CONTINUOUS-TIME Δ-Σ MODULATORS

SOME OF THE LOWEST POWER AND HIGHEST SPEED ΔΣ MODULATORS BUILT USING CONT-TIME LOOP FILTERS

Ex IN 2010 ISSCC PAPER 27.1

SIGNAL BANDWIDTH 125 MHZ (LOWPASS)
SAMPLE RATE 4GHZ (OSR = 16)
45 nm CMOS, $V_{DD} = 1.1, 1.8$
3'RD ORDER CT ADC, 4b QUANTIZER
POWER = 256 mW
DR = 70 dB THD = -74 dB FS
SNR = 65 dB SNDR = 65 dB

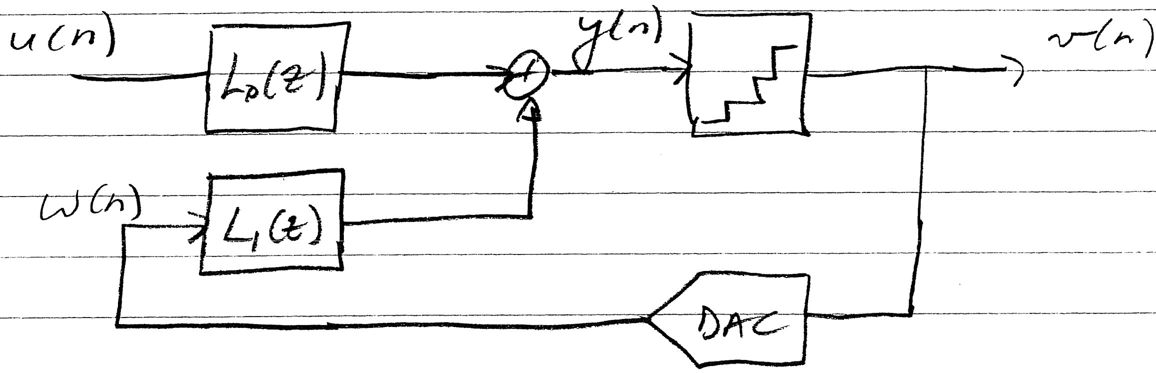
ADVANTAGES
- INHERENT ANTI-ALIASING BEFORE SAMPLING
- SAMPLING TAKES PLACE AT AN INSENSITIVE POINT IN LOOP
- CAN BE CLOCKED FASTER THAN SC COUNTERPART (PERHAPS 2→4 TIMES FASTER)
  CT ⇒ LIMITED BY QUANTIZER & DAC
  SC ⇒ LIMITED BY OPAMP SETTLING

DISADVANTAGES

- USUALLY POORER LINEARITY
- SENSITIVE TO CLOCK JITTER OF DAC

LOOP FILTER DESIGN

BASED ON MATCHING DISCRETE-TIME MODULATOR



DISCRETE-TIME SAMPLE-RATE, T

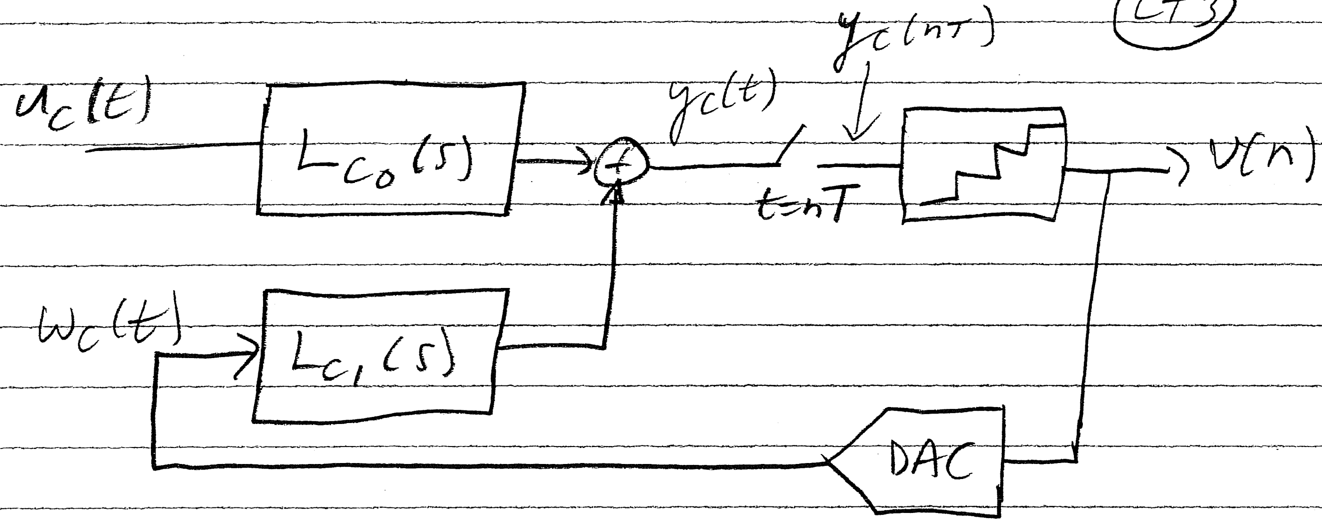
$T = \frac{1}{f_s}$

WANT

$$\mathcal{Z}^{-1}[L_1(z)] = \mathcal{L}^{-1}[R_c(s)\, L_{c1}(s)]\Big|_{t=nT}$$

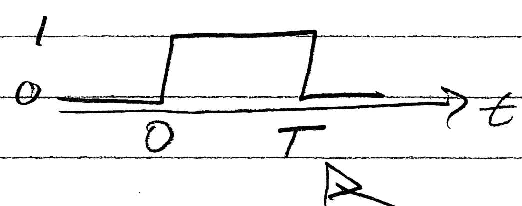
INVERSE Z TRANSFORM INVERSE LAPLACE TRANSFORM

=) IMPULSE INVARIANT METHOD



$R_c(s)$ REPRESENTS DAC PULSE TRANSFER FUNTION

FOR NRZ $r_c(t)$
IMPULSE RESPONSE



IN CASE OF NRZ $R_c(s)$ ABOVE

Z-DOMAIN	S-DOMAIN
$\frac{1}{z-1}$	$\frac{1}{Ts}$
$\frac{1}{(z-1)^2}$	$\frac{-Ts+2}{2T^2s^2}$
$\frac{1}{(z-1)^3}$	$\frac{2T^2s^2-3Ts+6}{6T^3s^3}$

IMPULSE INVARIANCE EQUIVALENCIES
TABLE 1

CT4

EXAMPLE

GIVEN $NTF(z) = (1 - z^{-1})^2$
& $STF(z) = z^{-1}$

FIND $L_{C0}(s)$, $L_{C1}(s)$

1) FIND $L_0(z)$, $L_1(z)$

$$L_0(z) = \frac{STF(z)}{NTF(z)} = \frac{z^{-1}}{1-2z^{-1}+z^{-2}}$$

$$L_0(z) = \frac{z}{z^2-2z+1} \qquad (1)$$

$$L_1(z) = 1 - \frac{1}{NTF(z)} = \frac{NTF(z)-1}{NTF(z)}$$

$$= \frac{-2z^{-2}+z^{-2}}{1-2z^{-1}+z^{-2}}$$

$$L_1(z) = \frac{-2z+1}{z^2-2z+1} \qquad (2)$$

2) BREAK $L_0(z)$ & $L_1(z)$ INTO PARTIAL FRACTION EXPANSIONS

$$L_0(z) = \frac{1}{z^2-2z+1} + \frac{1}{z-1}$$

CT5

$$L_0(z) = \frac{1}{(z-1)^2} + \frac{1}{z-1} \qquad (3)$$

$$L_1(z) = \frac{-1}{(z-1)^2} + \frac{-2}{z-1} \qquad (4)$$

3) USE TABLE (1) TO FIND $L_{c_0}(s), L_{c_1}(s)$

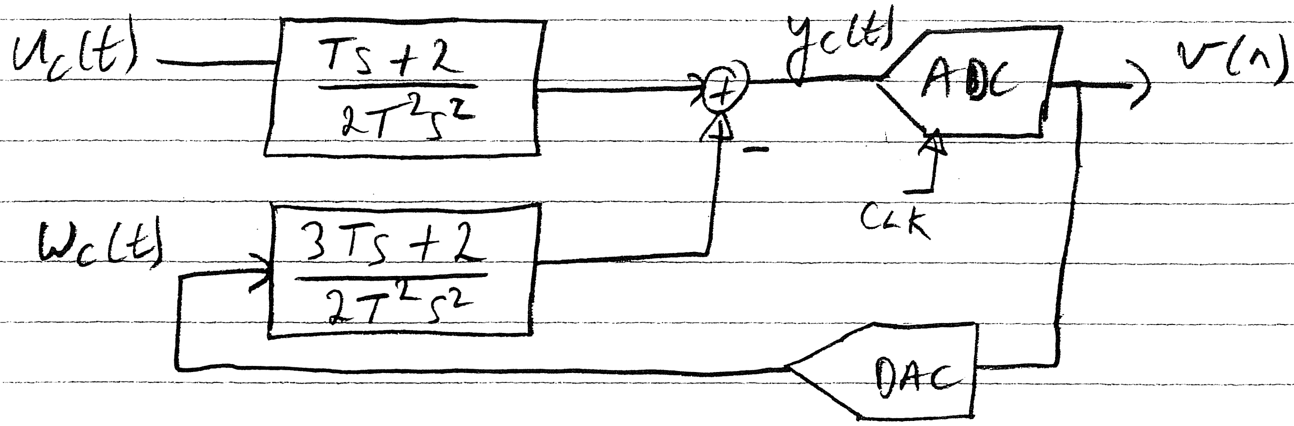
$$L_{c_0}(s) = \frac{-Ts+2}{2T^2s^2} + \frac{1}{Ts}$$

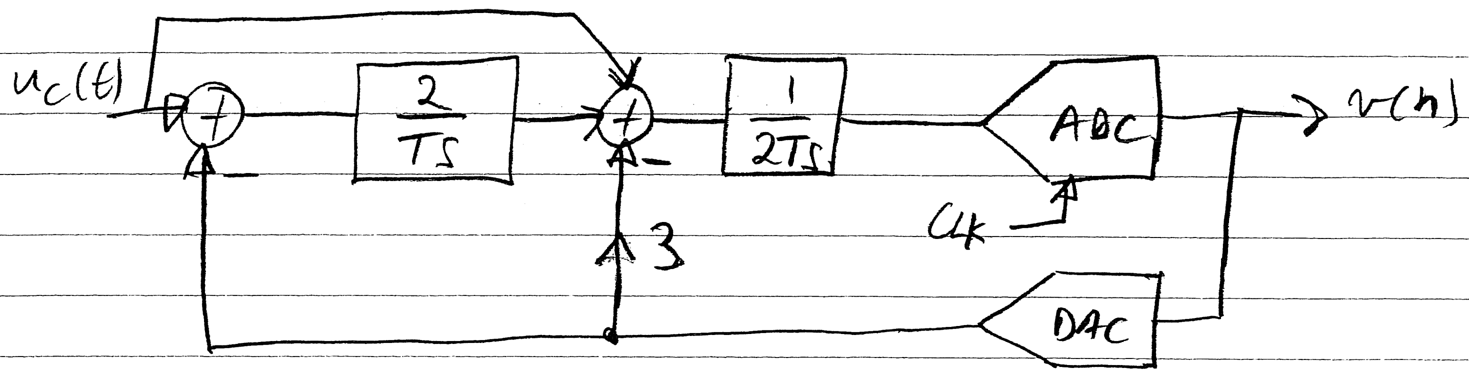
$$L_{c_0}(s) = \frac{Ts+2}{2T^2s^2} \qquad (5)$$

$$L_{c_1}(s) = \frac{Ts-2}{2T^2s^2} + \frac{-2}{Ts}$$

$$L_{c_1}(s) = \frac{-3Ts-2}{2T^2s^2} \qquad (6)$$

4) IMPLEMENTATION

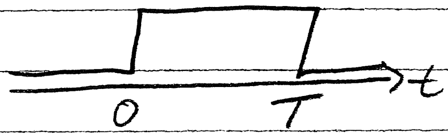




POSSIBLE IMPLEMENTATION

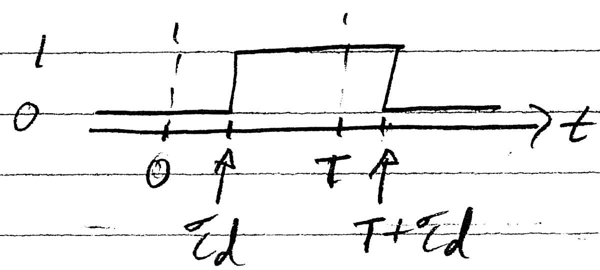
EXCESS LOOP DELAY

ABOVE WAS FOR CASE OF ADC/DAC IMPULSE RESPONSE



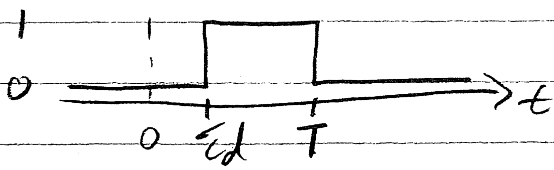
BUT THIS LEAVES NO TIME FOR ADC TO RESOLVE ITS DIGITAL OUTPUT & DAC TO TURN ON.

MORE PRACTICAL IS TO HAVE ADC/DAC IMPULSE RESPONSE OF

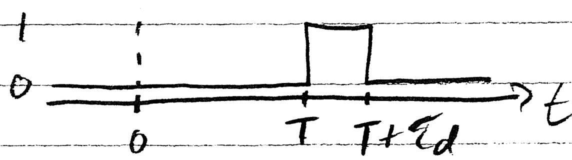


$r_c(t)$

CAN BE THOUGHT OF AS SUM OF 2 PULSE RESPONSES $r_c(t) = r_{c_1}(t) + r_{c_2}(t)$



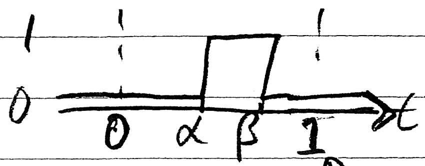
$r_{c_1}(t)$



$r_{c_2}(t)$

(CT8)

NEED TO FIND NEW IMPULSE INVARIANCE TABLE. CAN SHOW FOR



$r_{c(\alpha,\beta)}(t)$

WHERE $0 \le \alpha \le \beta \le 1$

NORMALIZED TO "$\underline{\underline{1}}$"

S-DOMAIN	Z-DOMAIN
$\dfrac{1}{Ts}$	$\dfrac{\beta-\alpha}{z-1}$
$\dfrac{1}{T^2 s^2}$	$\dfrac{\left[\beta-\frac{\beta^2}{2}-\alpha+\frac{\alpha^2}{2}\right]z+\left[\frac{\beta^2}{2}-\frac{\alpha^2}{2}\right]}{(z-1)^2}$

SO $r_{c_1}(t)$ HAS $\alpha = \tau_d/T$, $\beta = 1$

$r_{c_2}(t)$ HAS $\alpha = 0$, $\beta = \tau_d/T$

AND OCCURS IN NEXT SAMPLE

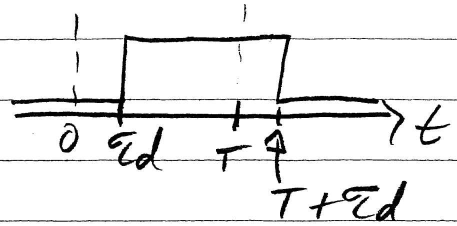
SO

$$r_c(t) = r_{c\left(\frac{\tau_d}{T},1\right)}(t) + r_{c\left(0,\frac{\tau_d}{T}\right)}(t-T)$$

FOR ABOVE, WE CAN FIND.

CT9

FOR $r_c(t)$



$$\tau_d' = \frac{\tau_d}{T}$$

TABLE 2

$\frac{1}{Ts}$	$\frac{1-\tau_d'}{z-1} + z^{-1}\frac{\tau_d'}{z-1}$
$\frac{1}{T^2 s^2}$	$\frac{(0.5-\tau_d'+0.5\tau_d'^2)z+0.5-0.5\tau_d'^2}{(z-1)^2} + z^{-1}\frac{(\tau_d'-0.5\tau_d'^2)z+0.5\tau_d'^2}{(z-1)^2}$

DUE TO PULSE EXTENDING INTO NEXT TIME INTERVAL $\Rightarrow$ MAKES Z-DOMAIN ORDER 1 HIGHER AND MORE UNSTABLE

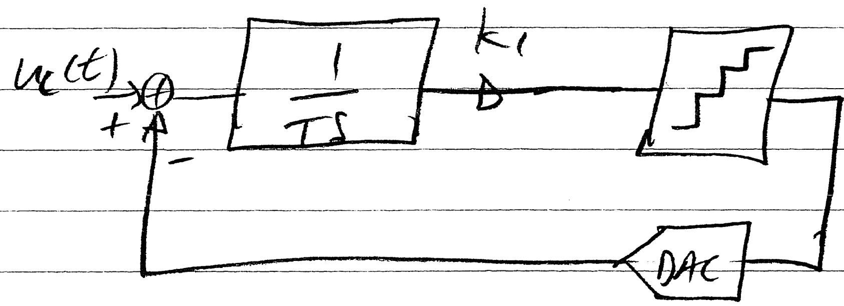
EXAMPLE

CONSIDER $NTF(z) = (1 - z^{-1})$

$$L_1(z) = \frac{1 - NTF(z)}{NTF(z)} = \frac{1 - (1 - z^{-1})}{1 - z^{-1}} = \frac{z^{-1}}{1 - z^{-1}}$$

$$L_1(z) = \frac{1}{z - 1}$$

$$\Rightarrow L_{c_1}(s) = \frac{1}{Ts}$$ IF NO EXCESS DELAY LOOP



BUT IF ADC HAS DELAY OF $\tau_d' = \frac{\tau_d}{T}$

$$L_{c_1}(s) = \frac{k_1}{Ts} \Rightarrow L_1(z) = k_1 \frac{(1 - \tau_d')}{z - 1} + \frac{k_1 z^{-1} \tau_d'}{z - 1}$$

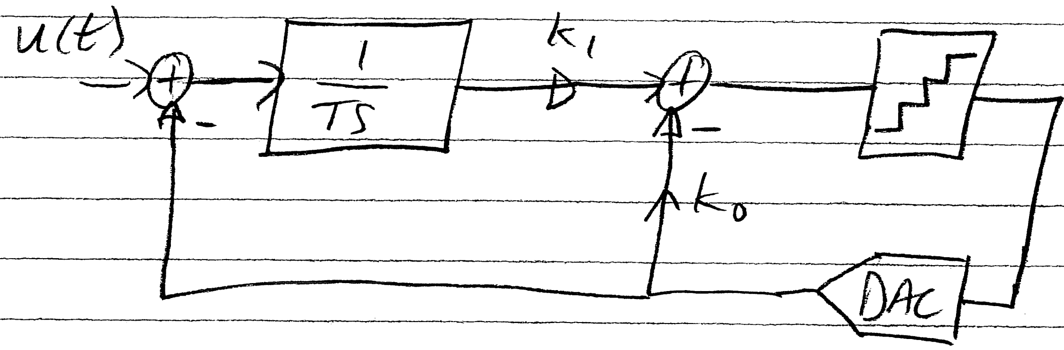
$$L_1(z) = \frac{k_1 (1 - \tau_d')}{(z - 1)} + \frac{k_1 \tau_d'}{z(z - 1)}$$

CT11

$$L_1(z) = \frac{k_1(1-\tau_d')z + k_1\tau_d'}{z(z-1)}$$

NO LONGER 1'ST ORDER!!

TO CORRECT ADD EXTRA FEEDBACK COEFF AROUND ADC/DAC LOOP



$$L_a(s) = k_0 + \frac{k_1}{Ts}$$

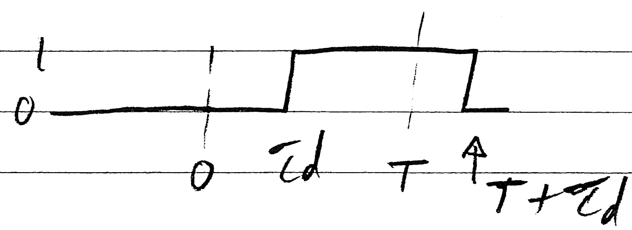
$$L_1(z) = k_0 z^{-1} + \frac{k_1(1-\tau_d')z + k_1\tau_d'}{z(z-1)}$$

$$= \frac{(k_1 - k_1\tau_d' + k_0)z + (k_1\tau_d' - k_0)}{z(z-1)}$$

WANT $L_1(z) = \frac{1}{z-1}$

REASON $L_1(z) = k_0 z^{-1} + \ldots$

DAC IMPULSE RESPONSE



SO DISCRETE-TIME IMPULSE RESPONSE IS

0 1 0 0 0 ...

(A SINGLE "1" AT $n=1$)

AND THAT MULTIPLIED BY k_0 AND ADDED TO INPUT OF ADC (NO FILTER)

So $k_1 \tau_d' - k_0 = 0 \Rightarrow k_0 = k_1 \tau_d'$

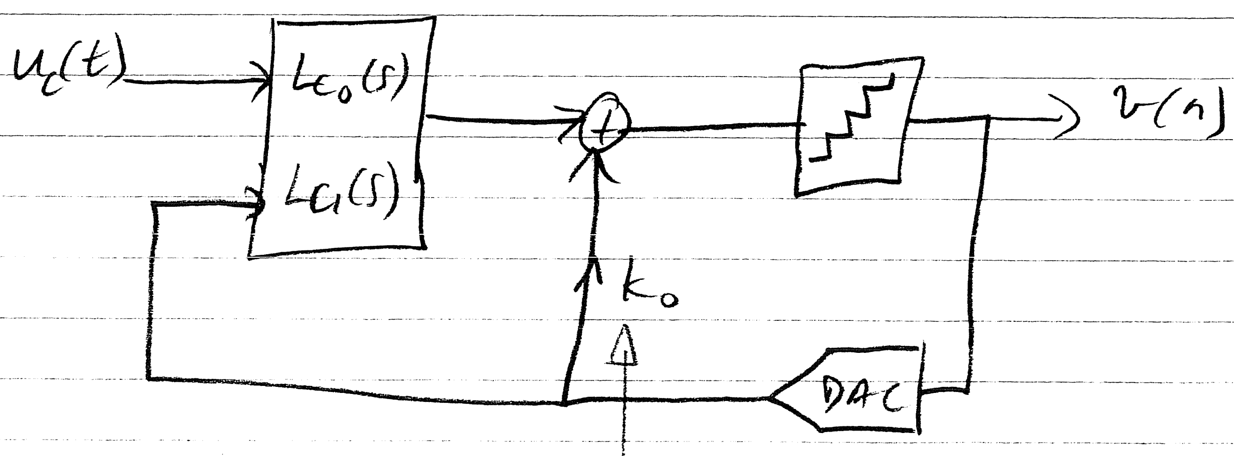
& $k_1 - k_1 \tau_d' + k_0 = 1$

$$k_1 - k_1 \tau_d' + k_1 \tau_d' = 1 \Rightarrow \boxed{\begin{aligned} k_1 &= 1 \\ k_0 &= \tau_d' \end{aligned}}$$

$$L_1(z) = \frac{1}{z-1}$$

MORE COMPLICATED FOR HIGHER-ORDER SYSTEMS, USE SYMBOLIC EQN SOLVER.

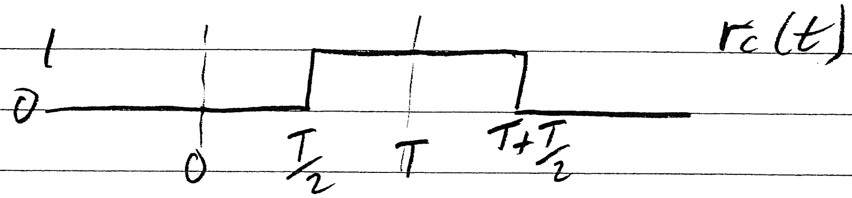
IN GENERAL,



ADDED FOR EXCESS LOOP DELAY

A COMMON CASE IS $\tau_d = \frac{T}{2}$

SO $\tau_d' = \frac{1}{2}$



CLOCK ADC AT TIME 0 AND THEN PUT DAC PULSE OUT AT TIME $\frac{T}{2}$

HERE

CONT-TIME	DISCRETE-TIME
$\frac{1}{Ts}$	$\frac{0.5 + 0.5z^{-1}}{(z-1)}$
$\frac{1}{T^2 s^2}$	$\frac{\frac{1}{8}z + \frac{3}{4} + \frac{1}{8}z^{-1}}{(z-1)^2}$