Temperature Measurement of Simulated Annealing Placements

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Abstract

One way to reduce the computational requirements of Simulated Annealing placement algorithms is to use a faster heuristic to replace the early phase of Simulated Annealing. Such systems need to know a starting temperature for the annealing phase that makes the best use of the existing structure, yet does an appropriate amount of improvement. This paper presents a method for *measuring* the temperature of an existing placement based on analysis of the probability distribution of the *change* in cost function. Using this view a new definition of equilibrium is given and the *equilibrium temperature* of a placement is defined. Temperatures of placements produced both by a Simulated Annealing and a Min-Cut placement algorithm are measured.

1 Introduction

The success of the Simulated Annealing algorithm for automatic placement [Sech85] has been hindered by its excessive computational requirements. Recent work on standard cell placement algorithms [Rose86, Grov87, Rose88] has suggested alleviating this by using a two-stage approach: begin with a good heuristic such as the Min-Cut algorithm [Dunl85] and follow it with a Simulated Annealing-based approach for more fine optimization. This allows a tradeoff between execution time and quality. A critical issue in this approach is to decide the starting temperature of the Simulated Annealing phase. If it is too high, then some of the structure created by the first phase will be destroyed and unnecessary extra work will have to be done in the Simulated Annealing phase. If the temperature is too low then solution quality is lost, similar to the case of a quenching cooling schedule [Whit84].

This paper presents a technique for *measuring* the temperature of a placement for use in such two-stage systems. To do so, we present a new view of Simulated Annealing state different from those articulated in [Rome84, Whit84, Aart85]. The principal difference is that we look at probability distributions of the *change* in cost function of a Simulated Annealing state, rather than the absolute cost function. Using this view we give a definition of equilibrium from which follows the notion of the *equilibrium temperature* of a placement.

From this we develop a measure that quantifies the nearness of a Simulated Annealing placement to equilibrium and give experimental evidence of its ability to detect equilibrium. This leads to a method for measuring the equilibrium temperature of a placement, and we show that it works both for placements produced by a Simulated Annealing and a Min-Cut placement algorithm.

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The determination of starting temperature for Simulated Annealing in two-stage systems has not been seriously addressed before. Both [Rose86,Rose88] and [Grov87] introduce the question but avoid answering it by choosing a starting temperature based simply on prior experience.

2 Definition of Equilibrium and Temperature

In previous discussions of cooling schedules and convergence [Rome84, Whit84, Aart85], the Simulated Annealing *state* has been represented either as the probability distribution of the absolute cost $P(C)$, or the set of transition probabilities from every state i to every other state j, T_{ij}. We suggest a different view that gives more information about equilibrium dynamics: the probability distribution of the *change* in cost function *from* the current state. $P(\Delta C)$ is the probability that a given state under a Simulated Annealing process with a particular generation function [Rome84] will generate a move with a change in cost function of ΔC. $P(\Delta C)$ varies with temperature (T) and as moves are made.

We can use this view to give a different perspective on the equilibrium of a Simulated Annealing process. Since at equilibrium the absolute cost function no longer changes, this implies that the expected value of the *change* in cost function is zero:

$$E(\Delta C) = 0 \qquad (1)$$

An expression for $E(\Delta C)$ can be formed assuming that $P(\Delta C)$ is known:

$$E(\Delta C) = \int_{-\infty}^{\infty} \Delta C \; P(\Delta C)\, P_{Accept}(\Delta C)\, d\Delta C \qquad (2)$$

$P_{Accept}(\Delta C)$ is the probability that the acceptance function will accept a move with cost ΔC [Rome84]. It commonly has the value 1 for $\Delta C \le 0$ and $e^{\frac{-\Delta C}{T}}$ for $\Delta C > 0$ [Sech85]. We note here that $P(\Delta C)$ in equation (2) must be the distribution measured on a *running* Simulated Annealing process *at* the equilibrium temperature. This distribution is difficult to measure, and will be discussed further in Section 3.1.

Using this $P_{Accept}(\Delta C)$ we can split equation (2) into two parts and, at equilibrium from equation (1) we can equate it to zero:

$$\int_{-\infty}^{0} \Delta C\, P(\Delta C)\, d\Delta C \; + \int_{0}^{\infty} \Delta C\, P(\Delta C)\, e^{\frac{-\Delta C}{T}}\, d\Delta C = 0 \qquad (3)$$

Thus *equilibrium* can now be defined as the state where, at a given $T = T_{eq}$, the distribution $P(\Delta C)$ satisfies equation (3).

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Conversely, the ***equilibrium temperature*** of a placement with a distribution $P(\Delta C)$ is the temperature, T_{eq}, for which equation (3) is satisfied.

2.1 An Equilibrium-Nearness Measure

Using equation (3) we can invent a measure of the nearness of a given Simulated Annealing state to equilibrium. Define E_- to be the absolute value of the first term in the equation, that is

$$E_- = \left| \int_{-\infty}^{0} \Delta C \; P(\Delta C) d\Delta C \right|$$

Similarly let E_+ be the second term of equation (3):

$$E_+ = \int_{0}^{\infty} \Delta C \; P(\Delta C) \; e^{\frac{-\Delta C}{T_m}} \, d\Delta C$$

Where T_m is the temperature of the Simulated Annealing process. We can now define the *Cost Force Ratio*, (CFR) as:

$$CFR = \frac{E_-}{E_+ + E_-} \times 100 \qquad (4)$$

The closer CFR is to 50% (the expected value of the good moves being equal to the expected values of the bad moves, $E_- = E_+$) the closer the system is to equilibrium.

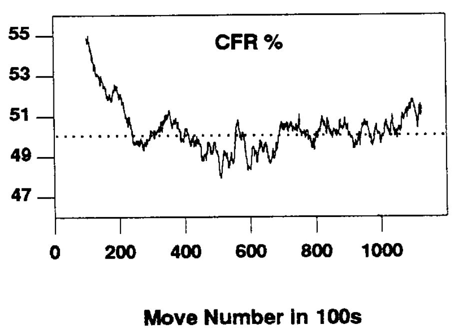

Figure 1 - *CFR vs Move as Process Achieves Equilibrium*

All experiments in this paper use a placement of the 833 standard cell Primary1 circuit from the Preas-Roberts standard cell benchmark suite [Prea87]. The placement was produced by the SALTOR Simulated Annealing placement program [Rose86,Rose88], which is based on the ideas of the Timberwolf standard cell placement program [Sech85]. Figure 1 is a plot of CFR versus generated move number for a Simulated Annealing process running on circuit Primary1, as it goes from non-equilibrium to equilibrium at temperature 400 changing to 300. CFR is determined by keeping a window of ΔC values multiplied by the P_{Accept} function and using this to calculate E_+ and E_-. In this figure the CFR comes down from an initial value of 55% and hovers around 50%. This shows that the CFR indicates when equilibrium has been achieved. It varies about the 50% point due to the stochastic nature of the algorithm and the approximation of measuring the CFR in a finite window.

3 Measuring Temperature

As defined in Section 2, the temperature of a placement is the temperature at which the Simulated Annealing process running on the placement is in equilibrium. In this section we present a method for measuring the temperature of an arbitrary placement.

The method is called the CFR Binary Search and has the following outline:

1. Measure $P(\Delta C)$ for the given circuit under the Simulated Annealing process. This is discussed in detail in Section 3.1.

2. Set the starting search temperature, T_m, arbitrarily.

3. Based on the current T_m, calculate $P_{Accept}(\Delta C) = e^{\frac{-\Delta C}{T_m}}$ for $\Delta C > 0$ and $= 1$ for $\Delta C \leq 0$.

4. Calculate the Cost Force Ratio, CFR, using $P_{Accept}(\Delta C)$ and equation (4).

5. If CFR < 50, reduce T_m according to a binary search and go to step 3;
   If CFR > 50, increase T_m according to a binary search and go to step 3.

6. When CFR = 50, T_m is the equilibrium temperature, T_{eq}. Finish.

Each iteration of the CFR Binary Search requires only the recalculation of the positive portion of the acceptance function probability, $P_{Accept}(\Delta C)$, and subsequently E_+ and CFR since E_- does not change with T_m. Note also that $P(\Delta C)$ need only be generated once. This is important since it takes many moves (10^4 to 10^5) to get an accurate picture of the probability distribution.

3.1 Measurement of the Probability Distribution

A key and difficult step in the CFR Binary Search temperature measurement procedure is the measurement of the distribution $P(\Delta C)$. There are two possible methods:

1. Static Measurement. $P(\Delta C)$ is measured by generating (but not accepting) moves in the Simulated Annealing process on the placement, and recording the frequency with which each cost occurs. These *virtual* moves do not change the placement.

2. Dynamic Measurement. $P(\Delta C)$ is measured by generating *and* accepting moves on the placement. Here the placement does change as the measurement is made.

For the general case of any Simulated Annealing application a static measurement will not give the correct distribution. This is because a static measurement of $P(\Delta C)$ could be taken when the system was at a local (but not global) optimum. In this case there would be no good (negative) moves generated and since

E_- would thus be 0 the temperature would appear to be 0, which is incorrect in the case of a local optimum. Similar problems can occur when the state is at or near discontinuities in the energy landscape.

The dynamic measurement approach must run the Simulated Annealing process *at* its equilibrium temperature. Using a different temperature would cause the placement's temperature to change. Unfortunately the equilibrium temperature is the quantity we are seeking, and is not known. This is a dilemma not unlike the Heisenberg uncertainty principle: the act of measuring the temperature this way can cause the temperature to change.

An alternative is to measure $P(\Delta C)$ using the static method, and to determine how accurate this is as an approximation. The accuracy is entirely problem dependent - it depends on the energy landscape of the underlying Simulated Annealing formulation. We have experimented to determine the accuracy for the standard cell placement problem and have found that the static measurement of $P(\Delta C)$ is almost exactly the same as the dynamic measurement. Figure 2 shows a plot of a static distribution and a dynamic distribution measured on circuit Primary1 at temperature 300. Measurements and numerical comparisons on this and several other circuits at various temperatures have shown very small differences between the static and dynamic measurements. Thus we will use the static measurement of $P(\Delta C)$ in the temperature measurement algorithm.

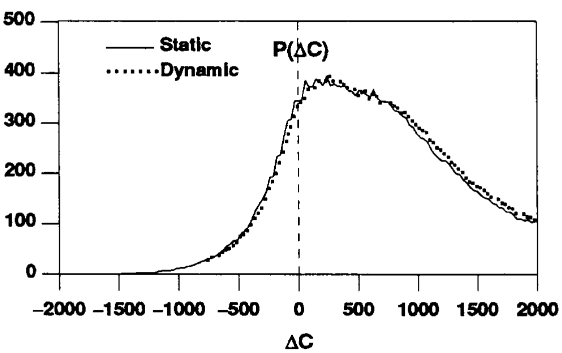

Figure 2 - *Comparison of Static and Dynamic Measurement*

3.2 Measurement of Annealing Placements

The CFR Binary Search was used to measure the temperature of a set of Primary1 placements produced by the SALTOR Simulated Annealing placement program [Rose86,Rose88]. Each placement was measured statically using N = 100,000 virtual moves to experimentally determine $P(\Delta C)$. Table 1 gives the temperature at which each placement's Simulated Annealing process was terminated (while in equilibrium), and the measured temperature using the CFR Binary Search.

The measured temperature is quite accurate at the higher temperature, usually less than 7% error. The lower temperature measurements are *proportionately* less accurate. The error is due to two effects: First, there is a slight difference, as discussed above, between the static and the (more correct) dynamic measurement of $P(\Delta C)$. Second, at lower temperatures, there are fewer negative moves, and so the accuracy of E_- decreases, decreasing the accuracy of CFR and hence the temperature measurement.

SA Produced Temperature	CFR Binary Search Measured Temp	Difference
500	496	-4
405	420	+15
294	285	-11
213	215	+2
153	164	+11
99	97	-2
57	60	+3
28	28	0
9	15	+6
2	4	+2

Table 1 - *Temperature Measurement of Annealing Placements*

This last point can be seen experimentally: figure 3 is a plot of the percentage standard deviation of the measured temperature as a function of the number of virtual moves, N, for temperatures 28, 153 and 405. The standard deviation was calculated from five runs at each number of virtual moves.

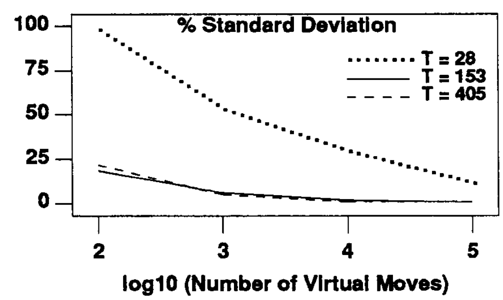

Figure 3 - *Variation of Temperature vs. Number of Moves*

The variation is a decreasing function of N, as would be expected. The figure illustrates the increase in percentage variation at lower temperature.

4 Measurement of Min-Cut Placements

Our goal is to determine the starting temperature when switching from a non-annealing placement algorithm to an annealing-based one. In this section we test the ideas presented above on the Min-Cut placement algorithm [Dunl85].

Several terms first need to be defined for Min-Cut placements, as shown in Figure 4. A Min-Cut placement algorithm is characterized by, among other things, the order and spacing of the cut lines applied. In Figure 4, the rectangle represents the entire placement, over which is laid a set of vertical and horizontal cut lines. If the spacing of the vertical cut lines is V and of the horizontal cut lines is H, then the *cut area*, A, is given by $A = V \times H$.

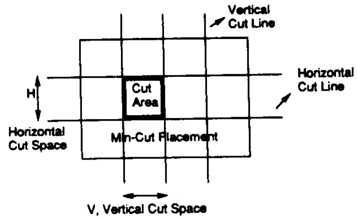

Figure 4 - *Definition of Cut-Area*

One difficulty with measuring the temperature of non-annealing produced placements is that the definition of temperature presented in Section 2 depends on the associated Simulated Annealing process being in equilibrium. It is clear, however, that a placement produced by a non-annealing algorithm is not in equilibrium. Thus we must make an approximation and assume that a min-cut placement can be thought of as being in equilibrium at some temperature. The effect of this approximation is measured in the next section where we compare the CFR Binary Search method with a more direct method.

4.1 Measurements

Using the CFR Binary Search method we measured the temperature of several Min-Cut placements with different cut areas. These placements were produced by the ALTOR standard-cell placement program [Rose85]. Table 2 gives the measured temperature for each placement and its cut area.

Cut Area $\mu m^2 \times 10^4$	Temperature Measured Binary Search	Delta Method	Difference
2021	398	374	+24
1011	234	200	+34
505.3	162	132	+30
252.6	124	96	+28
126.3	91	67	+24
63.22	73	50	+23
31.58	49	40	+9
25.24	40	32	+8
12.60	34	30	+4
7.697	29	27	+2
3.139	28	26	+2

Table 2 - *Temperature Measurement of Min-Cut Placements*

To check the CFR Binary Search measurements, the placements were measured using a different approach, called the *Delta Method*. It finds the temperature of a placement by running a dynamic annealing process on the placement over a range of temperatures. The percentage difference in absolute cost function after (100 moves per cell are made) is measured. When a temperature is found for which this difference is less than 2%, that is taken as the temperature of the placement. This is a direct way of experimentally finding the temperature at which the change in cost function is near 0. Table 2 gives the temperatures determined by the Delta Method, and the difference between the CFR Binary Search and the Delta Method. The CFR Binary Search measurement for Min-Cut placements is not as accurate as those for Annealing-produced placements, yet it does track the temperature reasonably well.

The CFR Binary Search method consistently overestimates the equilibrium temperature due to the fact that a min-cut placement is not in equilibrium, as discussed above.

5 Conclusions

We have presented a method for determining the *temperature*, in the Simulated Annealing sense, of an arbitrary placement. It uses a new view of Simulated Annealing state that is based on the probability distribution of the change in cost function. The temperature of several Simulated Annealing placements have been measured with good accuracy. The temperature of a set of Min-Cut placements has also been measured. This method is useful for determining the starting temperature when switching from a non-annealing based placement strategy to an annealing-based one.

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