University of Toronto

Department of Electrical and Computer Engineering

ECE241- Digital Systems

Midterm Examination

October 9, 1997

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Signature: ______________________________

Duration: **1 Hour**

Answer ALL questions on this test paper. There is extra space at the end if you need it.

EXAMINER's REPORT

1		/7
2		/9
3		/6
4		/10
5		/23
TOTAL:		/55

Question 1 — [7 marks]

Given the function $f = \bar{x}y + x\bar{y} + \bar{x}\bar{y}$ use algebraic manipulation to derive a minimal sum-of-products expression for its complement, $\bar{f}$. Show all your steps in the space below. Use as few steps in your derivation as possible.

For three input variables, are the following identities true or false (circle the right answer)?

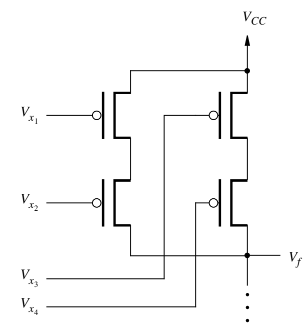
1. $\sum m(1, 2, 3, 4, 5, 6, 7) = M_0$ True or False

2. $\prod M(0, 1, 2, 3, 4, 5, 6) = m_7$ True or False

3. $x_1 x_2 \bar{x}_3 + \bar{x}_2 \bar{x}_3 + x_1 x_2 = x_1 \bar{x}_2 \bar{x}_3 + x_2 \bar{x}_3 + x_1 x_2$ True or False

Question 2 — [9 marks]

The figure below shows half of a CMOS circuit. Draw the other half that contains the NMOS transistors.



On the following page, show the truth table for f

$x_1\ x_2\ x_3\ x_4$	f
0 0 0 0	
0 0 0 1	
0 0 1 0	
0 0 1 1	
0 1 0 0	
0 1 0 1	
0 1 1 0	
0 1 1 1	
1 0 0 0	
1 0 0 1	
1 0 1 0	
1 0 1 1	
1 1 0 0	
1 1 0 1	
1 1 1 0	
1 1 1 1	

Question 3 — **[6 marks]**

In the Karnaugh map method of synthesis, you search for groups of 1's to generate product terms. Consider a valid group of N 1s in a map. For each 1 that is in the group, how many other 1s of the group are directly adjacent? (That is how many neighbors, in the K-map sense, are there for each 1 in the group? Your answer should be a function of N).

Is the following statement true or false?

STATEMENT: If a logic function has a unique minimum-cost sum-of-products form, then it must have a unique minimum-cost product-of-sums form. (By unique we mean that there is only *one* way to write the expression that has the minimum cost).

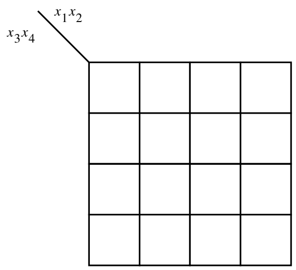
Question 4 — [10 marks]

Find the minimum-cost sum-of-products and product-of-sums forms of the function

$$f = \sum m(0, 2, 8, 9, 10, 15) + D(1, 3, 6, 7)$$

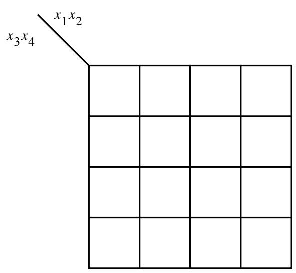
where D is the set of minterms that are don't-cares.

K-map for SOP:



Minimum-cost SOP form:

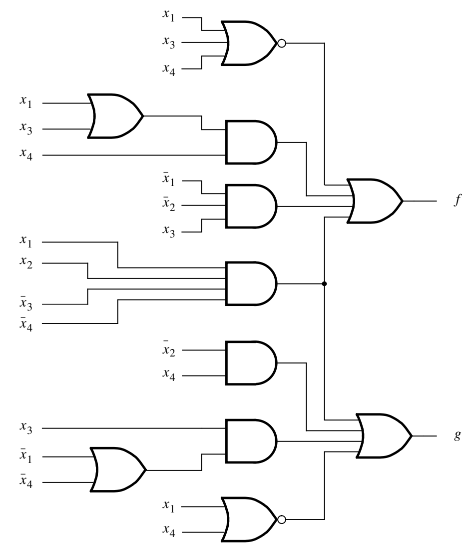
K-map for POS:



Minimum-cost POS form:

Question 5 — [23 marks]

Consider the circuit below that produces outputs f and g.



What is the cost of the above circuit? Use the measure Cost = # gates + # inputs to gates. Count the NOT gates required to complement the inputs. (NOTE: only 1 NOT gate is required to provide the complement of any variable — i.e. even though x_4 bar appears twice, you need only one NOT gate to produce it.)

ANSWER:

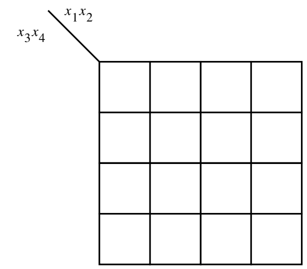
Re-design the circuit to reduce its cost. Use a SOP circuit for both f and g. Your circuit should minimize the total cost of f and g considered at the same time. Give your answer as indicated below.

Fill in the truth table for both f and g

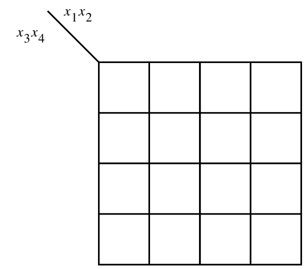
$x_1\ x_2\ x_3\ x_4$	f	g
0 0 0 0		
0 0 0 1		
0 0 1 0		
0 0 1 1		
0 1 0 0		
0 1 0 1		
0 1 1 0		
0 1 1 1		
1 0 0 0		
1 0 0 1		
1 0 1 0		
1 0 1 1		
1 1 0 0		
1 1 0 1		
1 1 1 0		
1 1 1 1		

Show separate K-maps for f and g (don't forget that you are to consider f and g at the same time when producing your optmized expressions from the K-maps).

K-map for f:



K-map for g:



sum-of-products expression for f:

sum-of-products expression for g:

Calculate the total cost of your two-output circuit:

EXTRA SPACE — USE ONLY IF NEEDED

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