Obtaining Digital Gradient Signals for Analog Adaptive Filters

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Motivation

- digital communications is an active area for research in academia and industry
- adaptive filters are important in digital communication applications (equalizers, interference cancellation, etc.)

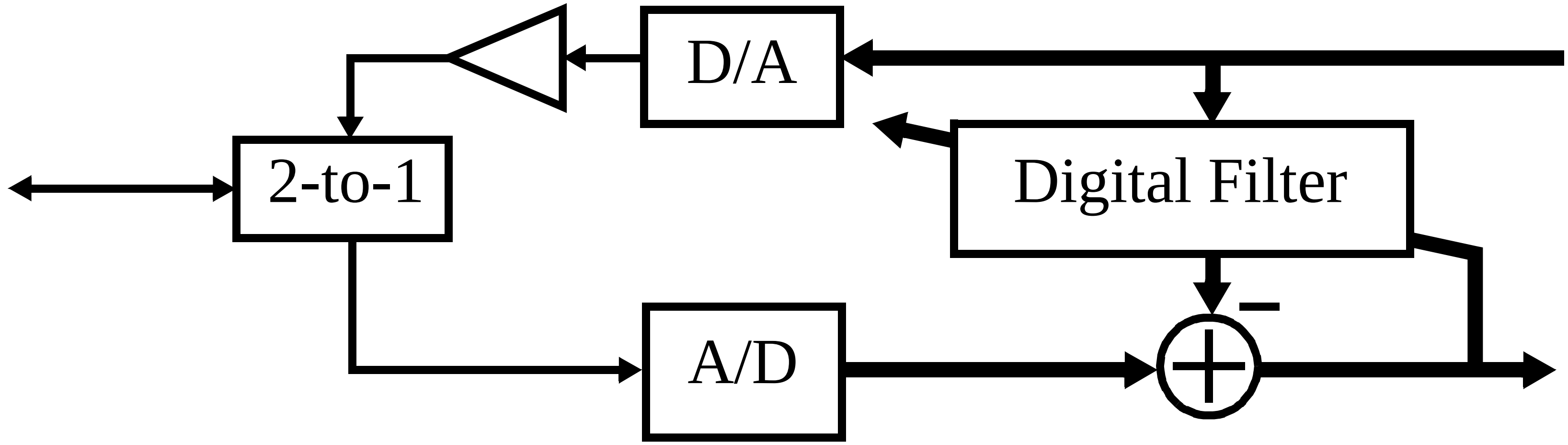
1. increased integration is sought to improve reliability and reduce cost
   $\Rightarrow$ digital & analog circuits must coexist
2. increased data rates pursued
   $\Rightarrow$ analog signal path
3. robust algorithm desired for adaptive signal processing functions
   $\Rightarrow$ implement the adaptation algorithm digitally

Motivation

	Analog	Digital	**Digitally-Programmable Analog**
power consumption	✓		✓
integrated circuit area	✓		✓
relaxed A/D specs	✓		✓
robust		✓	✓
scaleable		✓	
linear		✓	

Echo Cancellation Application

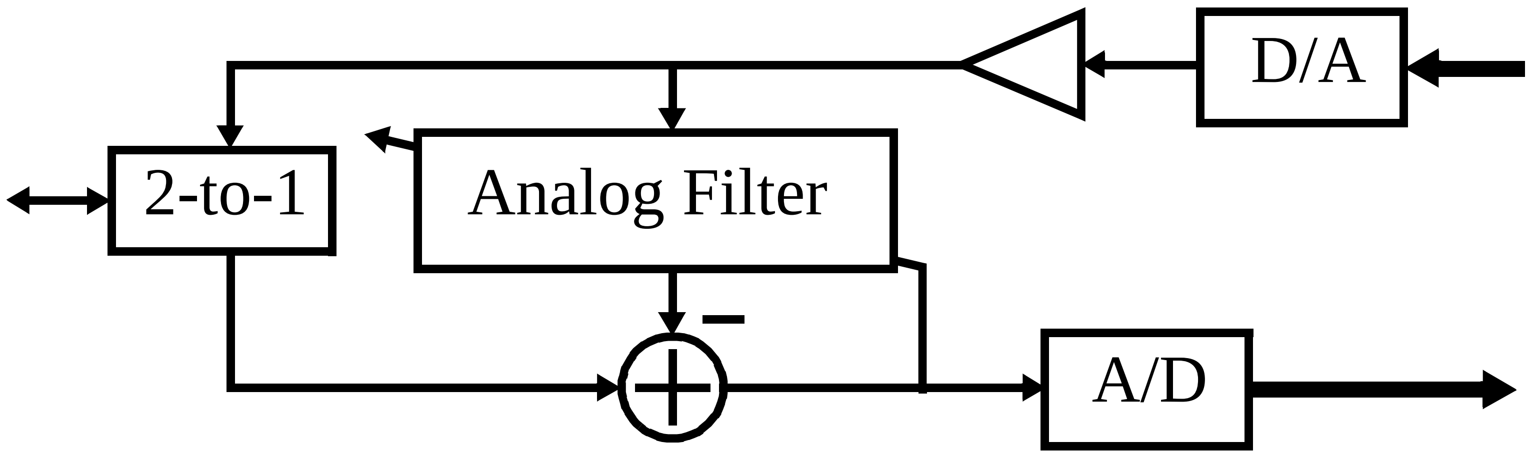
All Digital



- analog front-end still required
- high power consumption in digital logic & A/D at high speeds
- for linear echo cancellation, require highly linear D/A, line driver, and A/D

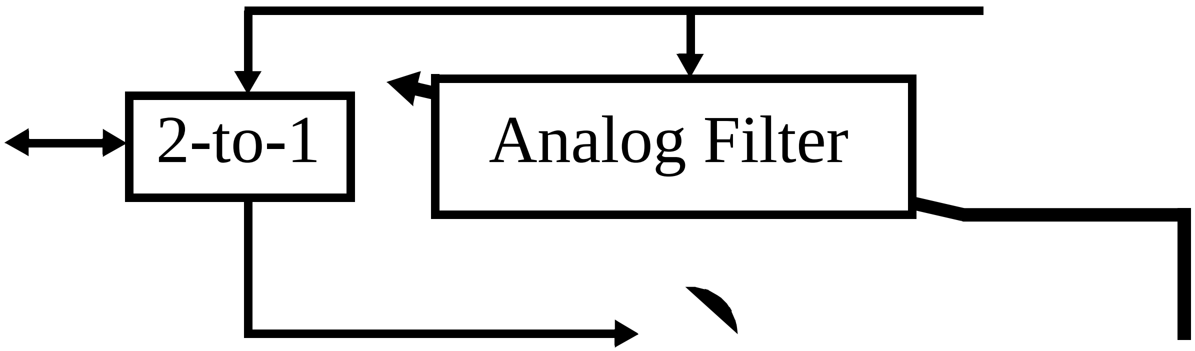
Echo Cancellation Application

All Analog



- considerable extra analog circuitry may be required to implement adaptation algorithm
- LMS adaptation is susceptible to the dc offsets present in analog circuits

Echo Cancellation Application



- low power
- relaxed linearity specifications on D/A, line driver, and A/D
- potentially robust with respect to dc offset effects

Background - LMS Algorithm

- LMS adaptation is popular due to easy hardware implementation
- filter parameters are updated according to:

$$p(k+1) = p(k) + 2\mu \cdot \phi(k) \cdot e(k)$$

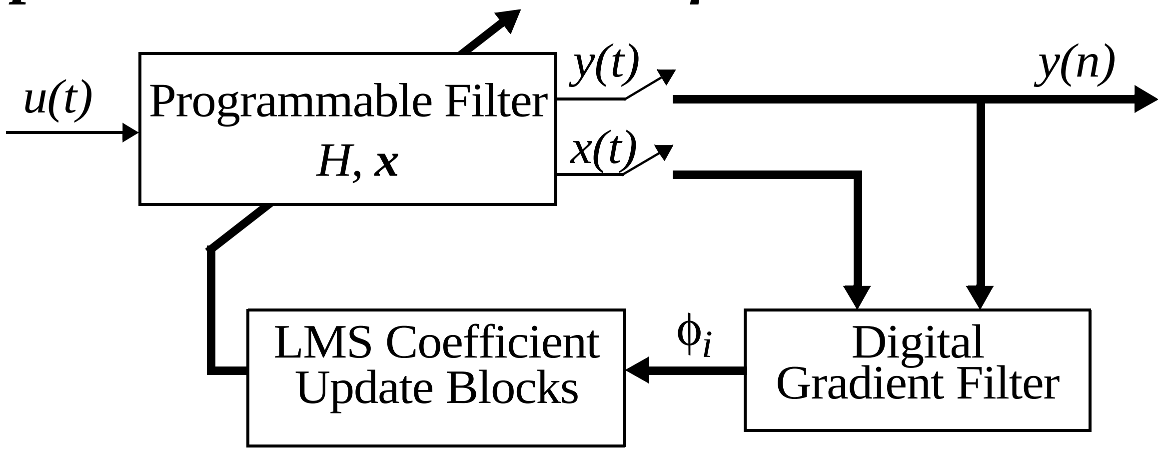
$$\phi = \frac{\partial y}{\partial p}$$

Problem: For an analog filter, how can we obtain the gradient signals, ϕ?

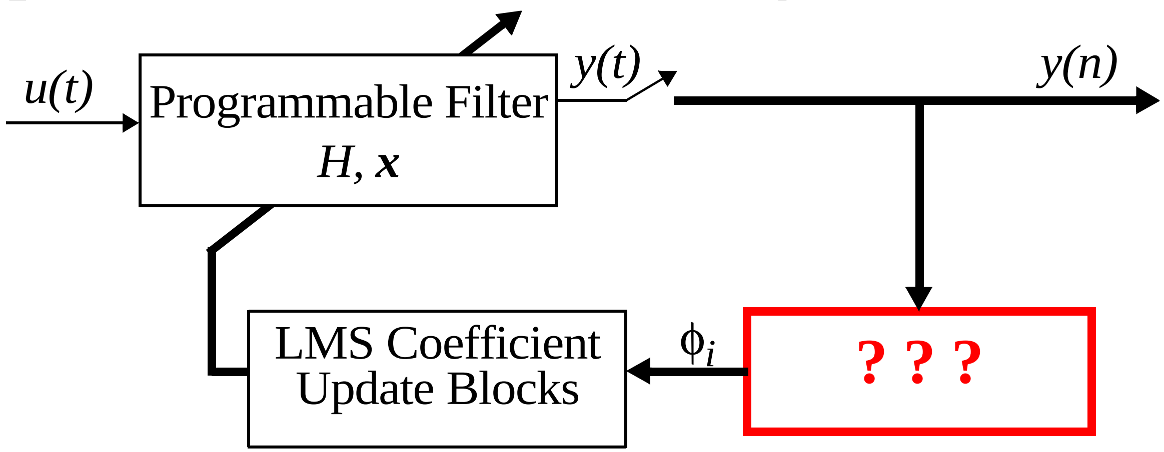
- all analog systems require additional filters to generate the gradient signals
- until now, digitally-programmable analog filters have required additional A/D converters to obtain the gradient signals

Obtaining Digital Gradient Signals

Filter adaptation with access to the filter's internal states

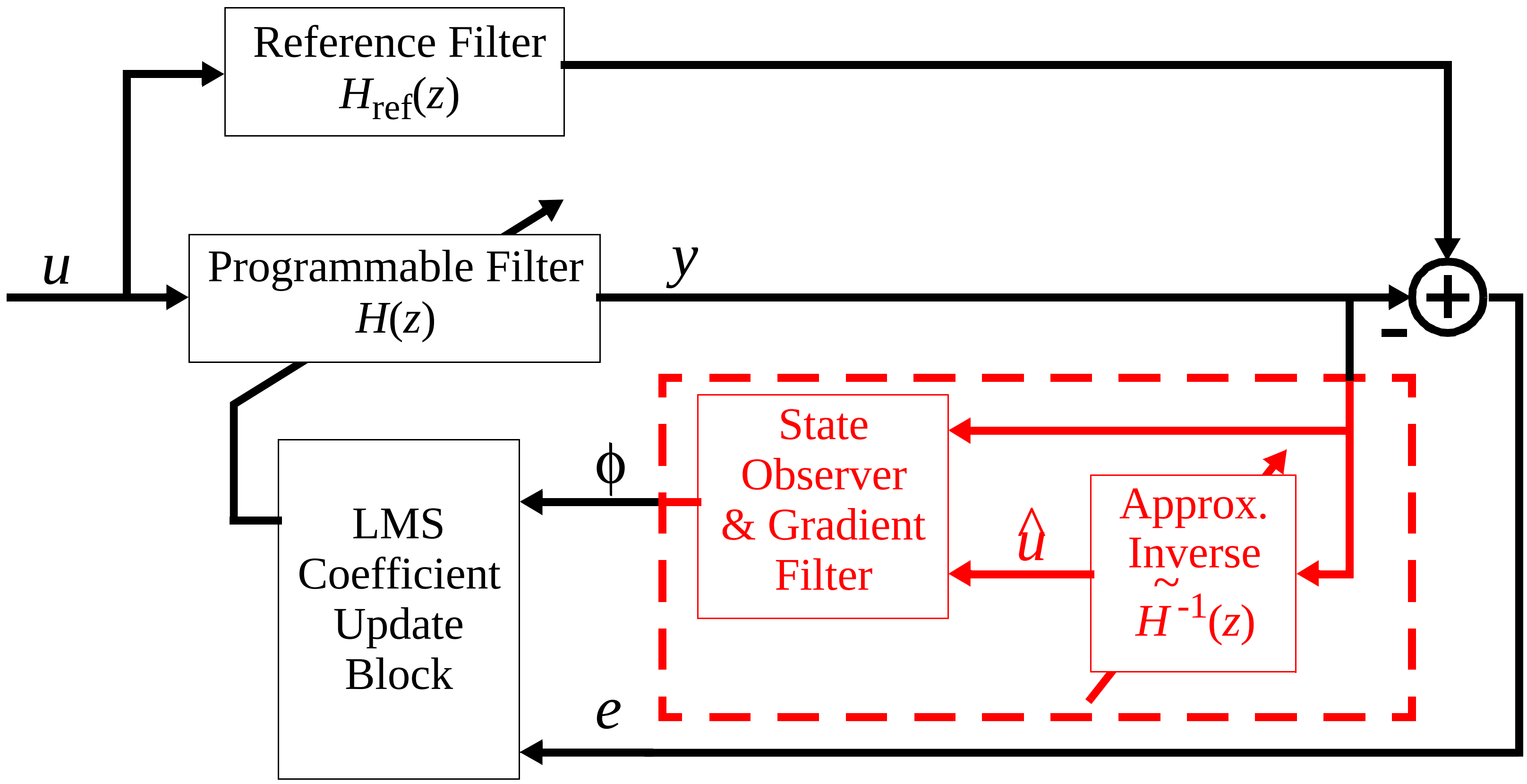


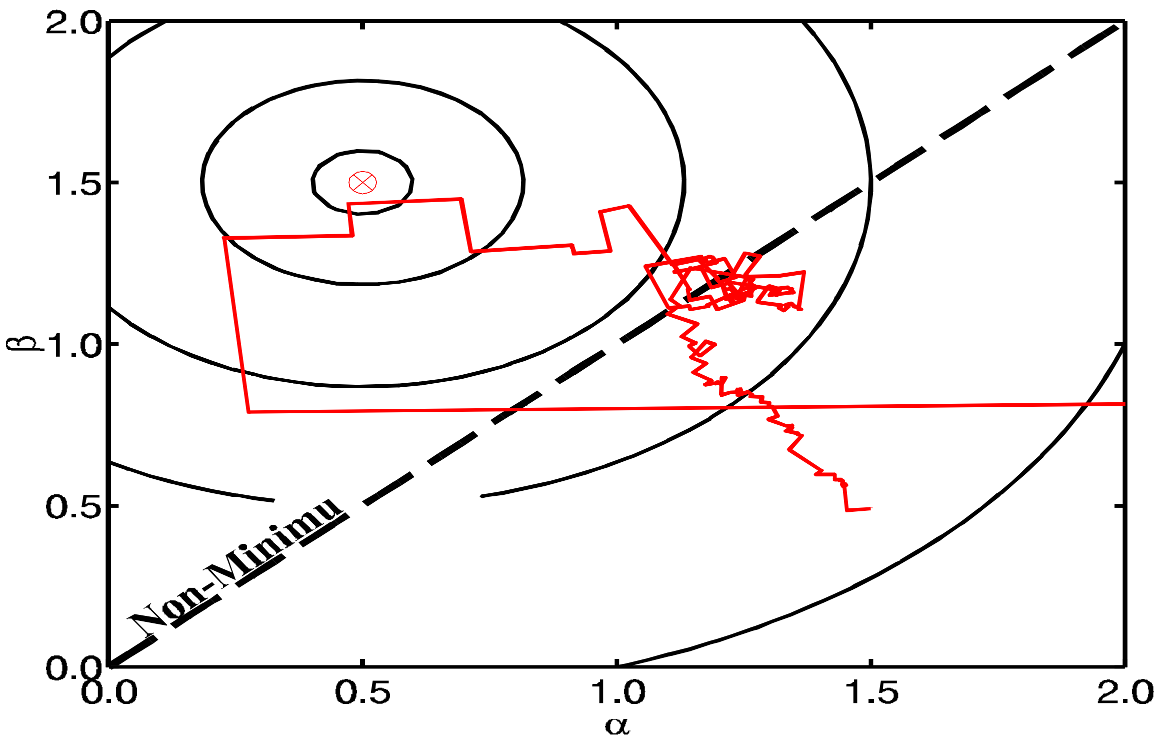
Filter adaptation without access to the filter's internal states



Obtaining Digital Gradient Signals

Model Matching Configuration





2.0
1.5
1.0
0.5
0.0
β
0.0
0.5
1.0
1.5
2.0
α
Non-Minimu

Obtaining Digital Gradient Signals

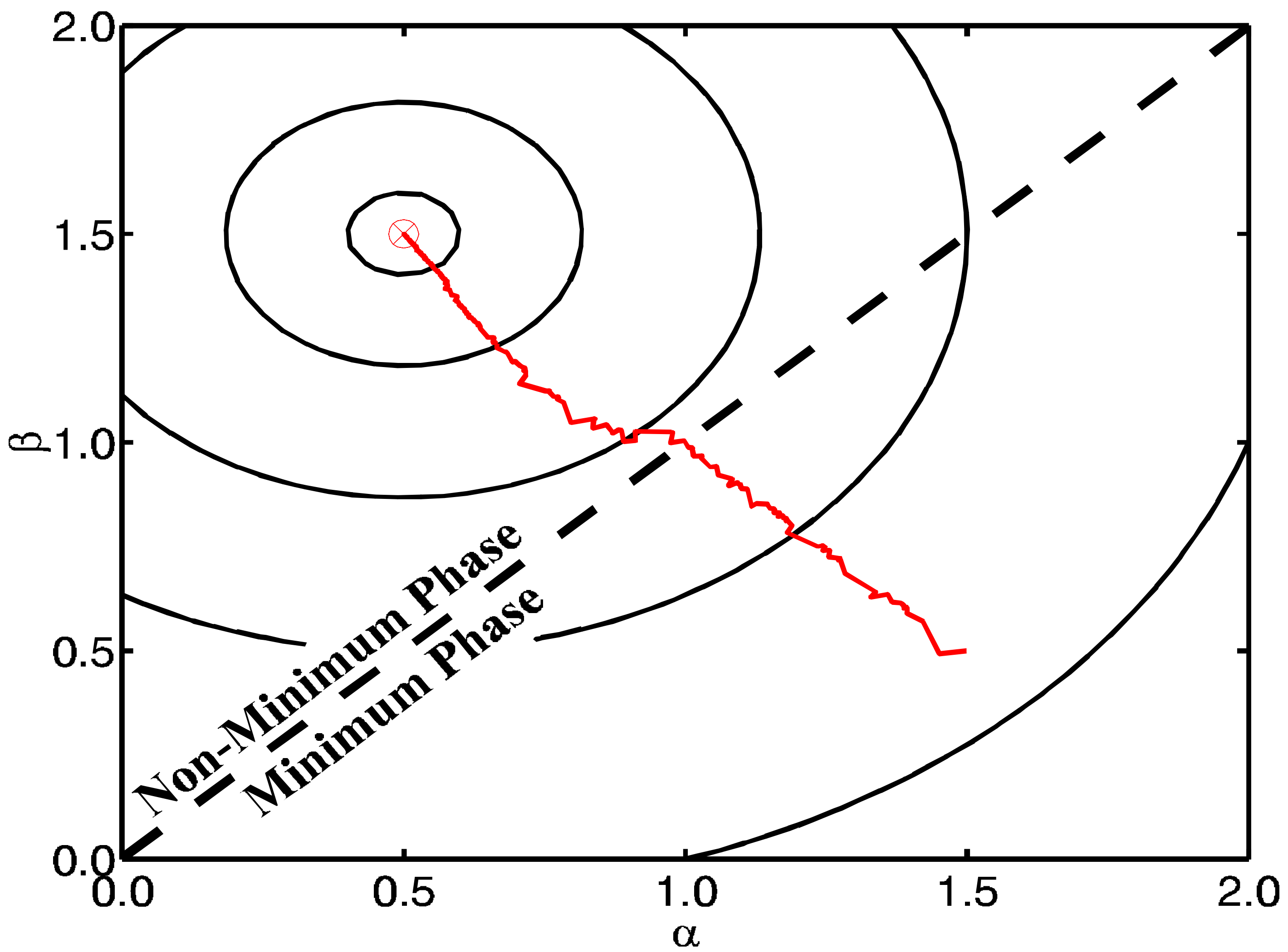
- first, separate $H(z)$ into binomial factors: $H(z) = H_1(z) \cdot H_2(z) \cdots$
- for each minimum phase factor, take the direct inverse: $1/H_i(z)$
- for each non-minimum phase factor, approximate the inverse by introducing delay and truncating the impulse response
- numerically, this is done by a Taylor Series expansion:

$$
\begin{aligned}
H_i(z) &= 1 - az^{-1} \\
\Rightarrow 1/H_i(z) &= 1/(1 - az^{-1}) \\
&= (-a^{-1}z)/(1 - a^{-1}z) \\
&= (-a^{-1}z)(1 + a^{-1}z + a^{-2}z^2 + \ldots) \qquad (*) \\
&= z^{d+1}(-a^{-1}z^{-d} - a^{-2}z^{-d+1} - \ldots - a^{-d-1}) \equiv \tilde{H}_i^{-1}(z)
\end{aligned}
$$

(*) the Taylor Series expansion is valid on the unit circle if $|a| > 1$ (i.e. for all non-minimum phase factors, $H_i(z)$)

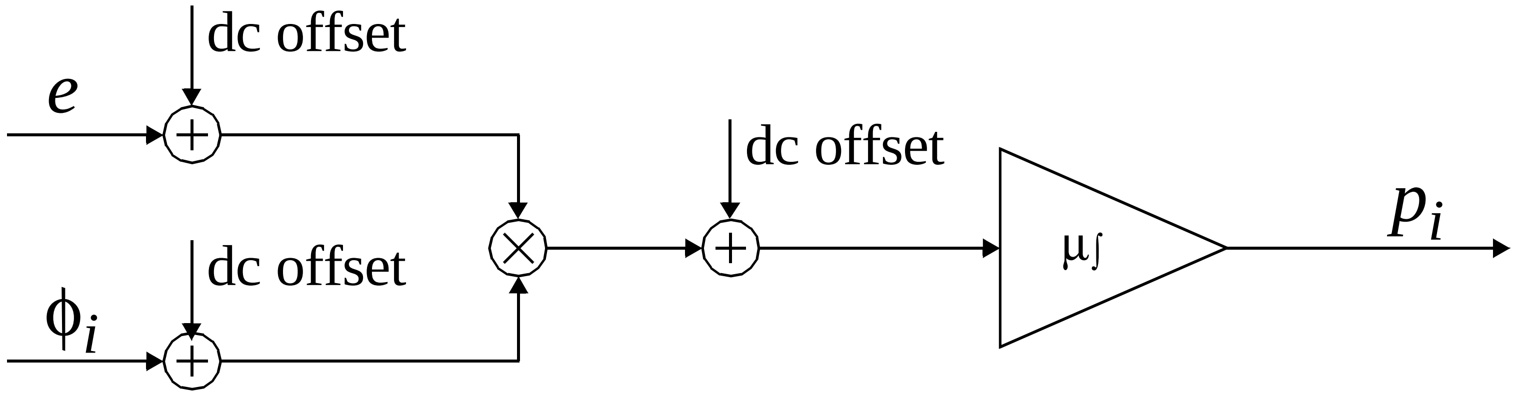
Obtaining Digital Gradient Signals

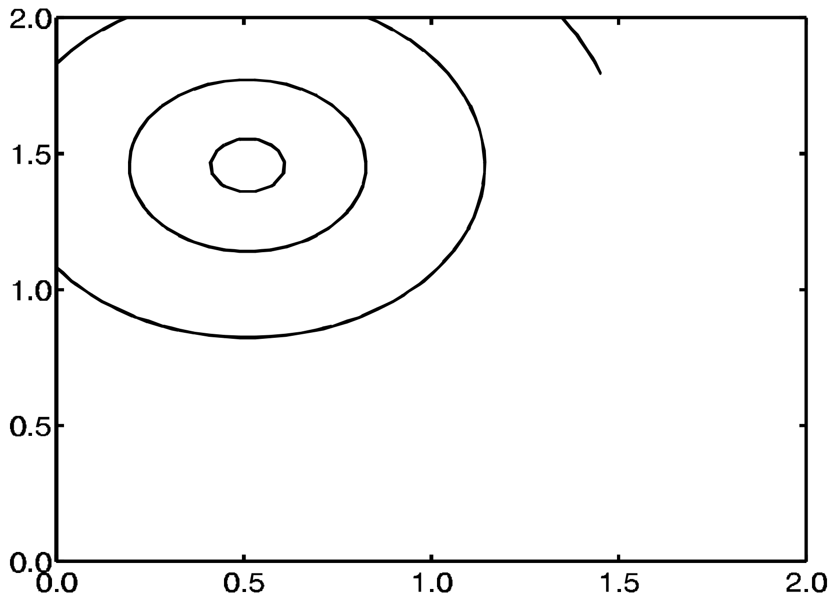
Model Matching Experiment Using Approximate Inverse



Dc Offset Effects

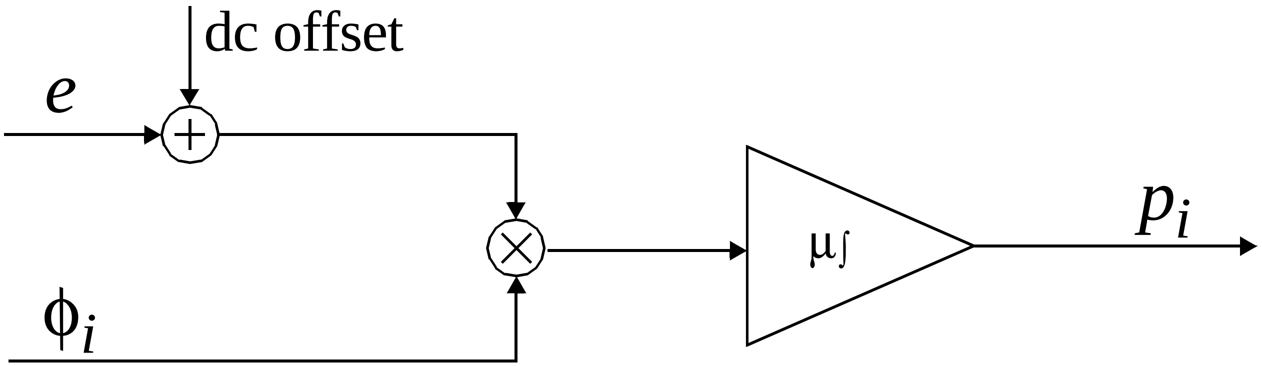
- dc offsets in analog LMS circuitry prevent convergence to optimal parameters

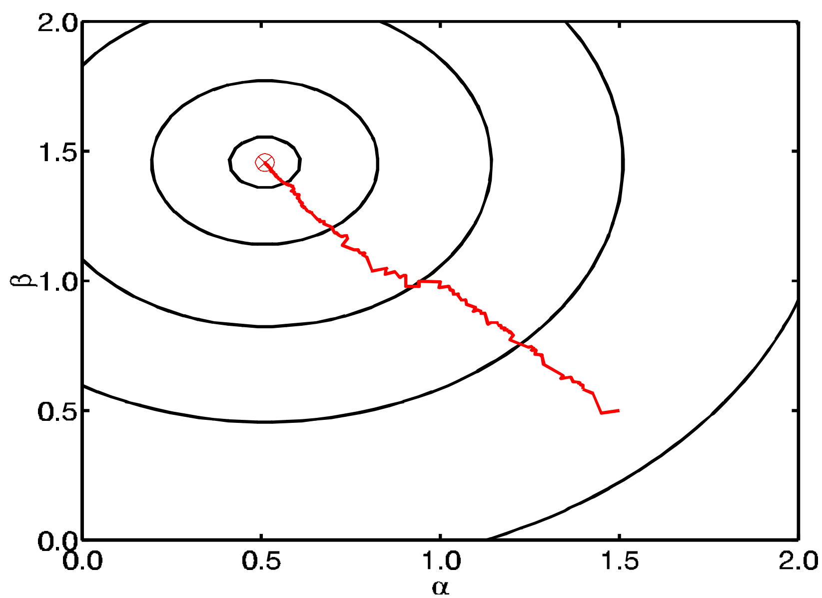




Dc Offset Effects

- by using digital estimates of the gradient signals, two sources of dc offsets are eliminated
  $\Rightarrow$ convergence to optimal parameter values is possible



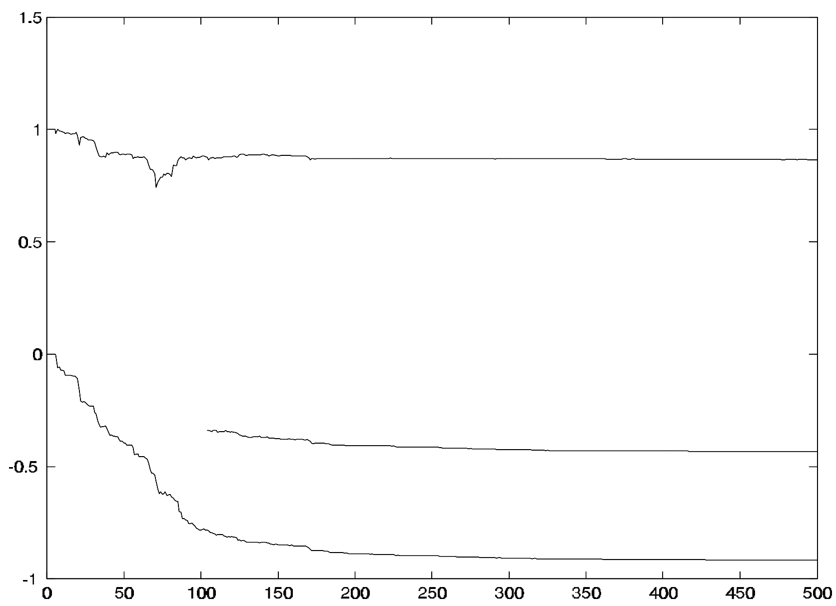


Dc Offset Effects

- 4th order model matching experiment with reference filter

$$H_{ref}(z) = 0.9 - 0.9z^{-1} - 0.4z^{-2} - 0.2z^{-3} + 0.1z^{-4}$$

- approximate inverse transfer function, $\tilde{H}^{-1}(z)$, implemented as 20-tap FIR filter



Conclusions

- a technique was presented for estimating the internal states of a filter with unknown inputs
- LMS adaptation is possible using these approximate state estimates
- useful for digital adaptation of analog filters since sampling the filter states and filter input is not required
- the adaptation is robust with respect to dc offsets