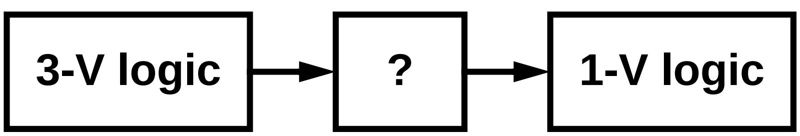
ECE1371 Advanced Analog Circuits

INTRODUCTION TO DELTA-SIGMA ADCS

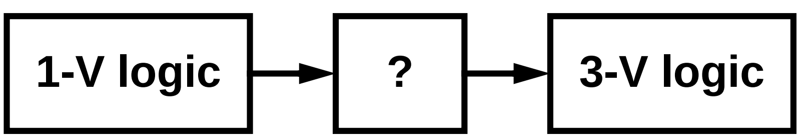
Richard Schreier
`richard.schreier@analog.com`

NLCOTD: Level Translator

VDD1 > VDD2, e.g.



- **VDD1 < VDD2, e.g.**



- **Constraints:** **CMOS**
  1-V and 3-V devices
  no static current

Highlights
(i.e. What you will learn today)

1 1^{st}-order modulator (MOD1)
   Structure and theory of operation

2 Inherent linearity of binary modulators

3 Inherent anti-aliasing of continuous-time modulators

4 2^{nd}-order modulator (MOD2)

5 Good FFT practice

0. Background
(Stuff you already know)

- **The SQNR* of an ideal *n*-bit ADC with a full-scale sine-wave input is (6.02*n* + 1.76) dB**

  "6 dB = 1 bit."

- **The PSD at the output of a linear system is the product of the input's PSD and the squared magnitude of the system's frequency response**

  i.e. $X \rightarrow \boxed{H(z)} \rightarrow Y$ $\quad S_{yy}(f) = |H(e^{j2\pi f})|^2 \cdot S_{xx}(f)$

- **The power in any frequency band is the integral of the PSD over that band**

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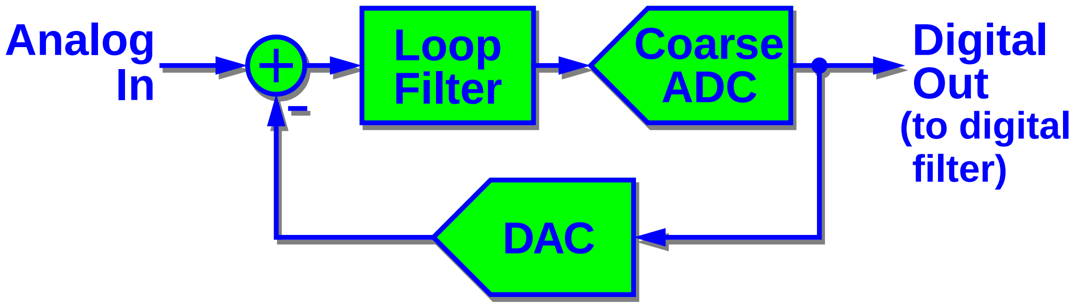
*. SQNR = Signal-to-Quantization-Noise Ratio

1. What is ΔΣ?

- ΔΣ is NOT a fraternity
- Simplified ΔΣ ADC structure:



- Key features: coarse quantization, filtering, feedback and oversampling

  Quantization is often *quite* coarse (1 bit!), but the effective resolution can still be as high as 22 bits.

What is Oversampling?

- Oversampling is sampling faster than required by the Nyquist criterion

  For a lowpass signal containing energy in the frequency range $(0, f_B)$, the minimum sample rate required for perfect reconstruction is $f_s = 2f_B$.

- The *oversampling ratio* is $OSR \equiv f_s/(2f_B)$

- For a regular ADC, $OSR \sim 2 - 3$

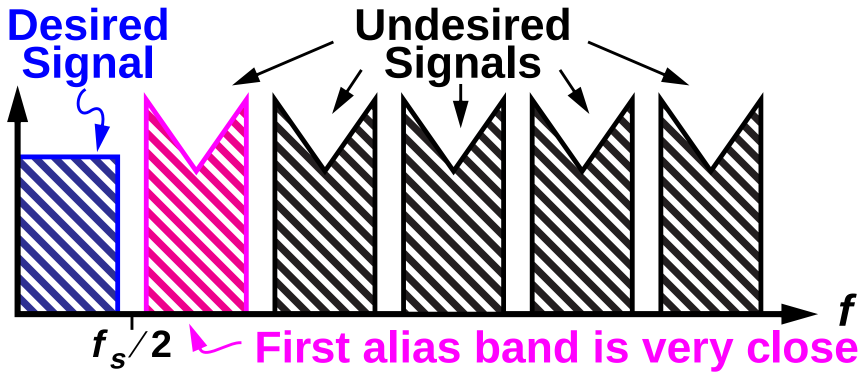
  To make the anti-alias filter (AAF) feasible

- For a ΔΣ ADC, $OSR \sim 30$

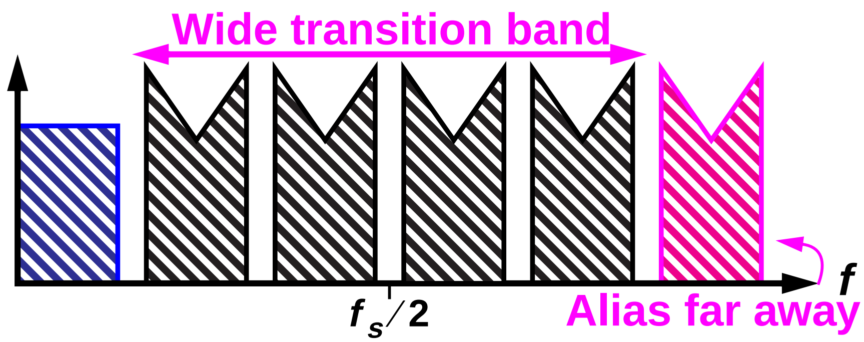
  To get adequate quantization noise suppression. Signals between f_B and $\sim f_s$ are removed digitally.

Oversampling Simplifies AAF

OSR ~ 1:

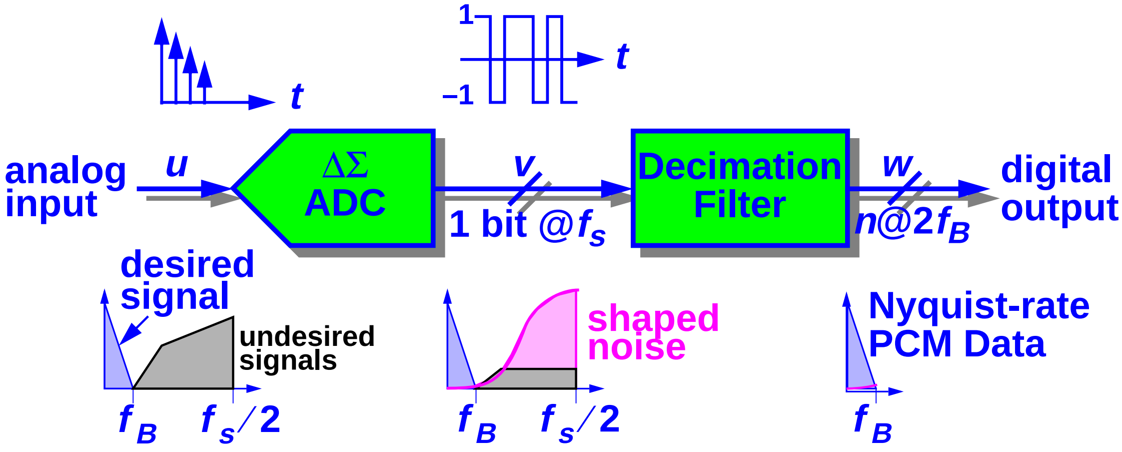


OSR = 3:

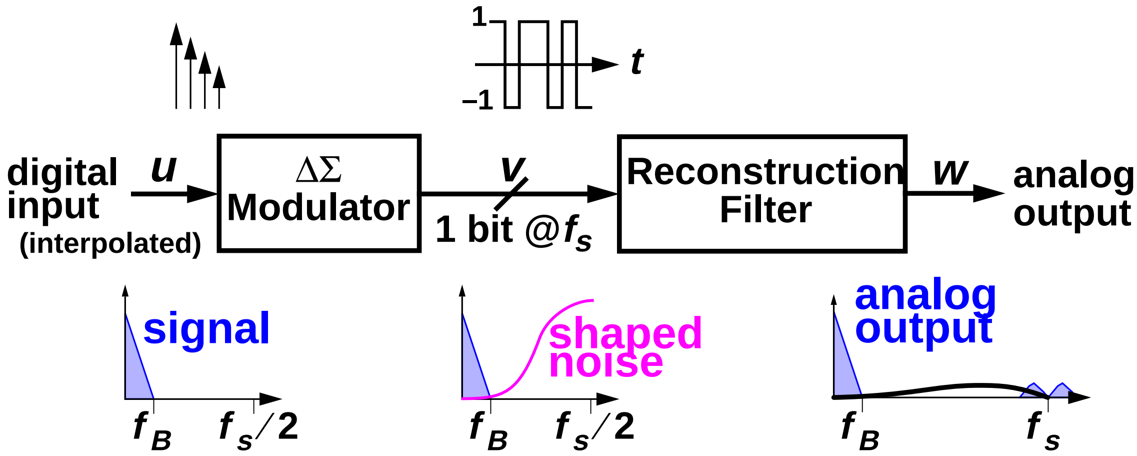


How Does A ΔΣ ADC Work?

- Coarse quantization ⇒ lots of quantization error. So how can a ΔΣ ADC achieve 22-bit resolution?
- A ΔΣ ADC spectrally separates the quantization error from the signal through *noise-shaping*



A ΔΣ DAC System



- **Mathematically similar to an ADC system**

  Except that now the modulator is digital and drives a low-resolution DAC, and that the out-of-band noise is handled by an analog reconstruction filter.

Why Do It The ΔΣ Way?

- **ADC: Simplified Anti-Alias Filter**

  Since the input is oversampled, only very high frequencies alias to the passband.
  A simple RC section often suffices.

  If a continuous-time loop filter is used, the anti-alias filter can often be eliminated altogether.

- **DAC: Simplified Reconstruction Filter**

  The nearby images present in Nyquist-rate reconstruction can be removed digitally.

\+ **Inherent Linearity**

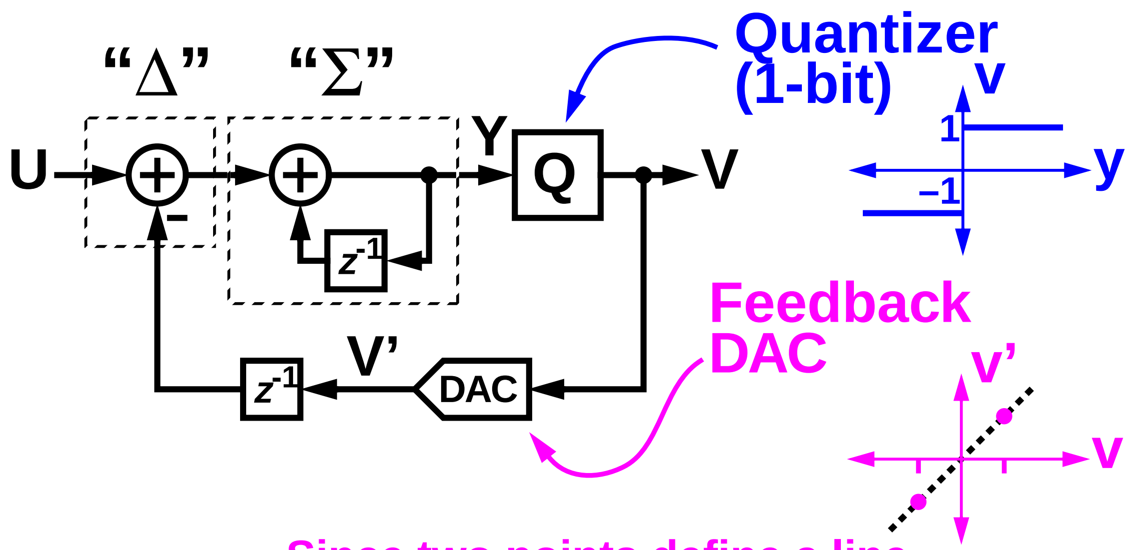
  Simple structures can yield very high SNR.

\+ **Robust Implementation**

  ΔΣ tolerates sizable component errors.

2. MOD1: 1^{st}-Order ΔΣ Modulator

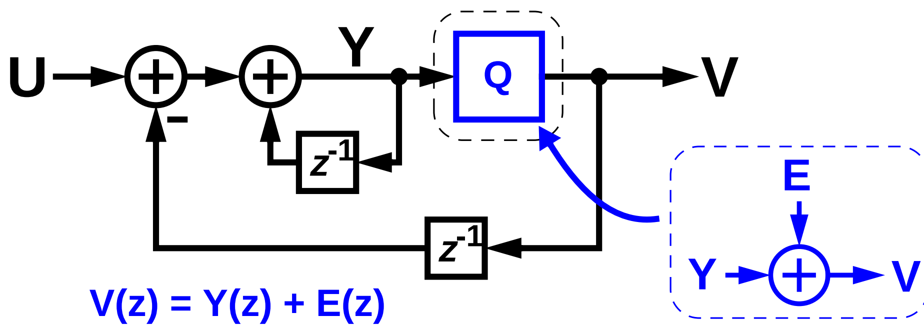
[Ch. 2 of Schreier & Temes]



Since two points define a line,
a binary DAC is <u>inherently linear</u>.

MOD1 Analysis

- **Exact analysis is intractable for all but the simplest inputs, so treat the quantizer as an additive noise source:**



$V(z) = Y(z) + E(z)$

$Y(z) = (U(z) – z^{-1}V(z)) / (1–z^{-1})$

$\Rightarrow (1–z^{-1}) V(z) = U(z) – z^{-1}V(z) + (1–z^{-1})E(z)$

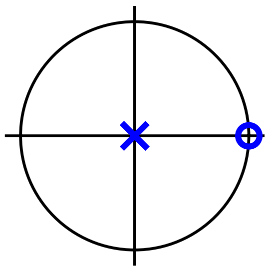
$$\boxed{V(z) = U(z) + (1–z^{-1})E(z)}$$

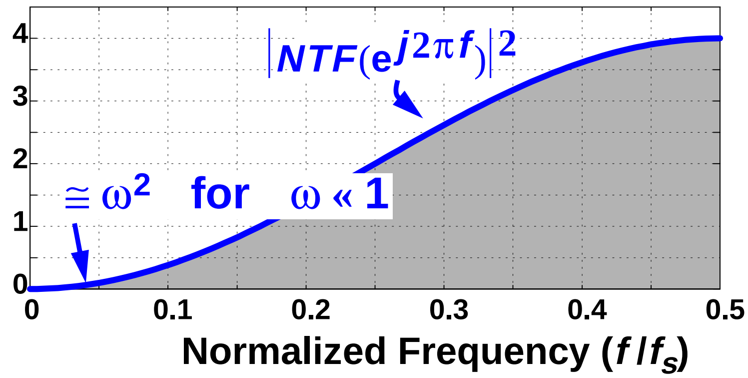
The Noise Transfer Function (NTF)

- **In general, V(z) = STF(z)•U(z) + NTF(z)•E(z)**
- **For MOD1, NTF(z) = $1 - z^{-1}$**

  The quantization noise has spectral shape!

Poles & zeros:





- **The total noise power increases, but the noise power at low frequencies is reduced**

In-band Quant. Noise Power

- **Assume that *e* is white with power σ_e^2**

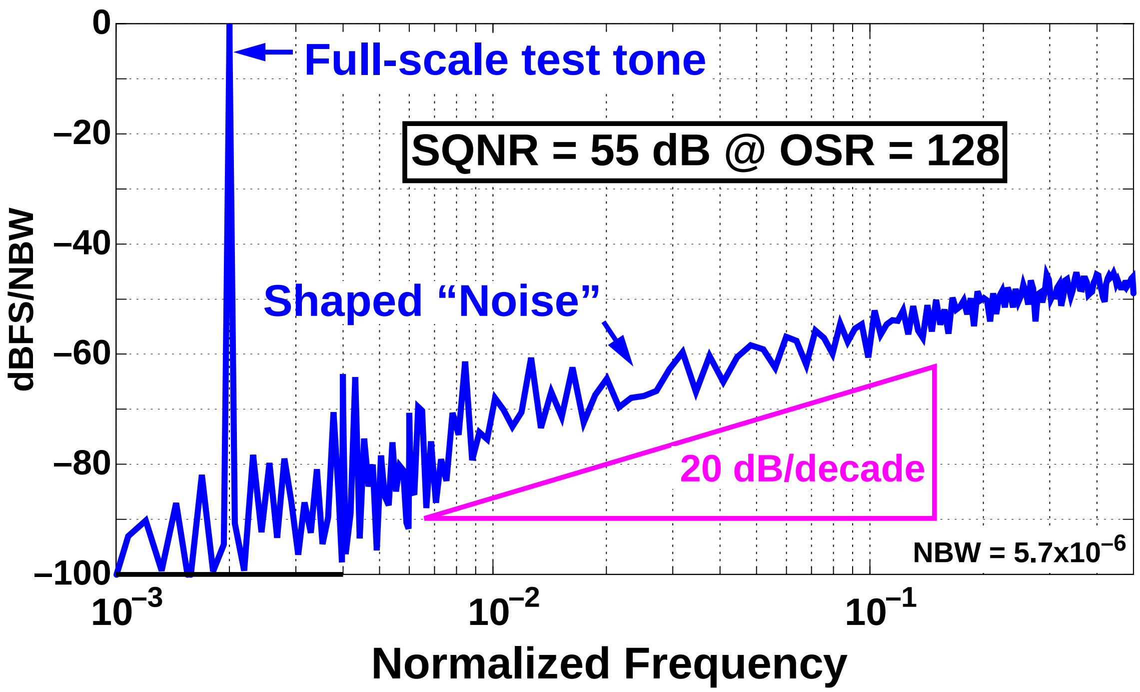
  i.e. $S_{ee}(\omega) = \sigma_e^2 / \pi$
- **The in-band quantization noise power is**

$$IQNP = \int_0^{\omega_B} |H(e^{j\omega})|^2 S_{ee}(\omega)\, d\omega \cong \frac{\sigma_e^2}{\pi} \int_0^{\omega_B} \omega^2 d\omega$$

- **Since** $OSR \equiv \dfrac{\pi}{\omega_B}$, $IQNP = \dfrac{\pi^2 \sigma_e^2}{3} (OSR)^{-3}$
- **For MOD1, an octave increase in *OSR* increases SQNR by 9 dB**

  "1.5-bit/octave SQNR-OSR trade-off."

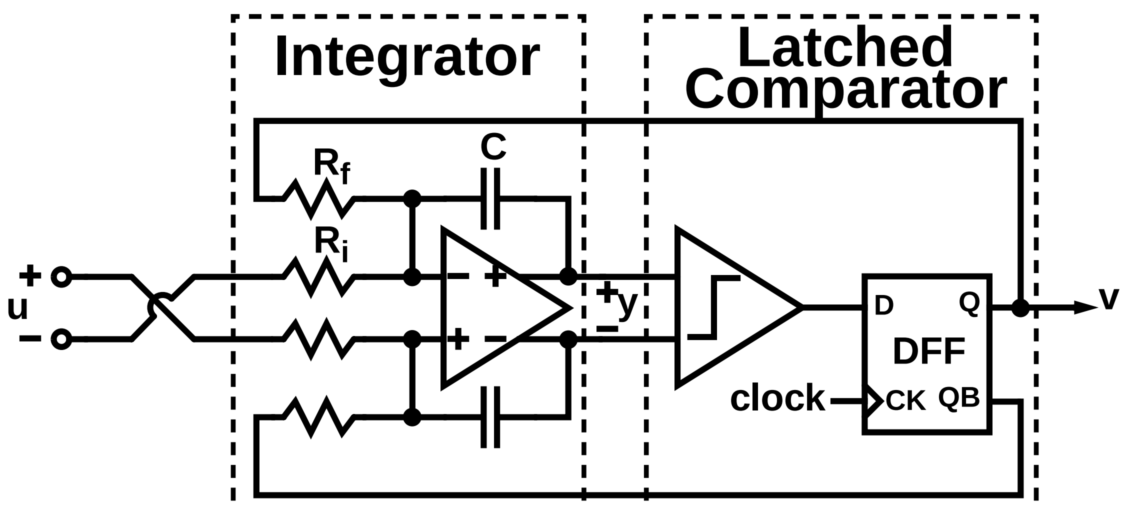
A Simulation of MOD1



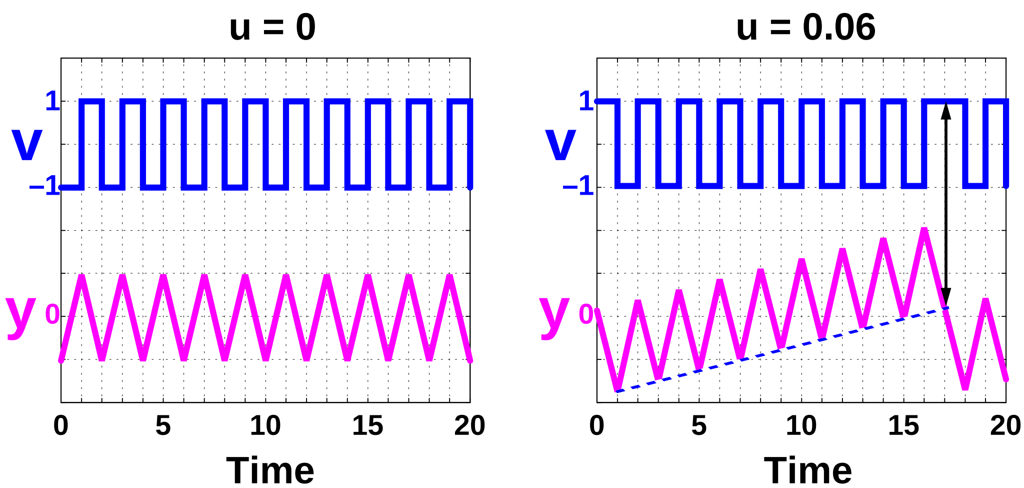
CT Implementation of MOD1

- **R_i/R_f sets the full-scale; C is arbitrary**

  Also observe that an input at f_s is rejected by the integrator— *inherent anti-aliasing*



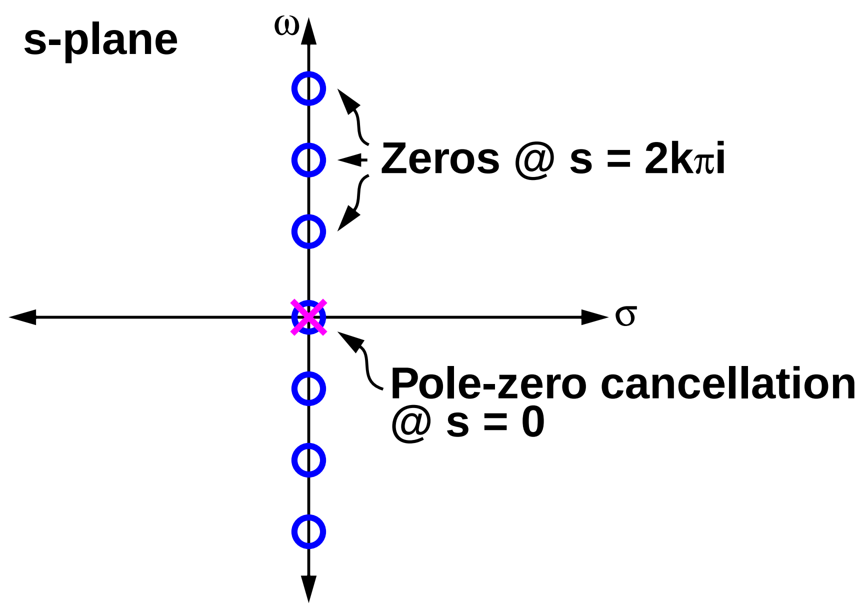
MOD1-CT Waveforms



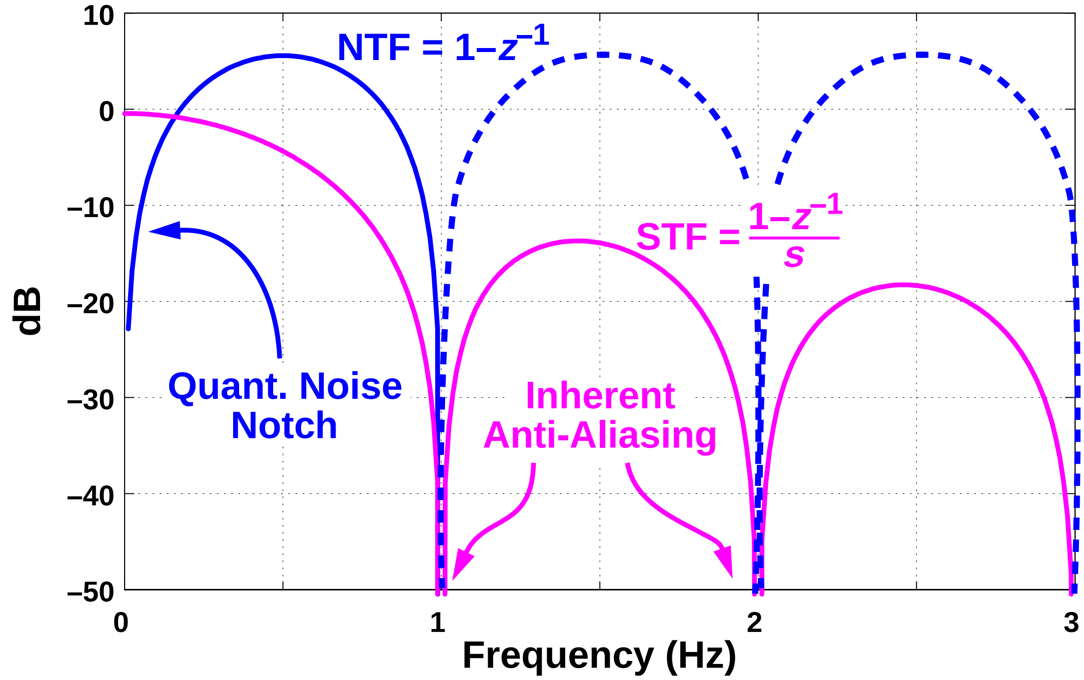
- With u=0, v alternates between +1 and –1
- With u>0, y drifts upwards; v contains consecutive +1s to counteract this drift

MOD1-CT STF = $\dfrac{1 - z^{-1}}{s}$

Recall $z = e^s$



MOD1-CT Frequency Responses



Summary

- **$\Delta\Sigma$ works by spectrally separating the quantization noise from the signal**

  Requires oversampling. $OSR \equiv f_s/(2f_B)$.

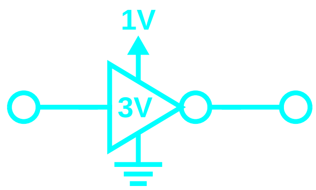
- **Noise-shaping is achieved by the use of *filtering* and *feedback***
- **A binary DAC is *inherently linear,* and thus a binary $\Delta\Sigma$ modulator is too**
- **MOD1 has $NTF(z) = 1 - z^{-1}$**

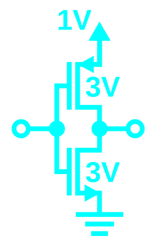
  $\Rightarrow$ Arbitrary accuracy for DC inputs.
  1.5 bit/octave SQNR-OSR trade-off.

- **MOD1-CT has *inherent anti-aliasing***

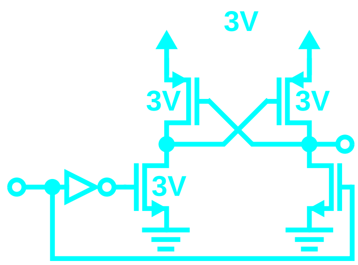
NLCOTD

3V → 1V:





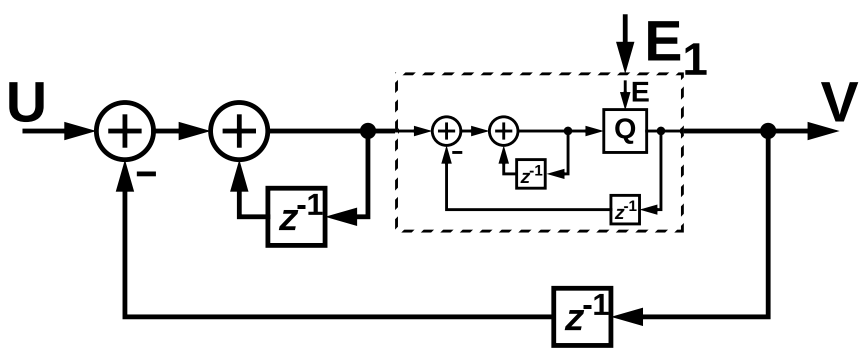
1V → 3V:



3. MOD2: 2^{nd}-Order ΔΣ Modulator

[Ch. 3 of Schreier & Temes]

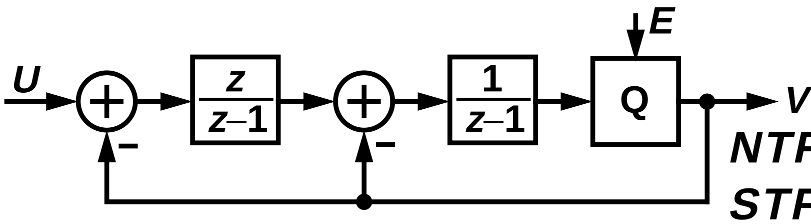
- **Replace the quantizer in MOD1 with another copy of MOD1 in a recursive fashion:**



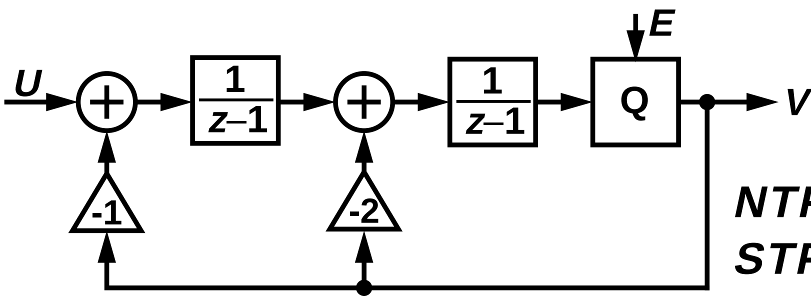
$$V(z) = U(z) + (1-z^{-1})E_1(z),\ E_1(z) = (1-z^{-1})E(z)$$

$$\Rightarrow V(z) = U(z) + (1-z^{-1})^2 E(z)$$

Simplified Block Diagrams

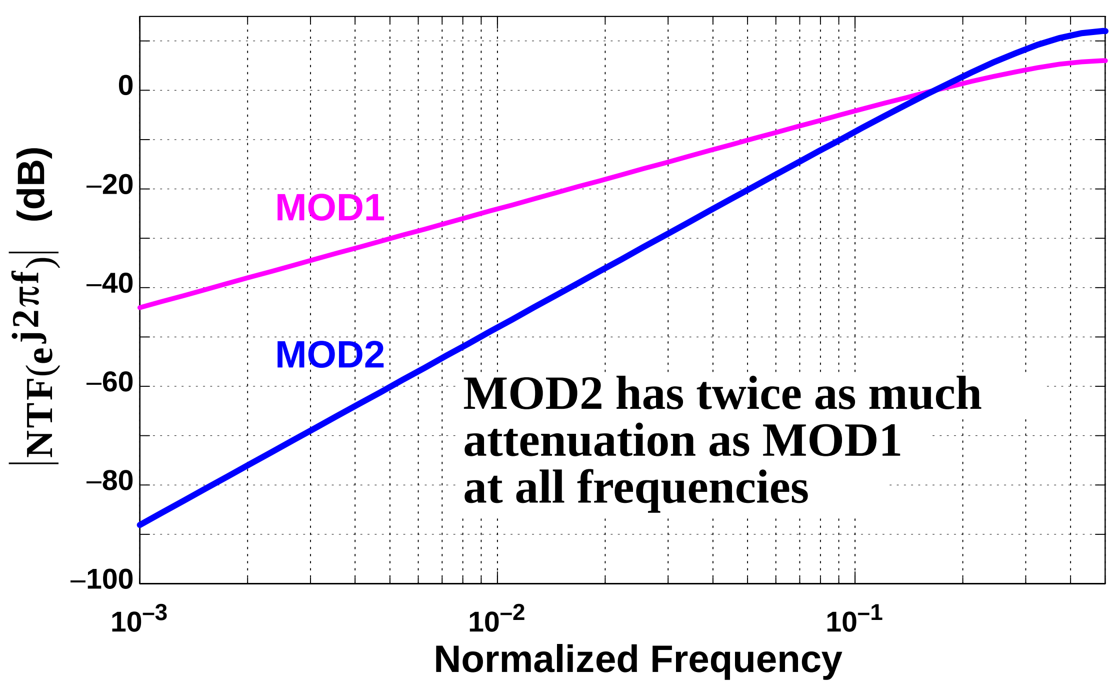


$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-1}$$



$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-2}$$

NTF Comparison

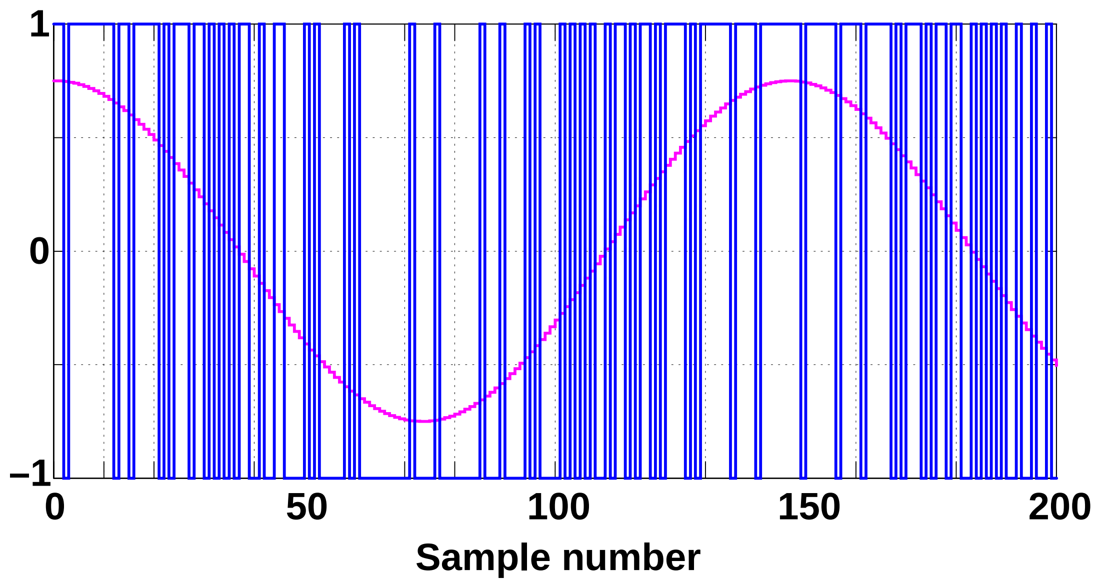


In-band Quant. Noise Power

- **For MOD2,** $|H(e^{j\omega})|^2 \approx \omega^4$
- **As before,** $IQNP = \int_0^{\omega_B} |H(e^{j\omega})|^2 S_{ee}(\omega)\, d\omega$ **and** $S_{ee}(\omega) = \sigma_e^2/\pi$
- **So now** $IQNP = \dfrac{\pi^4 \sigma_e^2}{5}(OSR)^{-5}$ $\ast$

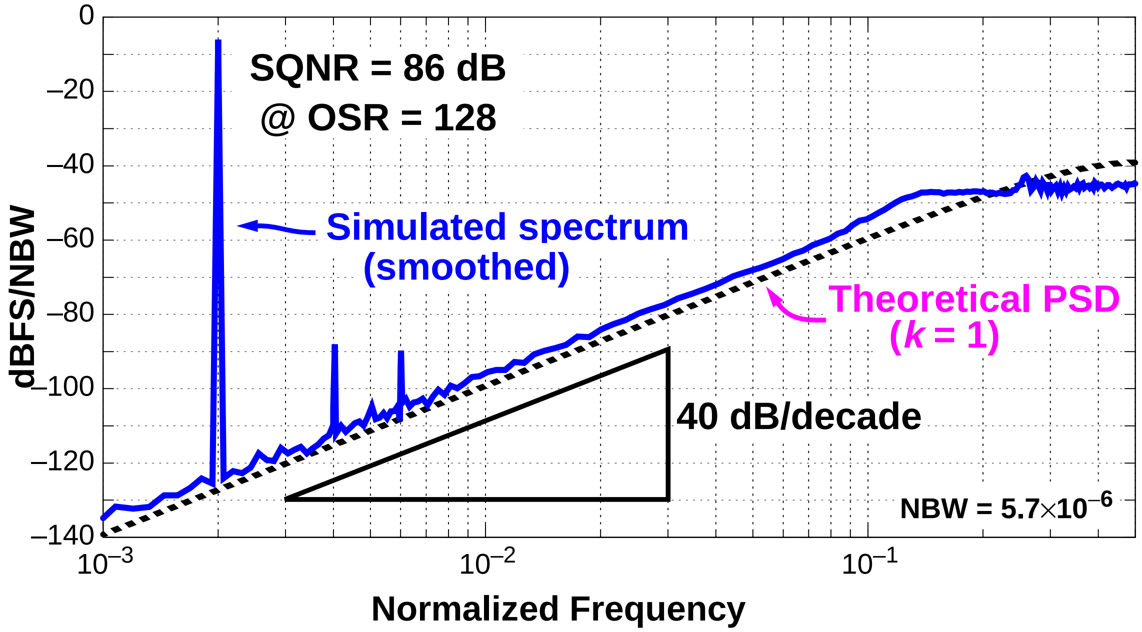
  With binary quantization to ± 1**,**
  $\Delta = 2$ **and thus** $\sigma_e^2 = \Delta^2/12 = 1/3$.
- **"An octave increase in *OSR* increases MOD2's SQNR by 15 dB (2.5 bits)"**

Simulation Example
Input at 75% of FullScale



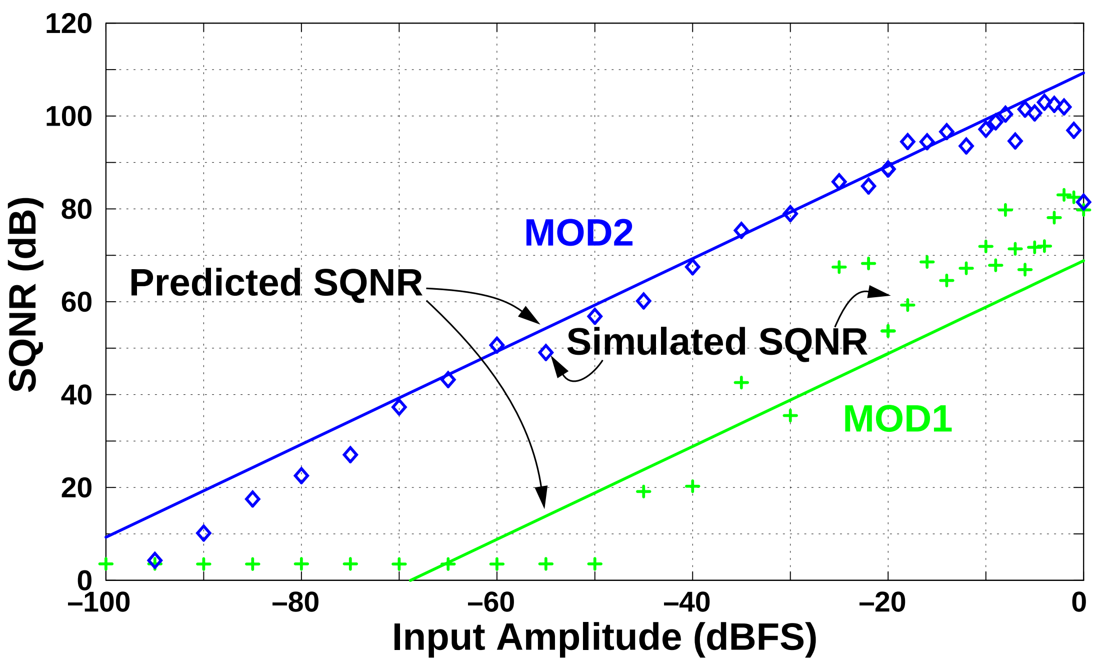
Simulated MOD2 PSD

Input at 50% of FullScale

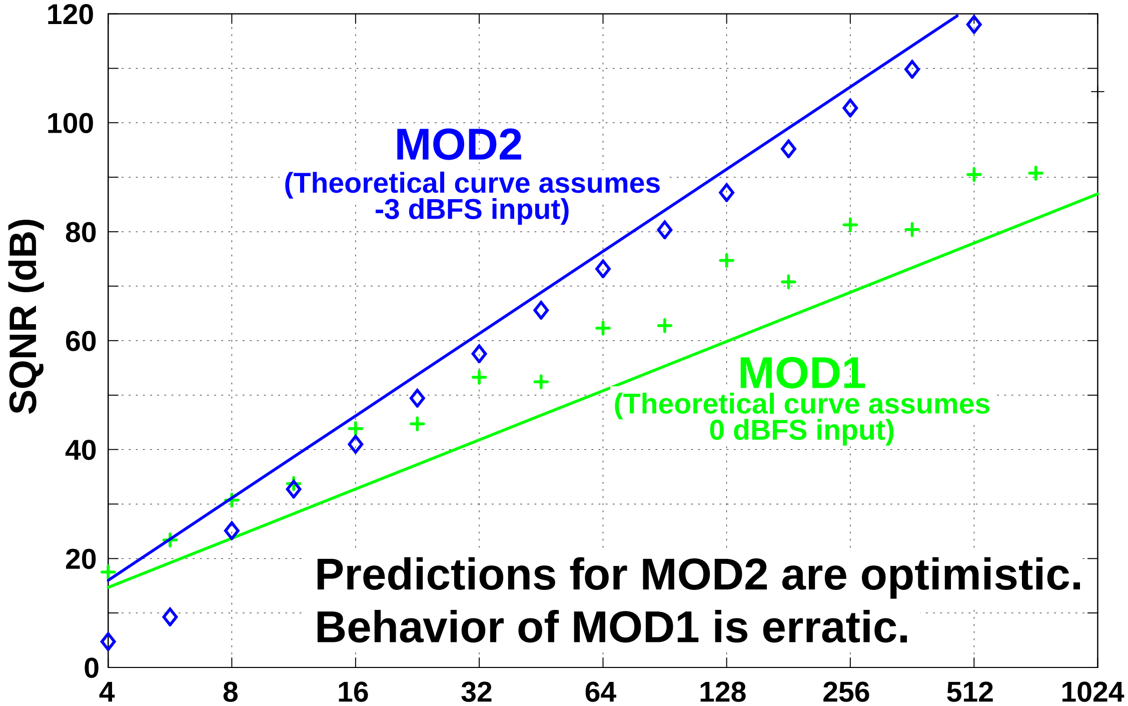


SQNR vs. Input Amplitude

MOD1 & MOD2 @ OSR = 256

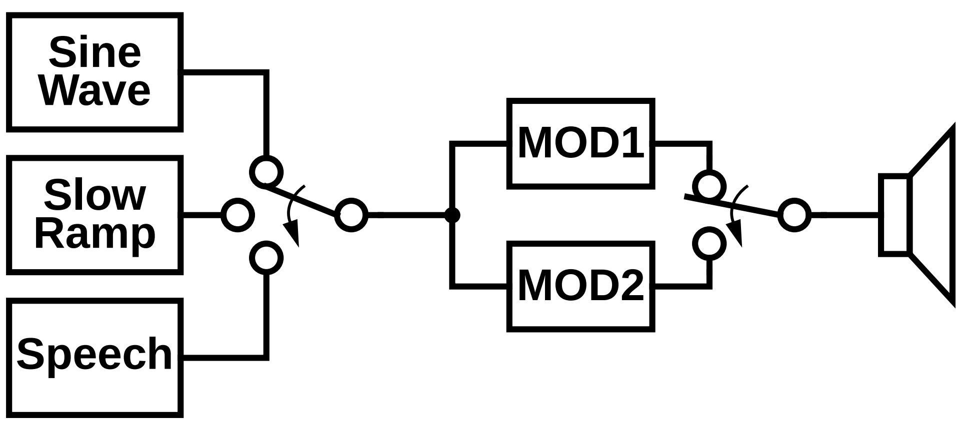


SQNR vs. OSR



Audio Demo: MOD1 vs. MOD2

[dsdemo4]

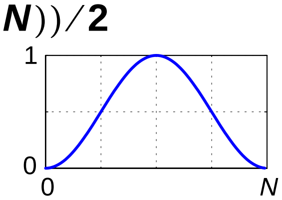


MOD1 + MOD2 Summary

- $\Delta\Sigma$ **ADCs rely on filtering and feedback to achieve high SNR despite coarse quantization**
  They also rely on digital signal processing.
  $\Delta\Sigma$ ADCs need to be followed by a digital decimation filter and $\Delta\Sigma$ DACs need to be preceded by a digital interpolation filter.
- **Oversampling eases analog filtering requirements**
  Anti-alias filter in an ADC; image filter in a DAC.
- **Binary quantization yields inherent linearity**
- **MOD2 is better than MOD1**
  15 dB/octave vs. 9 dB/octave SQNR-OSR trade-off.
  Quantization noise more white.
  Higher-order modulators are even better.

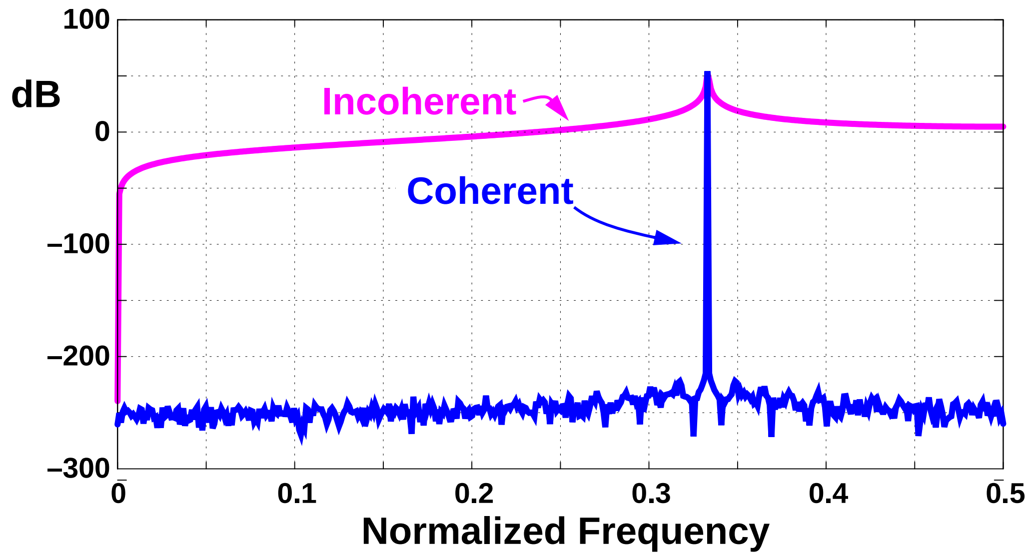
4. Good FFT Practice
[Appendix A of Schreier & Temes]

- **Use coherent sampling**
  I.e. have an integer number of cycles in the record.
- **Use windowing**
  A Hann window $w(n) = (1 - \cos(2\pi n / N)) / 2$ works well.



- **Use enough points**
  Recommend $N = 64 \cdot OSR$.
- **Scale (and smooth) the spectrum**
  A full-scale sine wave should yield a 0-dBFS peak.
- **State the noise bandwidth**
  For a Hann window, $NBW = 1.5 / N$.

Coherent vs. Incoherent Sampling

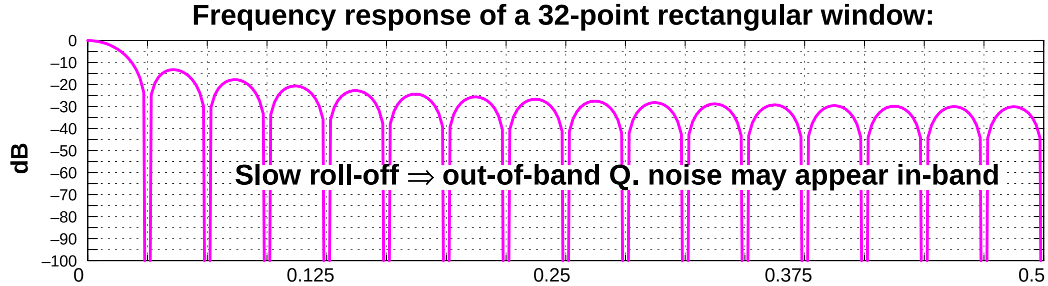


- **Coherent sampling: only one non-zero FFT bin**
- **Incoherent sampling: “spectral leakage”**

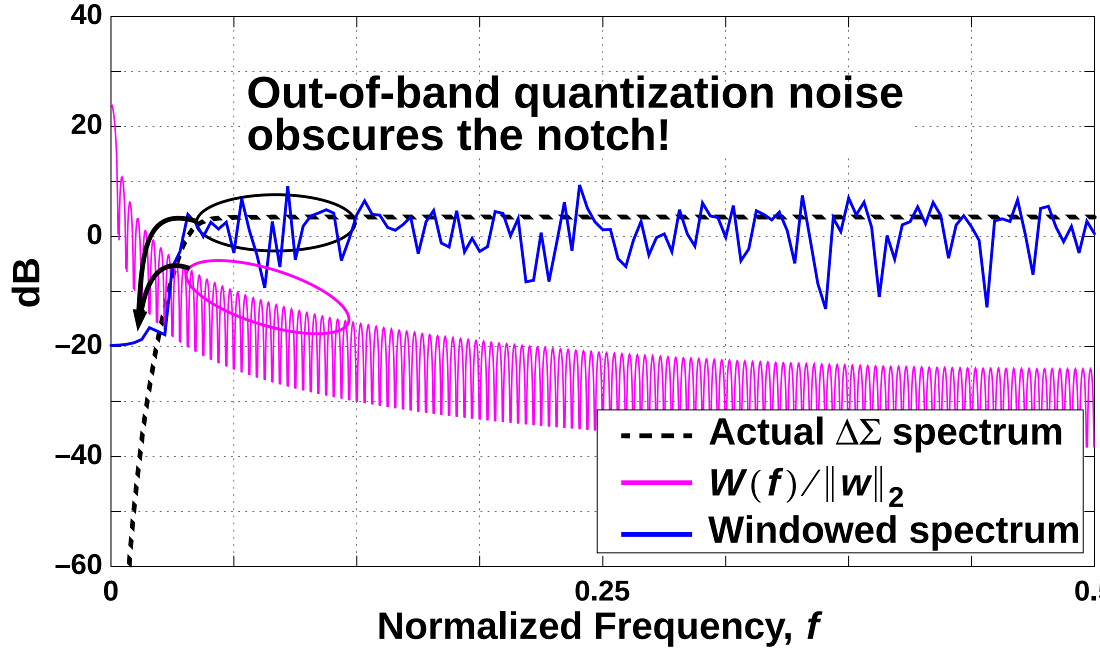
Windowing

- **$\Delta\Sigma$ data is usually not periodic**
  Just because the input repeats does not mean that the output does too!
- **A finite-length data record = an infinite record multiplied by a *rectangular window*:**
  $w(n) = 1, 0 \le n < N$
  Windowing is unavoidable.
- **“Multiplication in time is convolution in frequency”**

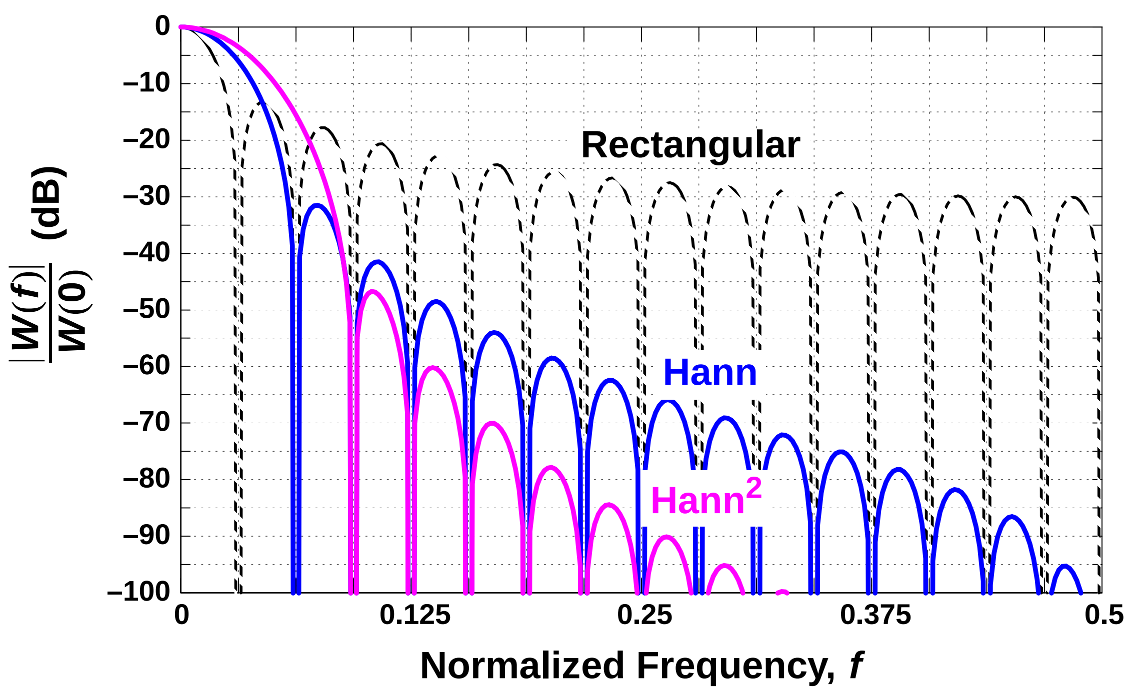
Frequency response of a 32-point rectangular window:



Example Spectral Disaster
Rectangular window, $N = 256$



Window Comparison ($N = 16$)



Window Properties

Window	Rectangular	Hann†	Hann^2
$w(n)$, $n = 0, 1, \ldots, N-1$ ($w(n) = 0$ otherwise)	1	$\dfrac{1 - \cos\dfrac{2\pi n}{N}}{2}$	$\left(\dfrac{1 - \cos\dfrac{2\pi n}{N}}{2}\right)^2$
Number of non-zero FFT bins	1	3	5
$\lVert w \rVert_2^2 = \sum w(n)^2$	N	$3N/8$	$35N/128$
$W(0) = \sum w(n)$	N	$N/2$	$3N/8$
$NBW = \dfrac{\lVert w \rVert_2^2}{W(0)^2}$	$1/N$	$1.5/N$	$35/18N$

†. MATLAB's "hann" function causes spectral leakage of tones located in FFT bins unless you add the optional argument "periodic."

Window Length, N

- Need to have enough in-band noise bins to

1 Make the number of signal bins a small fraction of the total number of in-band bins

<20% signal bins $\Rightarrow$ >15 in-band bins $\Rightarrow$ $N > 30 \cdot OSR$

2 Make the SNR repeatable

$N = 30 \cdot OSR$ yields std. dev. ~1.4 dB.

$N = 64 \cdot OSR$ yields std. dev. ~1.0 dB.

$N = 256 \cdot OSR$ yields std. dev. ~0.5 dB.

- $N = 64 \cdot OSR$ is recommended

FFT Scaling

- The FFT implemented in MATLAB is

$$X_M(k+1) = \sum_{n=0}^{N-1} x_M(n+1) e^{-j\frac{2\pi kn}{N}}$$

- If $x(n) = A\sin(2\pi fn/N)$†, then

$$|X(k)| = \begin{cases} \frac{AN}{2} & , k = f \text{ or } N-f \\ 0 & , \text{otherwise} \end{cases}$$

$\Rightarrow$ Need to divide FFT by $(N/2)$ to get A.

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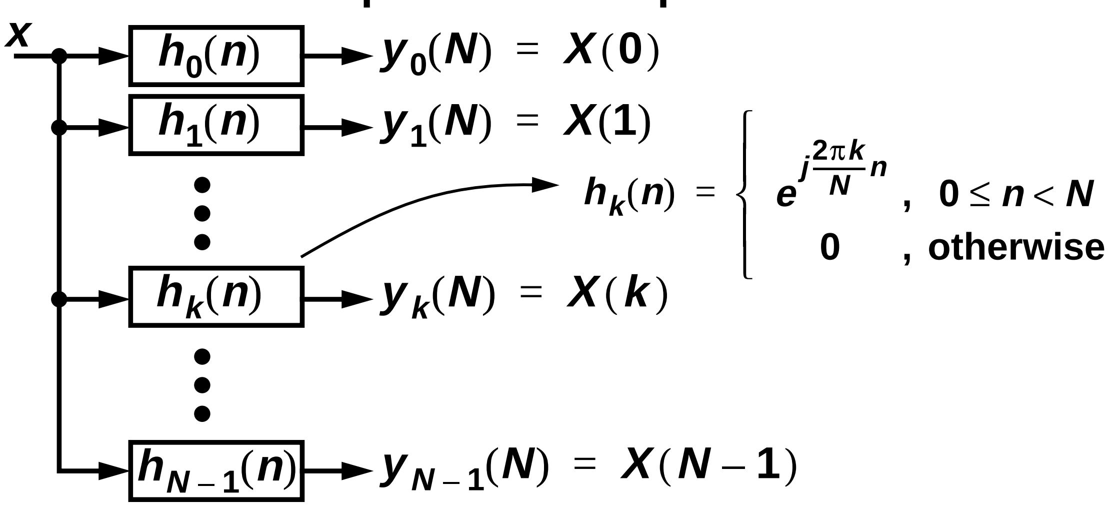
†. f is an integer in $(0, N/2)$. I've defined $X(k) \equiv X_M(k+1)$, $x(n) \equiv x_M(n+1)$ since Matlab indexes from 1 rather than 0.

The Need For Smoothing

- The FFT can be interpreted as taking 1 sample from the outputs of N complex FIR filters:



$\Rightarrow$ an FFT yields a high-variance spectral estimate

How To Do Smoothing

1. **Average multiple FFTs**

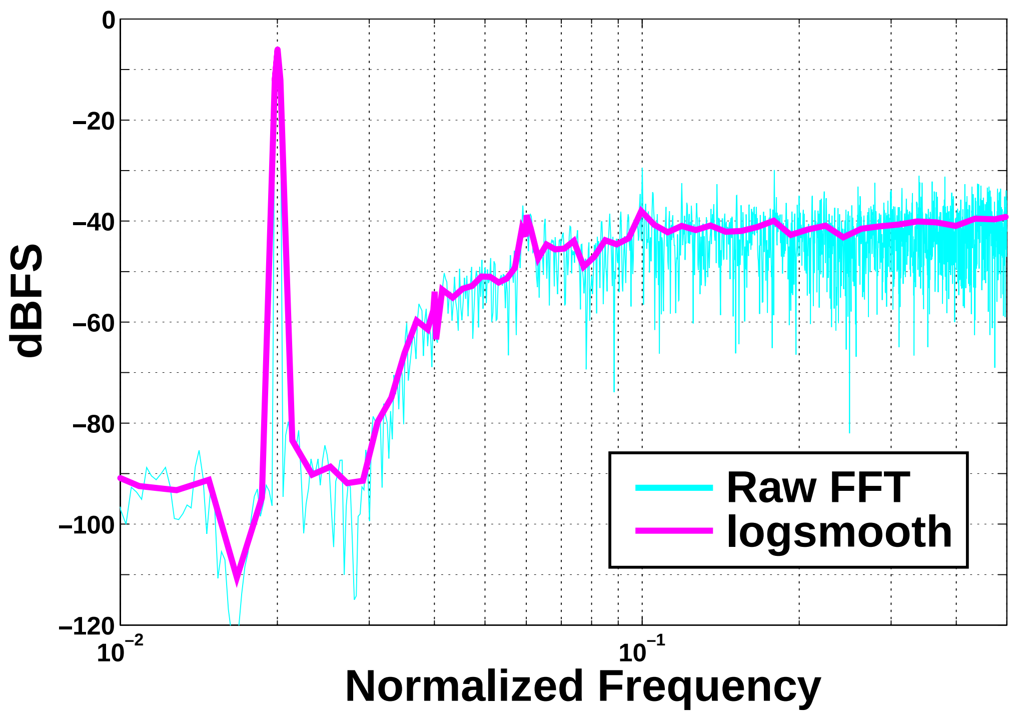
   Implemented by MATLAB's `psd()` function

2. **Take one big FFT and "filter" the spectrum**

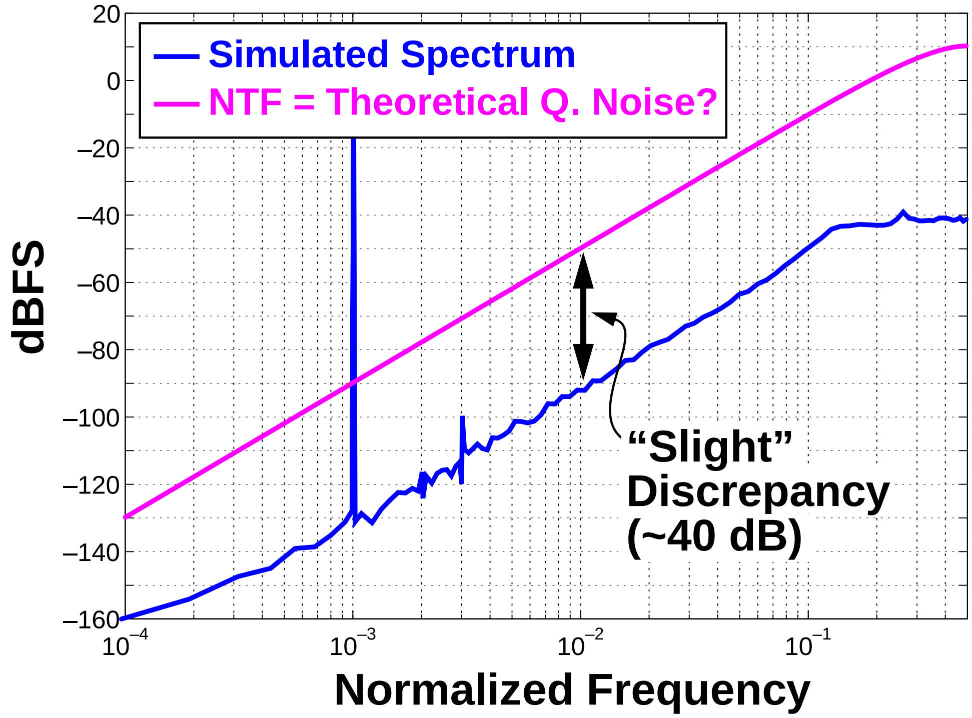
   Implemented by the $\Delta\Sigma$ Toolbox's `logsmooth()` function

   `logsmooth()` averages an exponentially-increasing number of bins in order to reduce the density of points in the high-frequency regime and make a nice log-frequency plot

Raw and Smoothed Spectra



Simulation vs. Theory (MOD2)



What Went Wrong?

1. **We normalized the spectrum so that a full-scale sine wave (which has a power of 0.5) comes out at 0 dB (whence the "dBFS" units)**

   $\Rightarrow$ **We need to do the same for the error signal.**

   i.e. use $S_{ee}(f) = 4/3$.

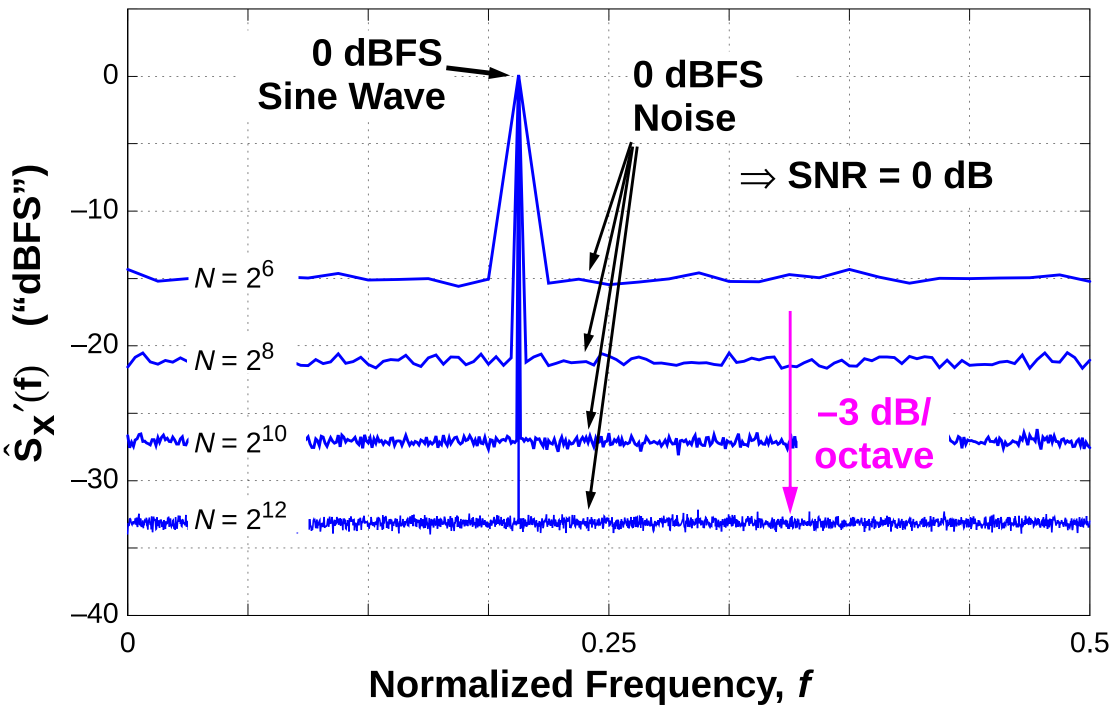
   But this makes the discrepancy 3 dB worse.

2. **We tried to plot a *power spectral density* together with something that we want to interpret as a *power spectrum***

- **Sine-wave components are located in individual FFT bins, but broadband signals like noise have their power spread over all FFT bins!**

  The "noise floor" depends on the length of the FFT.

Spectrum of a Sine Wave + Noise



Observations

- **The power of the sine wave is given by the height of its spectral peak**
- **The power of the noise is spread over all bins**

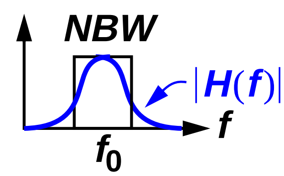
  The greater the number of bins, the less power there is in any one bin.
- **Doubling N reduces the power per bin by a factor of 2 (i.e. 3 dB)**

  But the total integrated noise power does *not* change.

So How Do We Handle Noise?

- **Recall that an FFT is like a filter bank**
- **The longer the FFT, the narrower the bandwidth of each filter and thus the lower the power at each output**
- **We need to know the *noise bandwidth* (NBW) of the filters in order to convert the power in each bin (filter output) to a power density**
- **For a filter with frequency response $H(f)$,**

$$NBW = \frac{\int |H(f)|^2 df}{H(f_0)^2}$$



FFT Noise Bandwidth
Rectangular Window

$$h_k(n) = \exp\left(j\frac{2\pi k}{N}n\right),\ H_k(f) = \sum_{n=0}^{N-1} h_k(n)\exp(-j2\pi fn)$$

$$f_0 = \frac{k}{N},\ H_k(f_0) = \sum_{n=0}^{N-1} 1 = N$$

$$\int |H_k(f)|^2 = \sum |h_k(n)|^2 = N \text{ [Parseval]}$$

$$\therefore NBW = \frac{\int |H_k(f)|^2 df}{H_k(f_0)^2} = \frac{N}{N^2} = \frac{1}{N}$$

Better Spectral Plot