper presents an accurate and efficient methodology for finding the impact of wireline transceiver parameters, such as equalizer coefficients, on post-FEC BER. The methodology relies on a statistical model proposed in [10] to accurately estimate post-FEC BER for high-speed wireline links subject to DFE error propagation. Section II establishes a system-level wireline transceiver model to estimate BER considering continuous-time-linear equalizer (CTLE) noise shaping and FFE noise enhancement. Section III uses a simplified CTLE and channel model to show that different equalizer settings are required to minimize pre-FEC and post-FEC BER. We then present simulation results with more practical CTLE and channel models across a variety of channels. Section IV will describe 1/(1+D) pre-coding and use the model to find its impact on pre-FEC and post-FEC BER and the optimal equalizer coefficients in each case. Finally, Section V concludes the paper.

II. Wireline Transceiver System Model

A. System Overview

Fig. 1 shows our proposed high-speed wireline system model communicating symbols b_k using pulse-amplitude modulation (PAM) signaling with time index k. The PAM symbols are filtered by an equalized channel response $\alpha(z) = \cdots \alpha_{-1}z^{1} + \alpha_0 + \alpha_1 z^{-1} + \cdots + \alpha_k z^{-k} + \cdots$ with main cursor α_0. The response $\alpha(z)$ is the physical channel's pulse response convolved with the impulse response of other components in the link, such as the transmitter (TX) FFE, TX driver, CTLE and receiver (RX) FFE. The physical channel is subject to additive white Gaussian noise (AWGN) filtered by the CTLE, thus

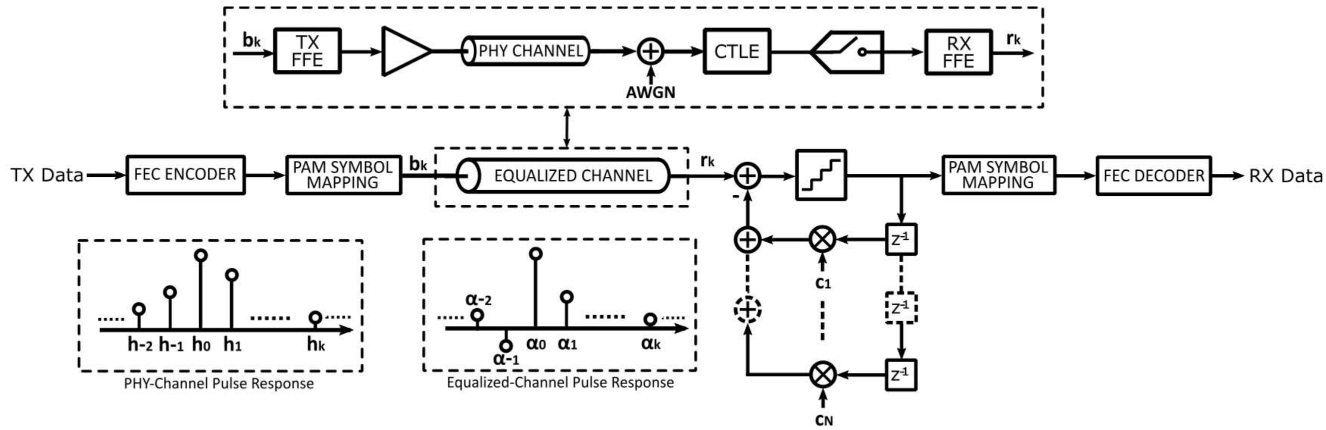

Fig. 1. Proposed wireline transceiver system model with an N-tap DFE at receiver, the equalized pulse response $\alpha(z)$ is generated by convolving PHY-channel pulse response $h(z)$ with the impulse response of other components in the equalized channel.

creating correlated noise samples. While ADC impairments can be modeled using the methods proposed by [12], we simplify the problem by assuming ideal ADC operation.

A detailed analysis of CTLE noise shaping and RX FFE noise enhancement will be provided in Section II.B and Section II.C, respectively. Once we have computed the equalized channel response $\alpha(z)$, we calculate the probability density function (pdf) of the received samples r_k at its output. These results are then applied to the statistical BER model proposed in [10] to obtain both the pre-FEC and post-FEC BER subject to the error propagation from the N-tap DFE in Fig. 1.

B. CTLE Noise Shaping

The AWGN becomes a colored noise through CTLE filtering. At the CTLE output, we require both the noise variance σ^2 and the noise autocorrelation function $R(\tau)$ to compute noise amplification in the RX FFE. To calculate the noise variance σ^2, we first define $P_n(f) = K$ as the constant power spectral density of the zero-mean AWGN process. Assuming the CTLE has an impulse response $b(t)$ and its Fourier transform is $B(f)$, we calculate the noise autocorrelation function using the Wiener-Khinchin theorem,

$$R(\tau) = K \int_{-\infty}^{\infty} |B(f)|^2 e^{i2\pi f\tau} df. \quad (1)$$

Since an AWGN process is wide-sense stationary, the output of the CTLE has a noise variance

$$\sigma^2 = R(0). \quad (2)$$

C. FFE Noise Enhancement

The optimal tap coefficients in a receiver FFE generally depend on the channel response and noise spectrum. The link's BER performance, in turn, depends on the RX FFE's noise amplification. Thus, in this subsection, we describe how to find the FFE noise enhancement.

First, we define $\boldsymbol{X}$ as a zero-mean random process describing the CTLE-filtered Gaussian noise defined by (1) and (2). The FFE output noise $\boldsymbol{Y}$ is the weighted sum of M correlated random variables ($\boldsymbol{X_1}, \boldsymbol{X_2}, \dots \boldsymbol{X_M}$) sampled at M unit intervals (UI). For a link communicating at a symbol rate $1/T_s$, the covariance $Cov(X_i, X_j)$ between $\boldsymbol{X_i}$ and $\boldsymbol{X_j}$ is

$$Cov(X_i, X_j) = R(T_s \cdot |i-j|). \quad (3)$$

We define β_i as the i^{th} FFE tap coefficient in an M-tap FFE. The total noise variance $Var(\boldsymbol{Y})$ at the FFE output node $Y = \sum_{i=1}^{M} \beta_i X_i$ is

$$Var(Y) = \sum_{i=1}^{M} \sum_{j=1}^{M} \beta_i \beta_j Cov(X_i, X_j). \quad (4)$$

III. The Impact of Varying FFE and DFE Tap Weights on Pre-FEC and Post-FEC BER

A. Performance Criteria

SNR, pre-FEC and post-FEC BER are three performance metrics that could be used to optimize the architecture and coefficients of a wireline transceiver. Currently, RX FFE and DFE coefficients are optimized using LMS adaptation algorithms where SNR is implicitly the optimization criteria. However, DFE error propagation is not captured by the SNR and the SNR-optimal FFE coefficients do not necessarily minimize pre-FEC BER [4] [10]. In addition, the FEC decoding process is much more sensitive to DFE error propagation rather than isolated random errors. Even using pre-FEC BER as the sole criteria for optimization fails to account for this. In this Section, we apply the transceiver model proposed in Section II and use standard FEC codes to compare pre-FEC and post-FEC BER as the criteria to optimize FFE and DFE.

B. Pre-FEC vs Post-FEC BER Optimum

We adopt a channel model with 30 dB insertion loss for a link communicating 4-PAM symbols at 56 GBaud/s subject to 0.55 $V_{P\text{-}P}$ swing at TX. At the receiver, we assume a simplified CTLE model having one zero at 3.77 GHz and two poles at 28.2 GHz and 31.2 GHz which together provide 12 dB peaking gain with 0 dB gain at DC. The CTLE-equalized pulse response is $h(z) = 0.1391\ z^1 + 0.4062 + 0.1876\ z^{-1} + 0.0237\ z^{-2} + 0.0009\ z^{-3}$ including both the CTLE and physical channel. The AWGN integrated rms noise is 4.58 mV_{rms}. State-of-art wireline links employ DFEs with 1-2 taps [1-2]. To illustrate the basic tradeoffs, we first assume a 1-tap DFE and a 7-tap FFE with 2 pre-cursor and four post-cursor taps. There is no pre-emphasis in the TX. When sweeping the 1^{st} post-cursor FFE tap, other FFE tap weights are chosen to minimize all pre-cursor and post-cursor ISIs using MMSE criterion. The post-FEC BER is calculated assuming the standard KP4 RS(544,514, 15) code.

In Fig.2, the pre-FEC BER and post-FEC BER performance surfaces are generated by sweeping the 1st post-cursor FFE tap weight and the DFE tap weight. The FFE main-cursor tap always maintains its amplitude at 1. The DFE tap weights in Figure 2 are normalized to α_0.

We obtain substantially different optimal points on the two performance surfaces plotted in Fig. 2. In Fig. 2 (a), the minimum pre-FEC BER is located at $\alpha_1/\alpha_0 = 0.80$. In Fig. 2 (b), the minimum post-FEC BER appears at $\alpha_1/\alpha_0 = 0.42$. The larger and positive FFE 1st post-cursor tap weight creates a low-pass FIR response that filters noise and improves SNR and pre-FEC BER at the DFE output. However, this implies a commensurately large DFE tap weight, which increases the frequency and length of error propagation bursts. The lower value of $\alpha_1/\alpha_0 = 0.42$ affords a pre-FEC BER 1.3 orders of magnitude higher, but a post-FEC BER that is 23.5 orders of magnitude lower. This suggests that DFE error propagation has a much greater impact on the post-FEC BER. Therefore, the tradeoff between FFE noise enhancement and DFE error propagation must be considered when architecting and optimizing wireline transceivers to minimize post-FEC BER. Unfortunately, LMS equalizer adaptation algorithms do not consider this effect.

C. *Simulation Results*

In this subsection, we provide more extensive simulation results using six measured channel responses to validate our methodology using post-FEC BER to find the optimal equalizer coefficients. The general simulation setup is similar to that used for Fig. 2 except that the TX now has a 1-tap FFE providing 5 dB pre-emphasis, and the RX FFE has 15 taps, including 3 pre-cursor taps and 11 post-cursor taps. An 8th-order CTLE model was applied to equalize all six channels. The equalized channel pulse responses including TX FFE, CTLE and PHY channel are tabulated in Table I. Fig. 3 plots the post-FEC BER of the link using two different criteria to optimize the equalizer coefficients: pre-FEC BER and post-FEC BER. The results are plotted for all 6 channel models at two integrated rms noise levels: 1.62 mV_{rms} and 2.42 mV_{rms}. Not surprisingly, the optimal post-FEC BER obtained by post-FEC BER optimization is always superior to the post-FEC BER obtained from pre-FEC BER optimization. The improvement is most dramatic at lower channel losses and/or lower noise levels. In higher-loss channels, the FFE provides more high-frequency boost and noise enhancement. When random errors dominate over long error bursts, the pre-FEC and post-FEC BER optima coincide.

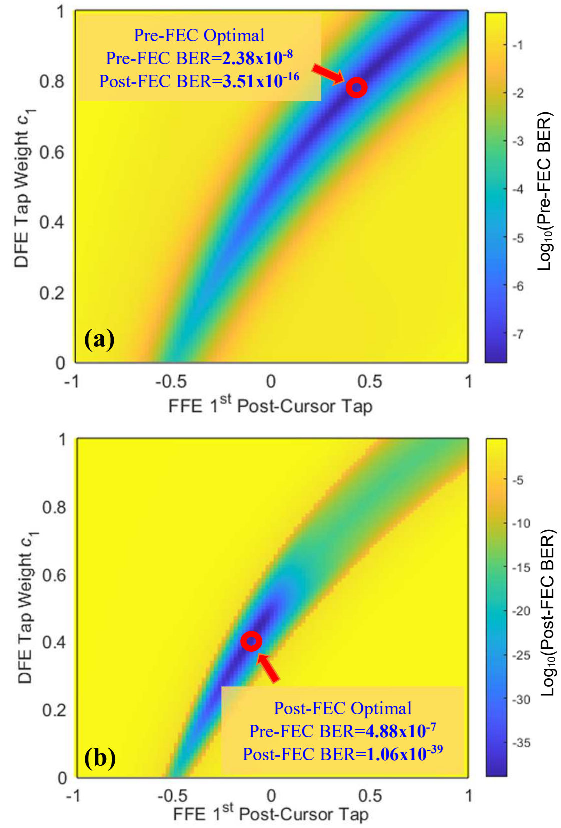

Fig. 2. BER performance surface generated by sweeping the FFE 1st post-cursor tap and the DFE tap weight using (a) Pre-FEC BER (b) Post-FEC BER.

IV. 1/(1+D) PRE-CODING

A well-known technique for mitigating error bursts is 1/(1+D) pre-coding (also referred to as MOD4 pre-coding). Fig. 4 shows a wireline transceiver model incorporating 1/(1+D) pre-coding. The MOD4 encoder accepts input t_k and generates transmitted symbols b_k. The RX decoder accepts the DFE decisions d_k as inputs and produces the outputs y_k, which are estimates of t_k. Fig. 4 also includes two example sequences illustrating how pre-coding mitigates error bursts. The MOD4 decoder removes burst errors because the error d_k-b_k in the current received symbol is added to the error d_{k-1}-b_{k-1} in the previously received symbol. For $c_1 > 0$, the burst error values arise due to DFE error propagation always take alternating signs in the form … +1 -1 +1 … [10]; as a result, consecutive error values cancel when added in the decoder. However, isolated

TABLE I. PULSE RESPONSE OF THE EQUALZIED CHANNEL BY INCLUDING TX FFE, CHANNEL AND CTLE

Case	Channel IL (dB)	Equalized Pulse Response at CTLE Output (mV)												
		α_{-3}	α_{-2}	α_{-1}	α_0	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
1	30	-1.39	-15.3	17.0	173	49.8	34.3	23.2	14.5	8.51	3.60	2.84	0.272	1.37
2	32	-1.45	-13.7	18.4	151	53.9	35.1	24.4	15.5	10.2	4.01	4.52	0.530	2.02
3	34	-1.49	-12.3	19.8	132	54.7	36.3	25.4	16.5	11.1	5.45	4.73	1.75	2.33
4	36	-1.50	-11.1	21.0	117	54.4	37.4	25.5	17.9	11.8	6.62	5.34	2.60	2.71
5	38	-1.50	-9.85	21.7	103	53.7	37.9	26.0	18.8	12.5	7.67	6.07	3.26	3.20
6	40	-1.46	-8.73	21.9	91.4	52.5	37.9	26.3	19.5	13.1	8.53	6.61	3.94	3.63

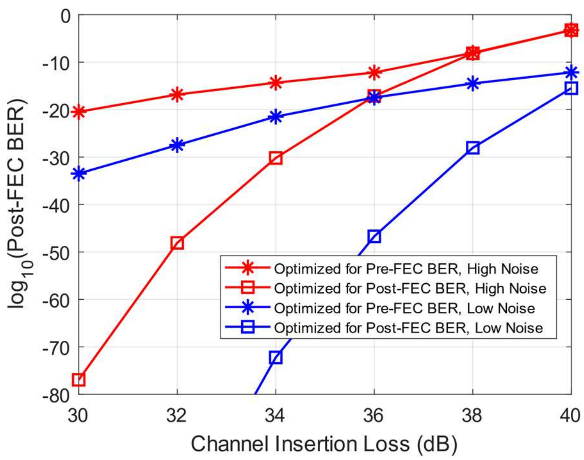

Fig. 3. Post-FEC BER using equalizers minimizing pre-FEC and post-FEC BER, simulated for 6 channel responses and two noise levels and RS(544, 514, 15) FEC.

individual symbol errors give rise to two consecutive symbol errors after decoding. Thus, the second example in Fig. 4 illustrates how a BER penalty arises from random isolated errors in the link. A method to model 1/(1+D) pre-coding using trellis dynamic programming appears in [13]. This method is used to generate the post-FEC BER results including 1/(1+D) pre-coding.

In Fig. 5(a), we plot both the pre-FEC BER and post-FEC BER as a function of α_1/α_0 for the 36dB-loss channel in Fig. 3. For each data point, the DFE tap weight c_1 is fixed at the indicated α_1/α_0 and the corresponding MMSE FFE tap weights are found. For both channels, post-FEC BER is minimized at a lower α_1/α_0 than pre-FEC. Thus, to minimize post-FEC BER, the FFE should be relied upon for more of the RX equalization than an MMSE (or LMS) criteria suggests.

Fig. 5(b) plots post-FEC BER with and without 1/(1+D) pre-coding. Since pre-coding eliminates long error bursts, the minimum post-FEC BER is lower than in Fig. 5(a) and occurs at larger values ofα_1/α_0. In fact, with pre-coding, both post-FEC and pre-FEC BER are minimized with the same equalizer coefficients. Thus, with pre-coding, FFE and DFE tap weights can be optimized using conventional methods without fear that error propagation will result in bursts that hurt post-FEC BER.

V. Conclusion

In this paper, we consider whether architecting and optimizing wireline links using SNR or pre-FEC BER as performance metrics is effective in minimizing post-FEC BER. Burst errors due to DFE error propagation hurt FEC

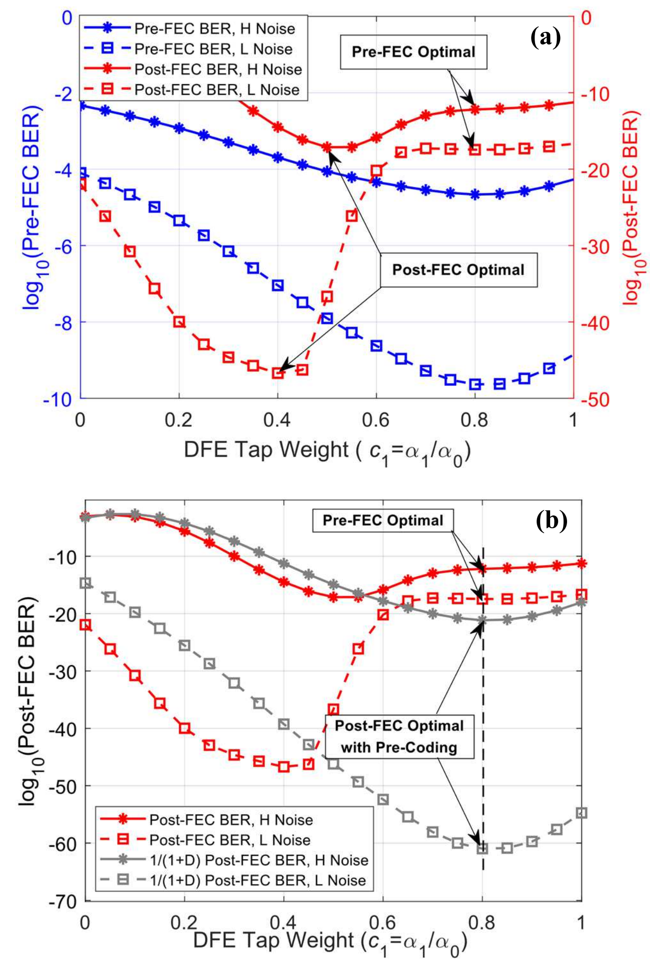

Fig. 5. Pre-FEC vs post-FEC BER as a function of α_1/α_0 simulated using the 36 dB channel case from Fig. 3: (a) without pre-coding (b) with 1/(1+D) pre-coding.

performance. But error propagation is not accurately accounted for when SNR or pre-FEC BER are used as the criteria for architecting and optimizing wireline links. Thus, we showed that, in general, links attain their minimum post-FEC BER with equalizer coefficients very different from those that minimize pre-FEC BER. However, the introduction of 1/(1+D) pre-coding mitigates the impact of error bursts, ensuring that both pre-FEC and post-FEC BER are minimized with the same equalizer coefficients. This analysis may have significant implications on the architecture and optimization of wireline transceivers.

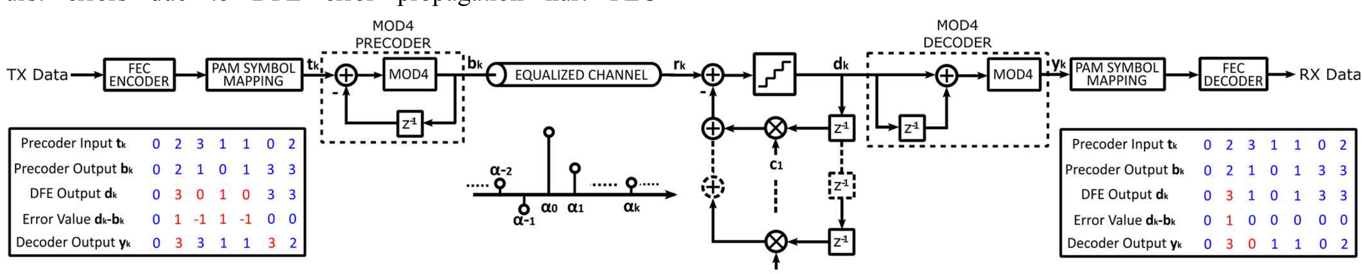

Fig. 4. A modified wireline transceiver system model with an N-tap DFE at receiver to consider 1/(1+D) pre-coding. Two examples are included in the figure illustrating: (1) a DFE burst error across four PAM symbols is mitigated to only two errors (2) a random error is duplicated with pre-coding.

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