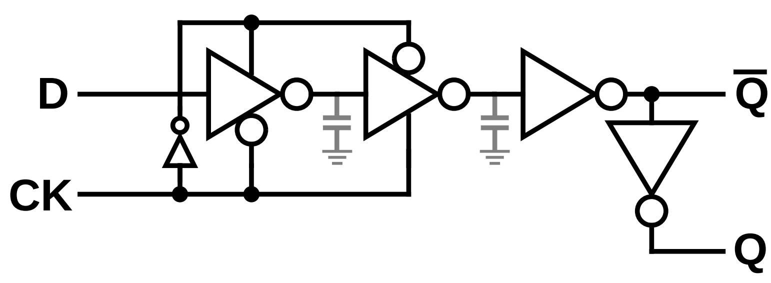
Higher-Order ΔΣ Modulators and the ΔΣ Toolbox

Richard Schreier
richard.schreier@analog.com

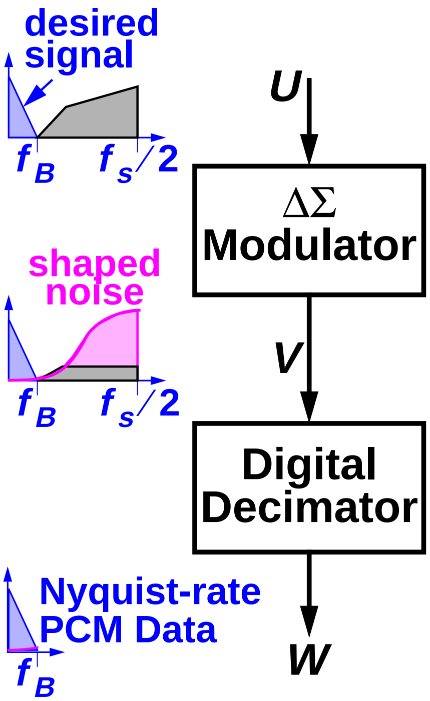
NLCOTD: Dynamic Flip-Flop

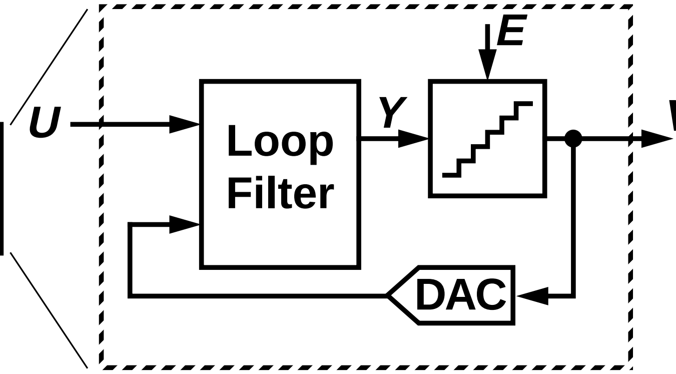
- Standard CMOS version



- Can the circuit be simplified?
  Is a complementarty clock necessary?

Review: A ΔΣ ADC System

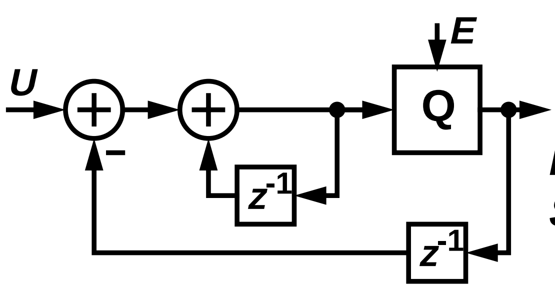




$$V(z) = STF(z)U(z) + NTF(z)E(z)$$

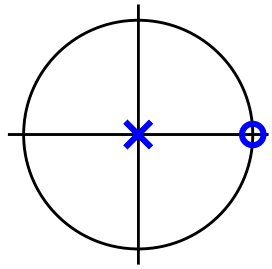
STF(*z*): signal transfer function
NTF(*z*): noise transfer function
E(*z*): quantization error

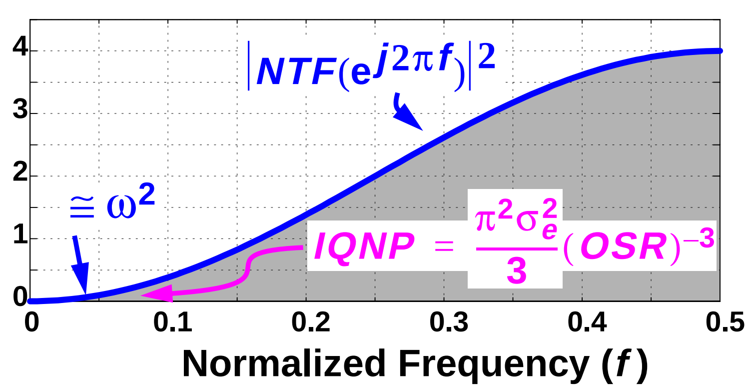
Review: MOD1



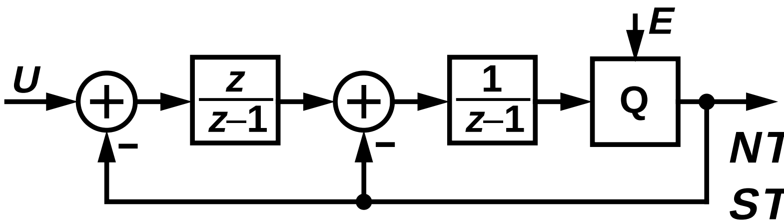
$$NTF(z) = (1 - z^{-1})$$
$$STF(z) = 1$$

NTF poles & zeros:



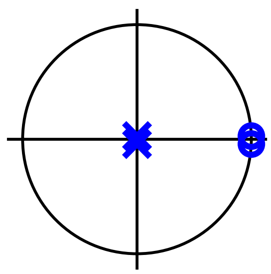


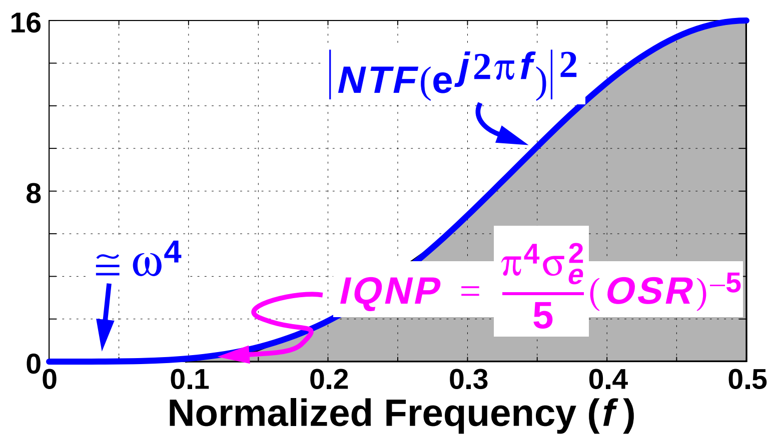
Review: MOD2



$$NTF(z) = (1 - z^{-1})^2$$
$$STF(z) = z^{-1}$$

NTF poles & zeros:



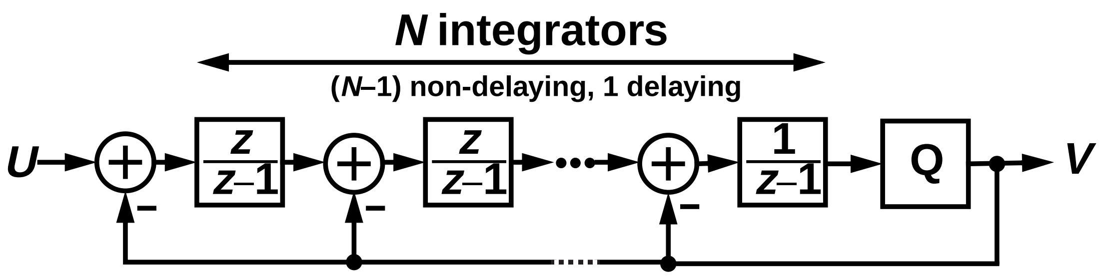


Review Summary

- **$\Delta\Sigma$ works by spectrally separating the quantization noise from the signal**
  - **Requires oversampling. $OSR \equiv f_s/(2f_B)$.**
  - **Achieved by the use of *filtering* and *feedback.***
- **A binary DAC is *inherently linear,* and thus a binary $\Delta\Sigma$ modulator is too**
- **MOD1-CT has *inherent anti-aliasing***
- **MOD1 has $NTF(z) = 1 - z^{-1}$**
  - $\Rightarrow$ **Arbitrary accuracy for DC inputs; 9 dB/octave SQNR-OSR trade-off.**
- **MOD2 has $NTF(z) = (1 - z^{-1})^2$**
  - $\Rightarrow$ **15 dB/octave SQNR-OSR trade-off.**

MOD*N*

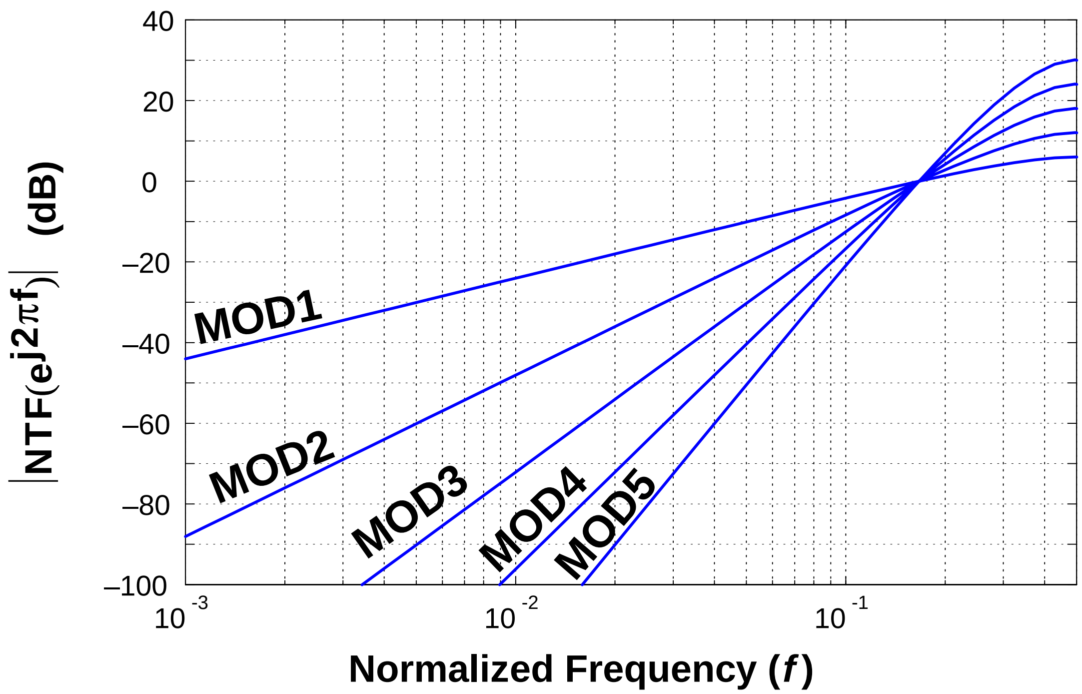
[Ch. 4 of Schreier & Temes]



$$STF(z) = z^{-1}$$
$$NTF(z) = (1 - z^{-1})^N$$

- **MOD*N*'s NTF is the *N*th power of MOD1's NTF**

NTF Comparison



Predicted Performance

- **In-band quantization noise power**

$$IQNP = \int_0^{0.5/OSR} |NTF(e^{j2\pi f})|^2 \cdot S_{ee}(f)\, df$$

$$\approx \int_0^{0.5/OSR} (2\pi f)^{2N} \cdot 2\sigma_e^2 df$$

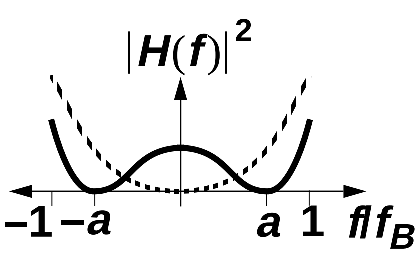
$$= \frac{\pi^{2N}}{(2N+1)(OSR)^{2N+1}}\sigma_e^2$$

- **Quantization noise drops as the (2*N*+1)th power of OSR— (6*N*+3) dB/octave SQNR-OSR trade-off**

Improving NTF Performance– NTF Zero Optimization

- **Minimize the integral of $|NTF|^2$ over the passband**

    Normalize passband edge to 1 for ease of calculation:



Need to find the a_i which minimize the integral

$$\int_{-1}^{1}(x^2 - a_1^2)^2 dx\ , \qquad n = 2$$

$$\int_{-1}^{1}x^2(x^2 - a_1^2)^2 dx\ , \qquad n = 3$$

$$\int_{-1}^{1}(x^2 - a_1^2)^2(x^2 - a_2^2)^2 dx\ , \qquad n = 4$$

$$\vdots$$

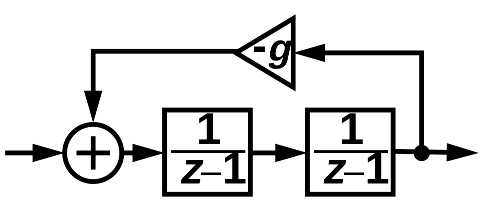
Solutions Up to Order = 8

Order	Optimal Zero Placement Relative to f_B	SQNR Improvement
1	0	0 dB
2	$\pm 1/\sqrt{3}$	3.5 dB
3	$0, \pm\sqrt{3/5}$	8 dB
4	$\pm\sqrt{3/7 \pm \sqrt{(3/7)^2 - 3/35}}$	13 dB
5	$0, \pm\sqrt{5/9 \pm \sqrt{(5/9)^2 - 5/21}}$ [Y. Yang]	18 dB
6	±0.23862, ±0.66121, ±0.93247	23 dB
7	0, ±0.40585, ±0.74153, ±0.94911	28 dB
8	±0.18343, ±0.52553, ±0.79667, ±0.96029	34 dB

Topological Implication

- **Feedback around pairs integrators:**

2 Delaying Integrators



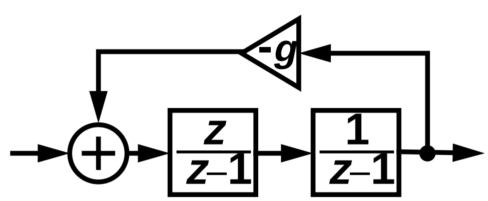
Poles are the roots of

$$1 + g\left(\frac{1}{z-1}\right)^2 = 0$$

i.e. $z = 1 \pm j\sqrt{g}$

Not quite on the unit circle, but fairly close if g<<1.

Non-delaying + Delaying Integrators (LDI Loop)



Poles are the roots of

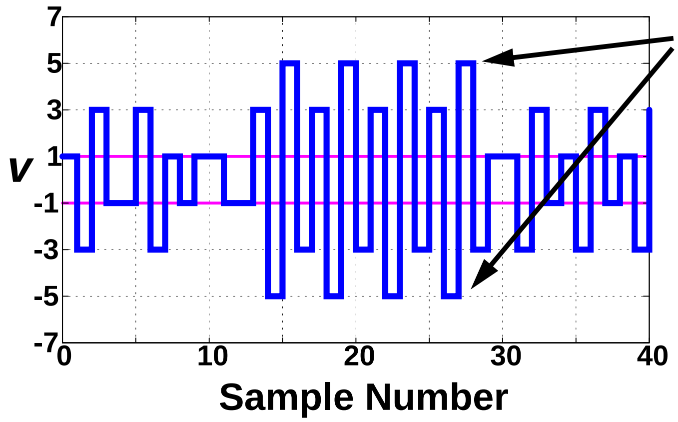
$$1 + \frac{gz}{(z-1)^2} = 0$$

i.e. $z = e^{\pm j\theta}$, $\cos\theta = 1 - g/2$

Precisely on the unit circle, regardless of the value of g.

Problem: A High-Order Modulator Wants a Multi-bit Quantizer

E.g. MOD3 with an Infinite Quantizer and Zero Input

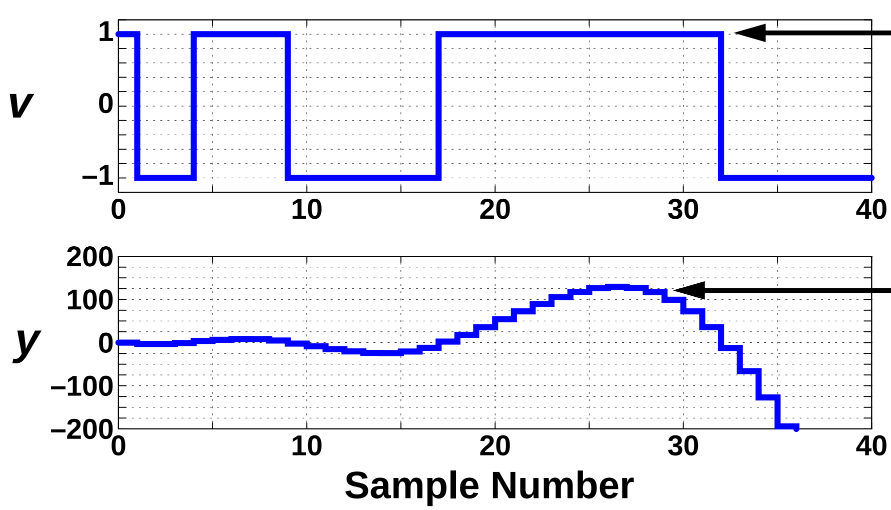


Quantizer input gets large, even if the input is small.

6 quantizer levels are used by a small input.

Simulation of MOD3-1b

(MOD3 with a Binary Quantizer)



- **MOD3-1b is unstable, even with zero input!**

Solutions to the Stability Problem

Historical Order

1 **Multi-bit quantization**

Initially considered undesirable because we lose the inherent linearity of a 1-bit DAC.

2 **More general NTF (not pure differentiation)**

Lower the NTF gain so that quantization error is amplified less.

Unfortunately, reducing the NTF gain reduces the amount by which quantization noise is attenuated.

3 **Multi-stage (MASH) architecture**

- **Combinations of the above are possible**

Multi-bit Quantization

A modulator with *NTF* = *H* and *STF* = 1 is guaranteed to be stable if $|u| < u_{max}$ at all times, where $u_{max} = nlev + 1 - \|h\|_1$ and $\|h\|_1 = \sum_{i=0}^{\infty} |h(i)|$

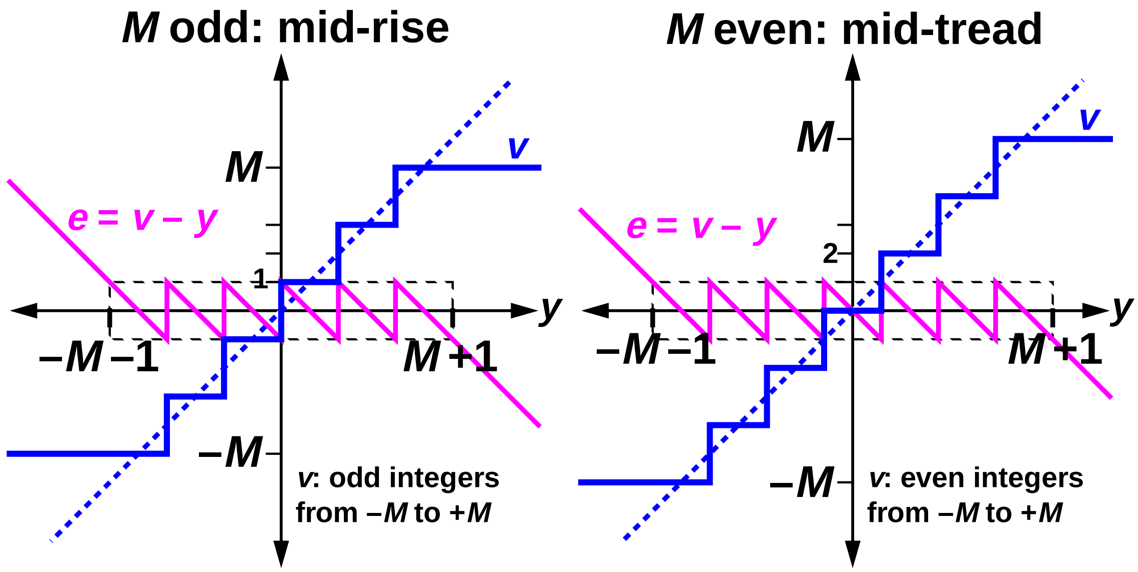
- **In MOD*N* $H(z) = (1 - z^{-1})^N$, so $h(n) = \{1, -a_1, a_2, -a_3, \ldots (-1)^N a_N, 0 \ldots\}$, $a_i > 0$ and thus $\|h\|_1 = H(-1) = 2^N$**
- **$nlev = 2^N$ implies $u_{max} = nlev + 1 - \|h\|_1 = 1$**

  MOD*N* is guaranteed to be stable with an *N*-bit quantizer if the input magnitude is less than $\Delta/2 = 1$.

  This result is quite conservative.
- **Similarly, $nlev = 2^{N+1}$ guarantees that MOD*N* is stable for inputs up to 50% of full-scale**

M-Step Symmetric Quantizer

$\Delta = 2$, ($nlev = M + 1$)



- **No-overload range:** $|y| \le nlev \Rightarrow |e| \le \Delta/2 = 1$

Inductive Proof of $\|h\|_1$ Criterion

- **Assume STF = 1 and** $(\forall n)(|u(n)| \le u_{max})$
- **Assume** $|e(i)| \le 1$ **for** $i < n$**.[Induction Hypothesis]**

$$|y(n)| = \left|u(n) + \sum_{i=1}^{\infty} h(i)e(n-i)\right|$$
$$\le u_{max} + \sum_{i=1}^{\infty} |h(i)||e(n-i)|$$
$$\le u_{max} + \sum_{i=1}^{\infty} |h(i)| = u_{max} + \|h\|_1 - 1$$

Then $u_{max} = nlev + 1 - \|h\|_1$

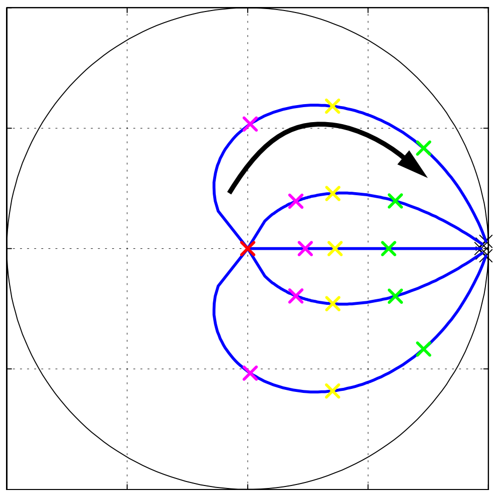
$\Rightarrow \quad |y(n)| \le nlev$

$\Rightarrow \quad |e(n)| \le 1$

- **So by induction** $|e(i)| \le 1$ **for all** $i > 0$

More General NTF

- **Instead of $NTF(z) = A(z)/B(z)$ with $B(z) = z^n$, use a more general $B(z)$**
  - **Roots of *B* are the poles of the NTF and must be inside the unit circle.**



Moving the poles away from $z = 1$ toward $z = 0$ makes the gain of the NTF approach unity.

The Lee Criterion for Stability in a 1-bit Modulator: $\|H\|_\infty \le 2$

[Wai Lee, 1987]

- **The measure of the "gain" of *H* is the maximum magnitude of *H* over frequency, aka the *infinity-norm* of *H*:** $\|H\|_\infty \equiv \max_{\omega \in [0, 2\pi]} |H(e^{j\omega})|$

Q: Is the Lee criterion <u>necessary</u> for stability?

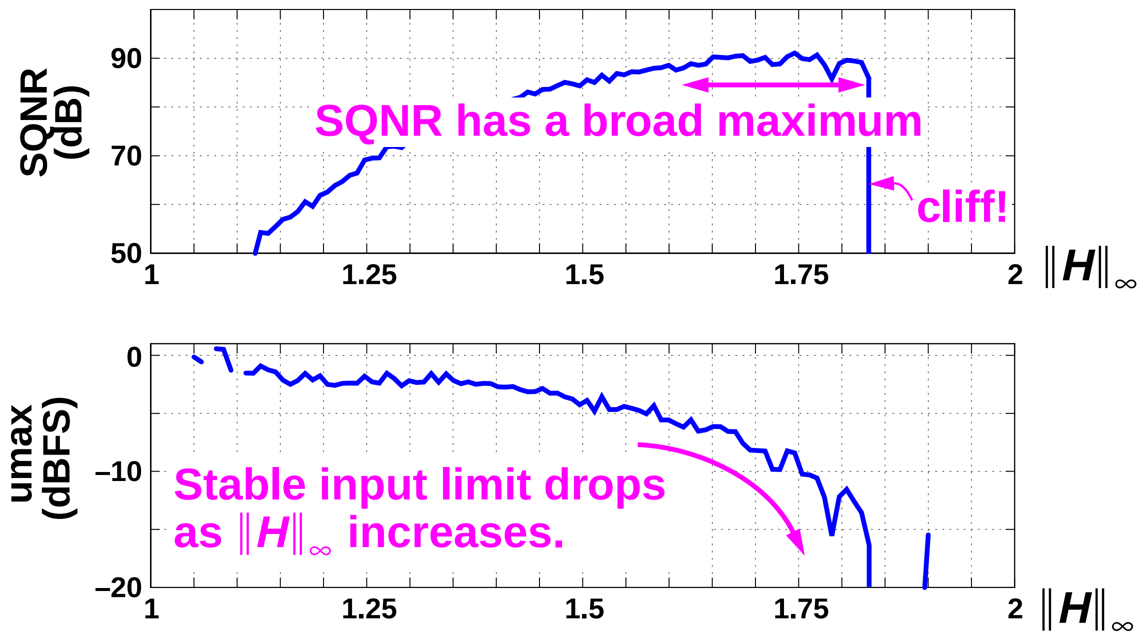
No. MOD2 is stable (for DC inputs less than FS) but $\|H\|_\infty = 4$.

Q: Is the Lee criterion <u>sufficient</u> to ensure stability?

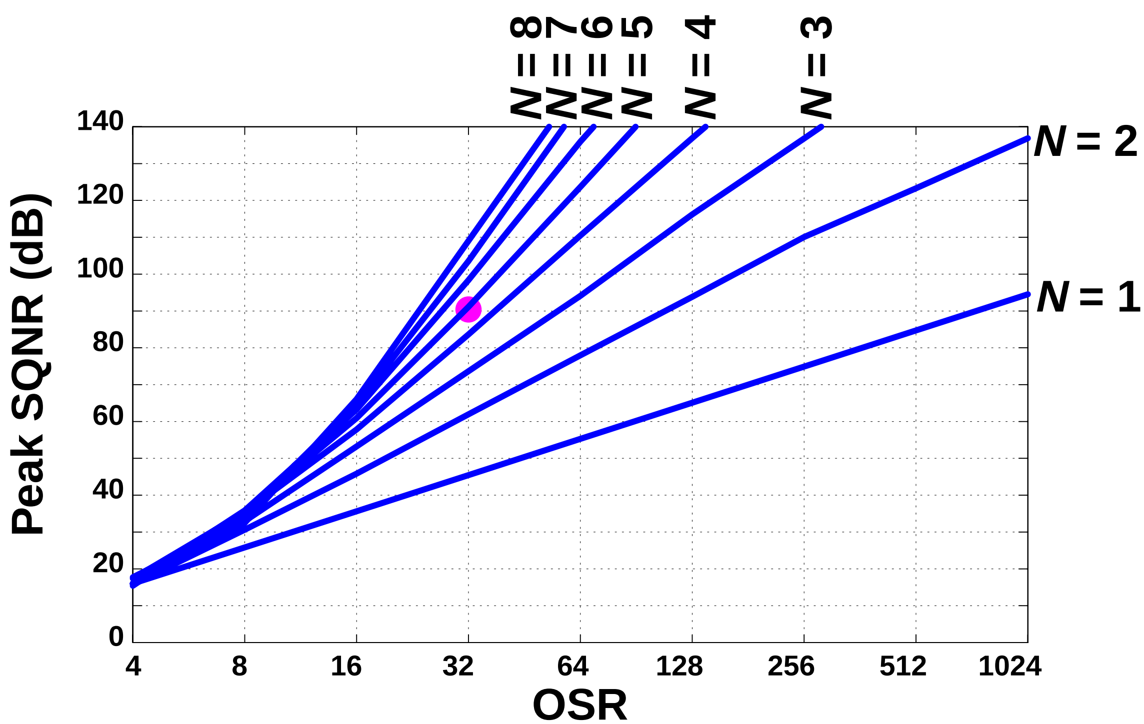
No. There are lots of counter-examples, but $\|H\|_\infty \le 1.5$ often works.

Simulated SQNR vs. $\|H\|_\infty$

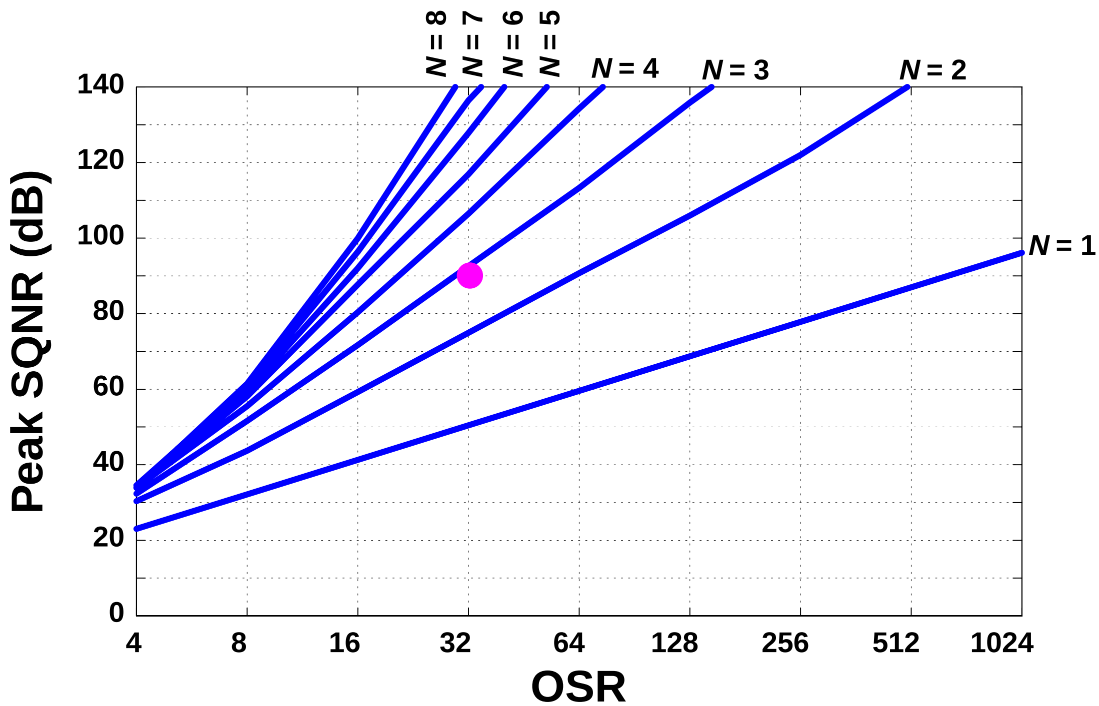
5^{th}-order NTFs; 1-b Quant.; OSR = 32



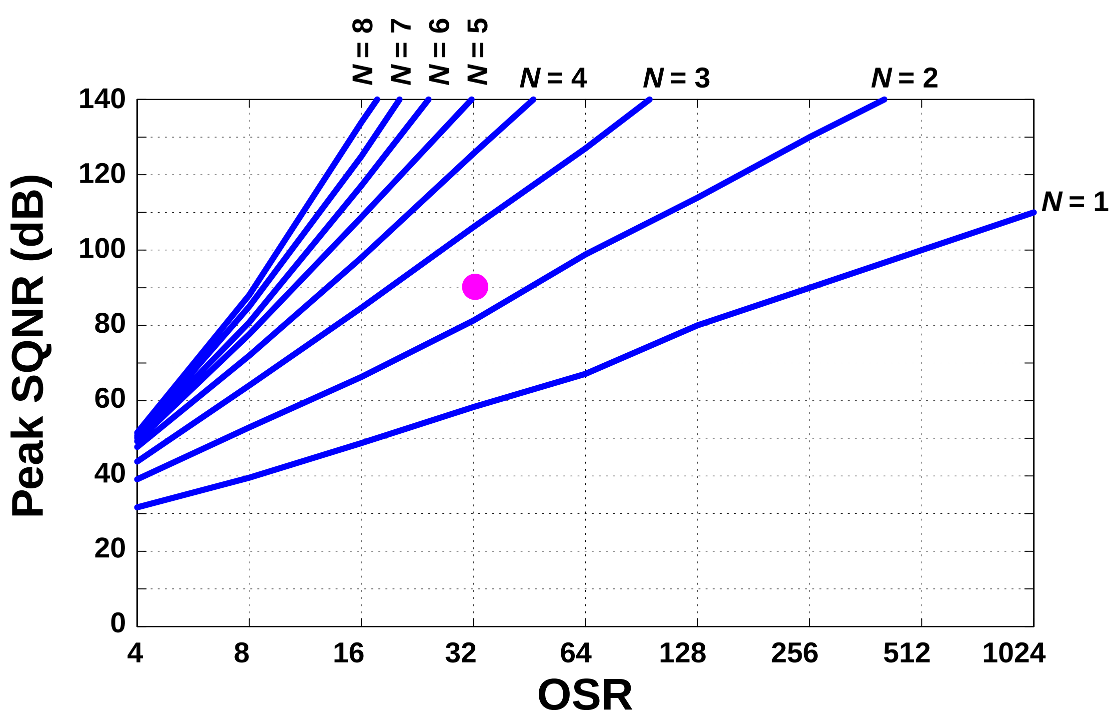
SQNR Limits— 1-bit Modulation



SQNR Limits for 2-bit Modulators

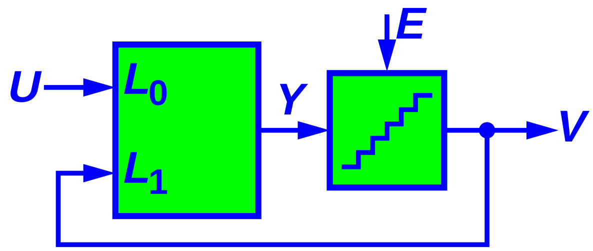


SQNR Limits for 3-bit Modulators



Generic Single-Loop ΔΣ ADC

- **Linear Loop Filter + Nonlinear Quantizer:**



$$Y = L_0 U + L_1 V$$
$$V = Y + E$$
$$\Rightarrow \quad V = STF \cdot U + NTF \cdot E, \text{ where}$$

$$NTF = \frac{1}{1 - L_1} \quad \& \quad STF = L_0 \cdot NTF$$

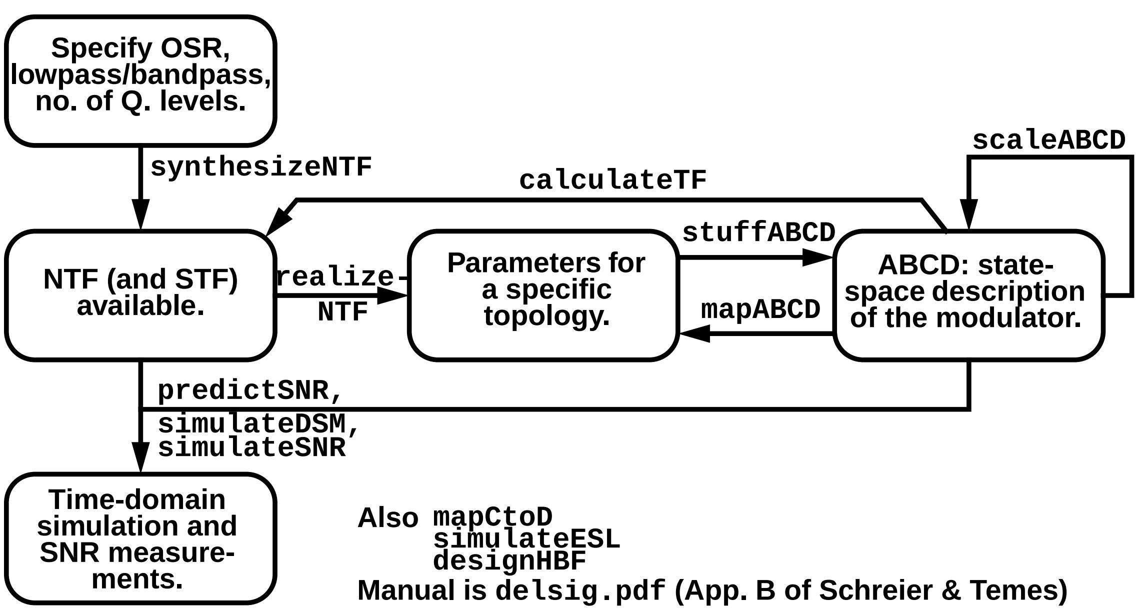
Inverse Relations:

$L_1 = 1 - 1/NTF$, $L_0 = STF / NTF$

ΔΣ Toolbox

`http://www.mathworks.com/matlabcentral/fileexchange`

Search for "Delta Sigma Toolbox"

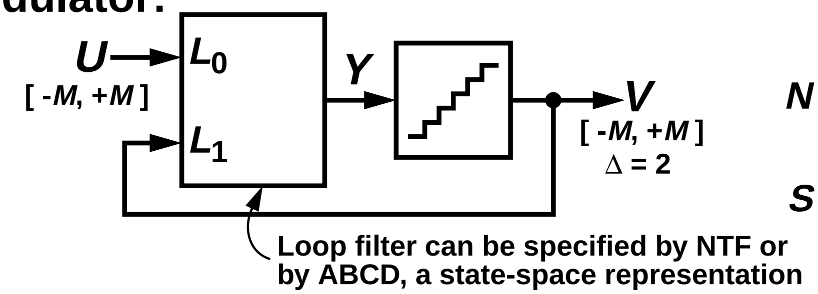


Also `mapCtoD`
`simulateESL`
`designHBF`

Manual is `delsig.pdf` (App. B of Schreier & Temes)

ΔΣ Toolbox Modulator Model

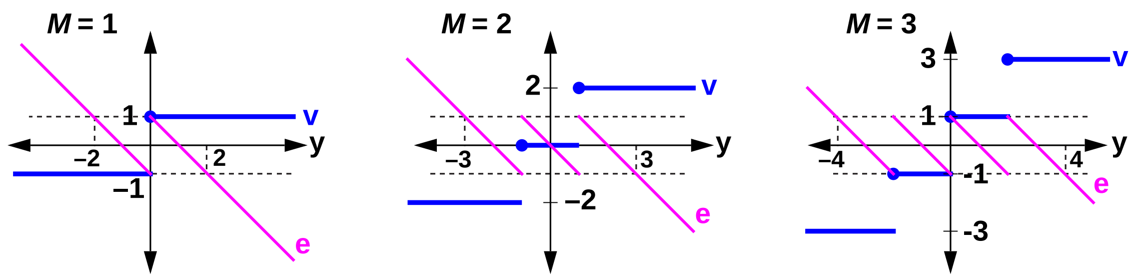
Modulator:



$$NTF = \frac{1}{1 - L_1}$$

$$STF = \frac{L_0}{1 - L_1}$$

Quantizer:



Mid-tread quantizer;
v: even integers [–*M*,+*M*]

Mid-rise quantizer;
v: odd integers [–*M*,+*M*]

NTF Synthesis

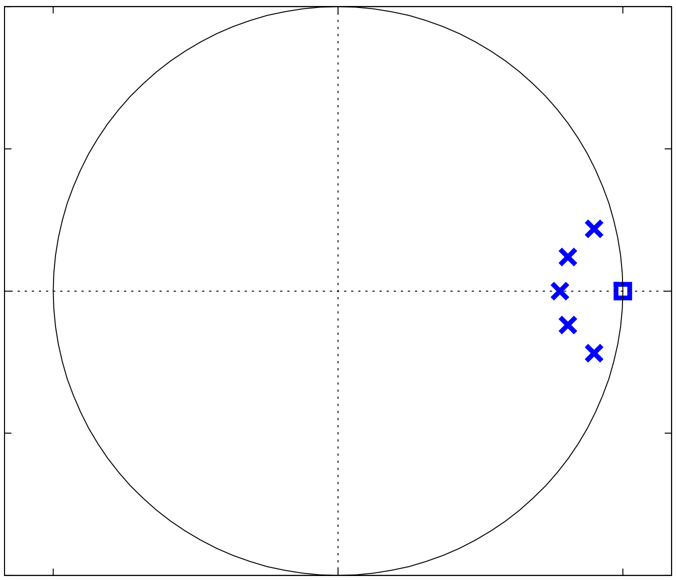
`synthesizeNTF`

- **Not all NTFs are realizable**
  Causality requires $h(0) = 1$, or, in the frequency domain, $H(\infty) = 1$. Recall $H(z) = h(0)z^0 + h(1)z^{-1} + \ldots$
- **Not all NTFs yield stable modulators**
  Rule of thumb for single-bit modulators:
  $\|H\|_\infty < 1.5$ [Lee].
- **Can optimize NTF zeros to minimize the mean-square value of *H* in the passband**
- **The NTF and STF share poles, and in some modulator topologies the STF zeros are not arbitrary**
  Restrict the NTF such that an all-pole STF is maximally flat. (Almost the same as Butterworth poles.)

Lowpass Example [dsdemo1]

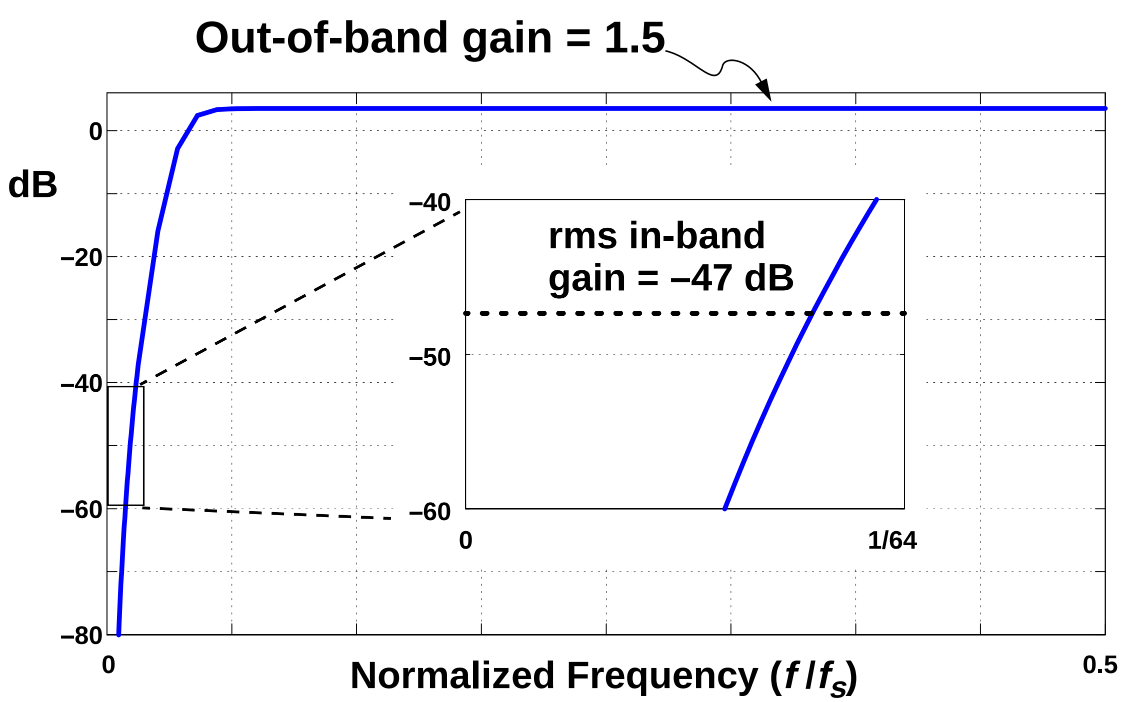
5th-order NTF, all zeros at DC

- **Pole/Zero diagram:**



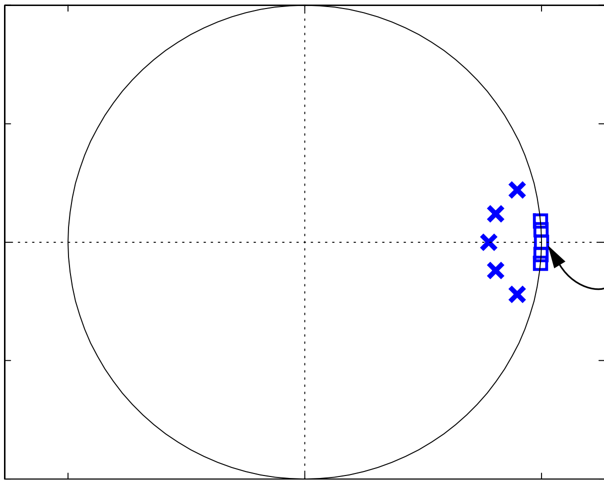
```
OSR = 32;
H = synthesizeNTF(5);
plotPZ(H);

f = linspace(0,0.5);
z = exp(2i*pi*f);
H_z = evalTF(H,z);
plot(f,dbv(H_z));
g = rmsGain(H,0,0.5/OSR)
```


Lowpass NTF



Improved 5th-Order Lowpass NTF

Zeros optimized for OSR=32

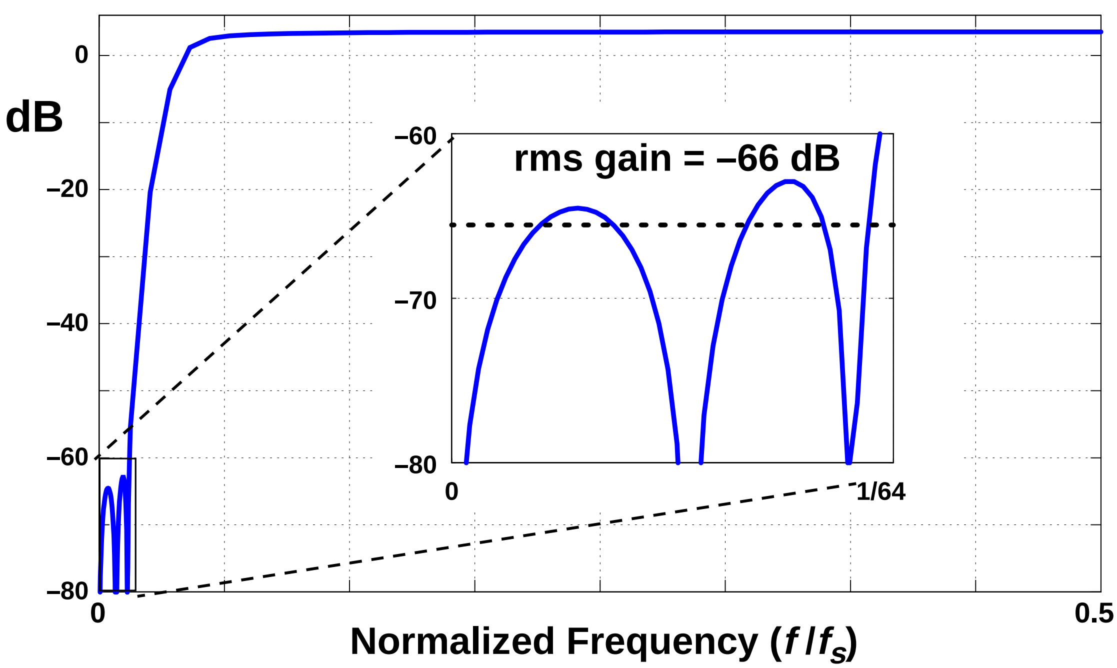


optimization flag

```
OSR = 32;
H = synthesizeNTF(5,OSR,1);
...
```

Zeros spread across the band-of-interest to minimize the rms value of the NTF.

Improved NTF

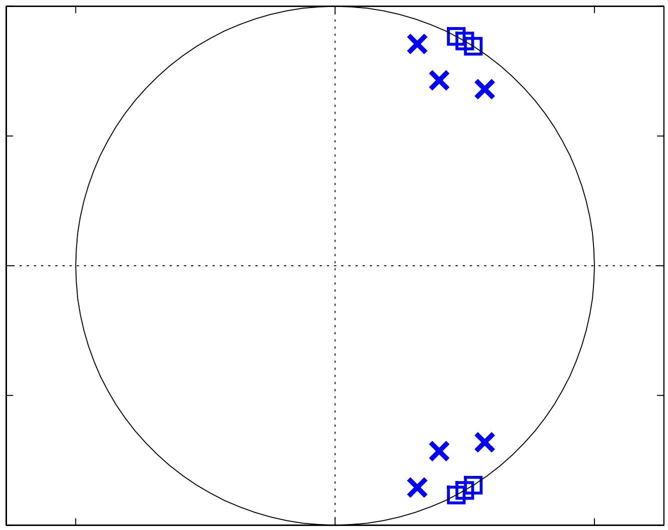


Bandpass Example

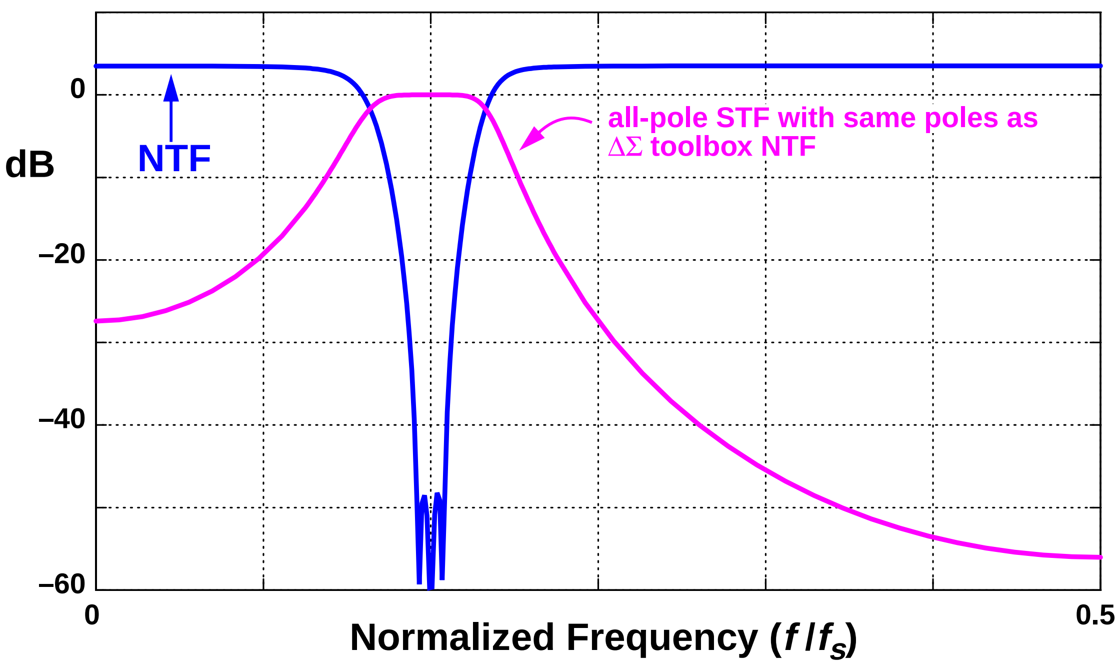
```
OSR = 64;
f0 = 1/6;
H=synthesizeNTF(6,OSR,1,[],f0);...
```

center frequency

[] or NaN means use default value, i.e. Hinf = 1.5



Bandpass NTF and STF



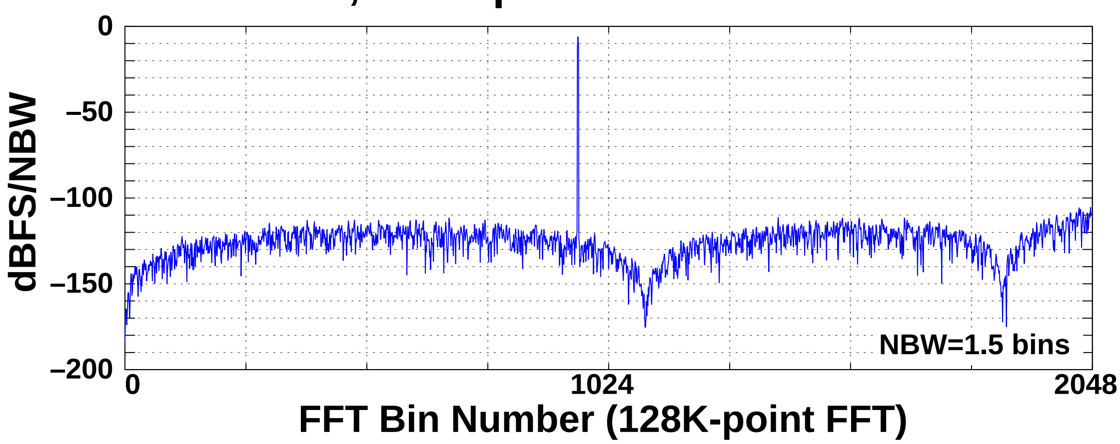
Summary: NTF Selection

- **If OSR is high, a single-bit modulator may work**
- **To improve SQNR,**
  - Optimize zeros,
  - Increase $\|H\|_\infty$, or
  - Increase order.
- **If SQNR is insufficient, must use a multi-bit design**
  - Can turn all the above knobs to enhance performance.
- **Feedback DAC assumed to be ideal**

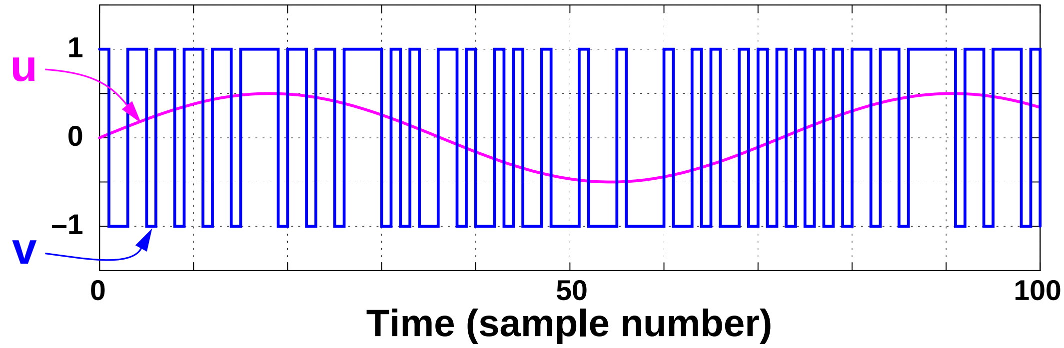
NTF-Based Simulation [dsdemo2]

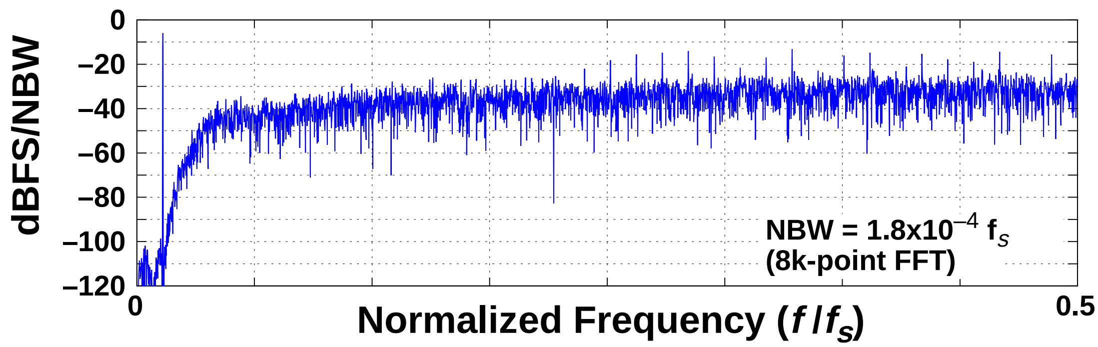
```
order=5; OSR=32;
ntf = synthesizeNTF(order,OSR,1);
N=2^17; fbin=959; A=0.5; % 128K points
input = A*sin(2*pi*fbin/N*[0:N-1]);
output = simulateDSM(input,ntf);
spec = fft(output.*ds_hann(N)/(N/4));
plot(dbv(spec(1:N/(2*OSR))));
```

- **In mex form; 128K points in < 0.1 sec**



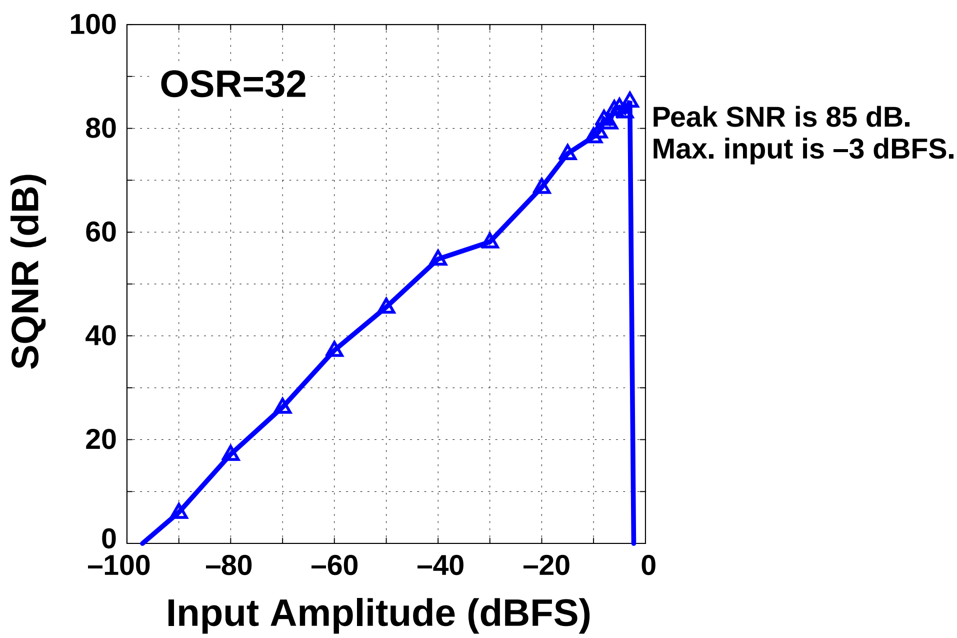
Simulation Example Cont'd



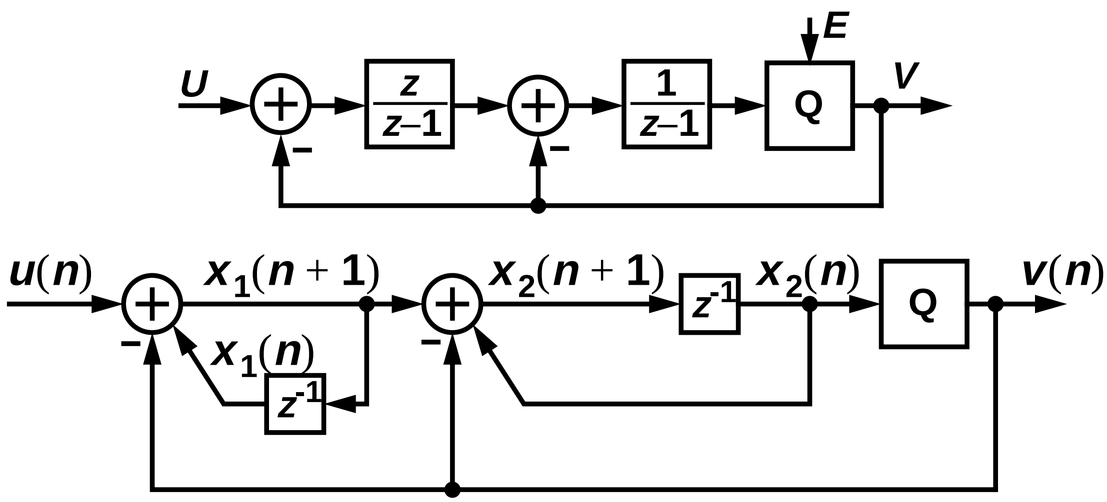


SNR vs. Amplitude: `simulateSNR`

```
[snr amp] = simulateSNR(ntf,OSR);
plot(amp,snr,'b-^');
```



MOD2 Expanded



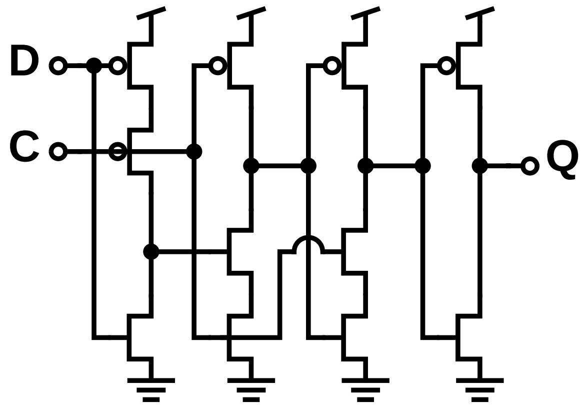
Difference Equations:

$$v(n) = Q(x_2(n))$$
$$x_1(n+1) = x_1(n) - v(n) + u(n)$$
$$x_2(n+1) = x_2(n) - v(n) + x_1(n+1)$$

Example Matlab™ Code

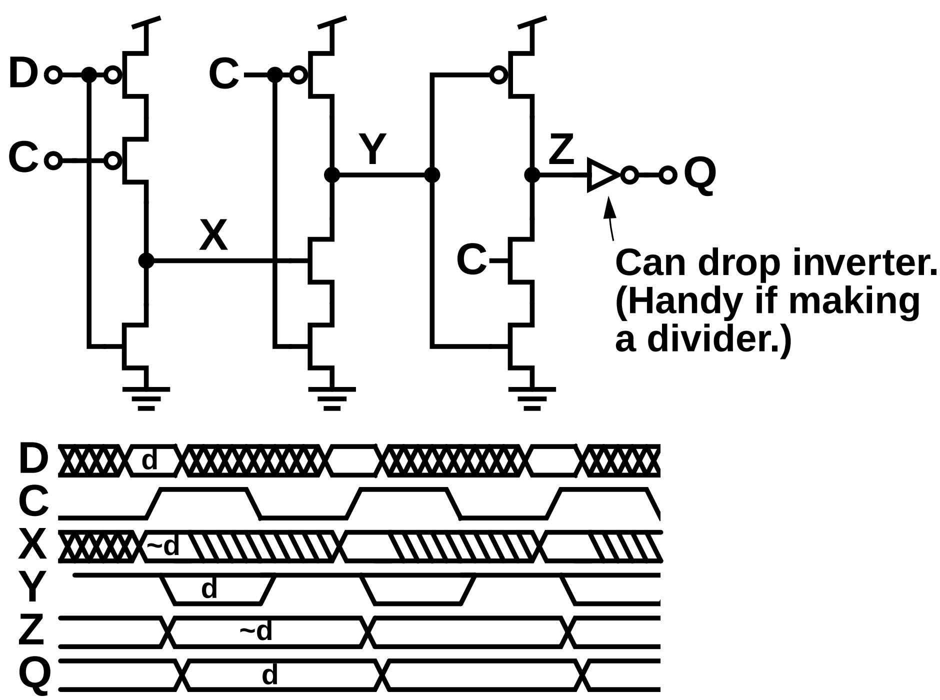
```
function [v] = simulateMOD2(u)
    x1 = 0;
    x2 = 0;
    for i = 1:length(u)
        v(i) = quantize( x2 );
        x1 = x1 + u(i) - v(i);
        x2 = x2 + x1 - v(i);
    end
return

function v = quantize( y )
    if y>=0
        v = 1;
    else
        v = -1;
    end
return
```

NLCOTD: True Single-Phase Dynamic FF



- + Clock not inverted anywhere
- + Small
- + Fast

TSPFF Operation



TSPFF Gotchas

- **Leakage:**
  Won't work if clock is too slow.
  Possible high current if clock is stopped.
  - **Need to add devices to hold the dynamic nodes at a safe value.**
- **No positive feedback**
  - **Vulnerable to metastability.**