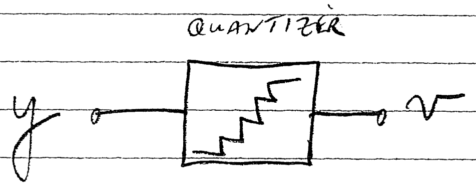
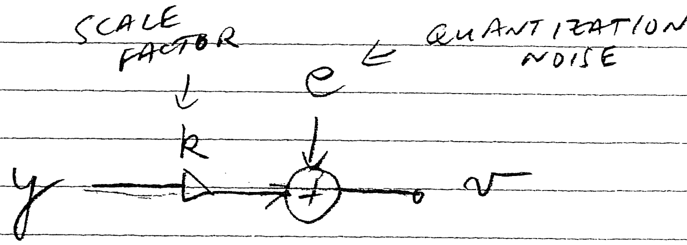
UNDERSTANDING DELTA-SIGMA CONVERTERS

BY SCHREIER + TEMES, 2005

CHAP 2

2.1 QUANTIZER





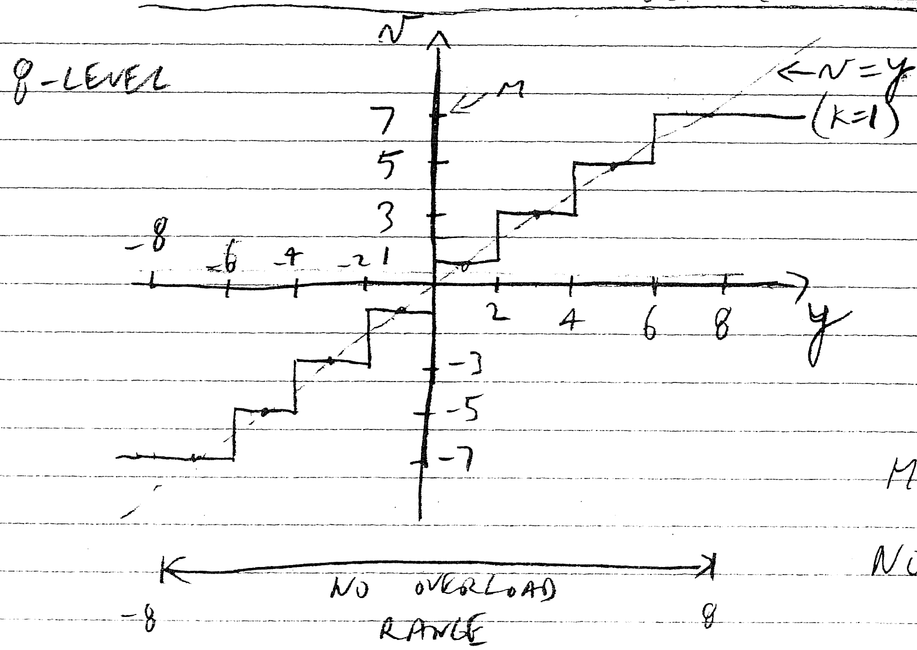
$$v = ky + e$$

k OFTEN EQUALS 1 FOR MULTI LEVEL QUANTIZER

$k \neq 1$ FOR 2 LEVEL QUANTIZER.

MULTI-LEVEL QUANTIZERS (k=1)

MID RISE QUANTIZER (SYMMETRIC)



M=7

M ⇒ ODD

OF LEVELS = M+1

$\Delta = 2$

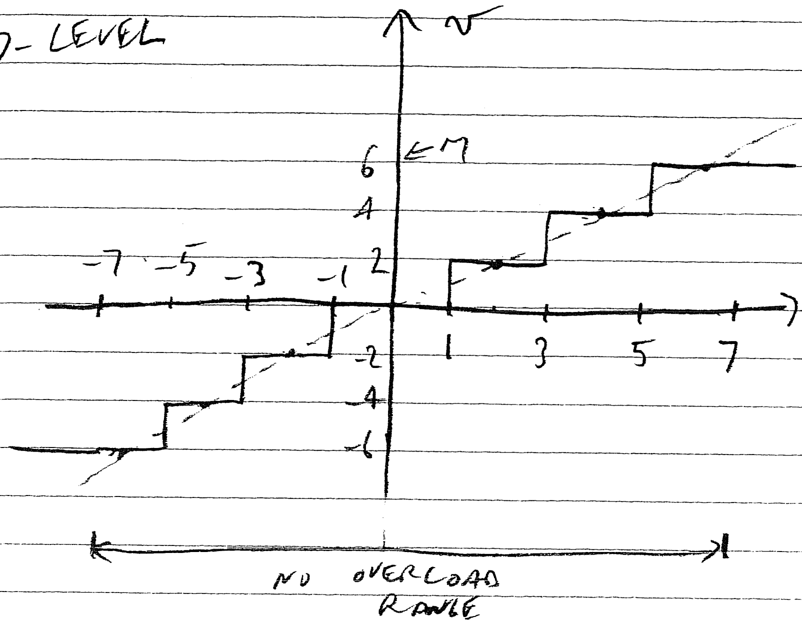
M = # OF STEPS

NO OVERLOAD RANGE = M+1

INPUT SIGNAL RANGE DEFINED AS $\pm M$ (I PREFER $\pm M+1$)

MID TREAD QUANTIZER (SYMMETRIC)

7- LEVEL



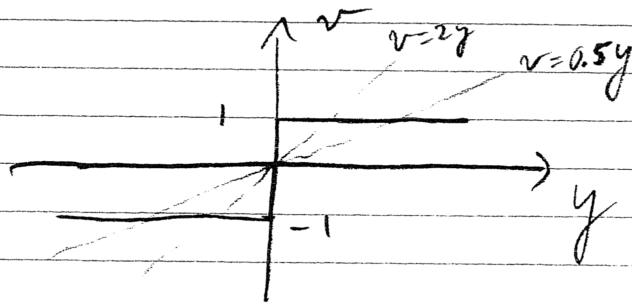
$M=6$

$M \Rightarrow$ EVEN

OF LEVELS $= M+1$

$\Delta = 2$

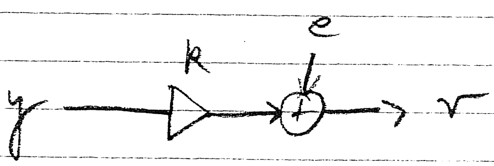
2 LEVEL QUANTIZER (K=?)



$M=1$

$K=?$

$\Delta = 2$



WANT TO FIND K SO THAT QUANTIZATION ERROR IS MINIMIZED.

$$\sigma_e^2 \equiv \lim_{N \to \infty} \left(\frac{1}{N}\right) \sum_{n=0}^{N} e(n)^2$$

DEFINE INNER PRODUCT AS

$$\langle a, b \rangle \equiv \lim_{N \to \infty} \left[\frac{1}{N} \sum_{n=0}^{N} a(n)\, b(n) \right] = E[ab]$$

$$v = ky + e \Rightarrow e = v - ky$$

$$\sigma_e^2 = \langle e, e \rangle$$

$$= \langle v - ky, v - ky \rangle$$

$$\sigma_e^2 = \langle v, v \rangle - 2k \langle v, y \rangle + k^2 \langle y, y \rangle$$

SIMILAR TO $z = ak^2 + bk + c$

WHERE $a \equiv \langle y, y \rangle$, $b \equiv -2 \langle v, y \rangle$, $c \equiv \langle v, v \rangle$

$$\frac{\partial z}{\partial k} = 2ak + b$$ FOR MINIMUM $$\frac{\partial z}{\partial k} = 0$$

$$\Rightarrow 0 = 2ak + b \Rightarrow k = -\frac{b}{2a}$$

HERE MIN OCCURS AT $k = \dfrac{+2 \langle v, y \rangle}{2 \langle y, y \rangle}$

$$\boxed{k = \frac{\langle v, y \rangle}{\langle y, y \rangle}}$$

BUT $v = \mathrm{SGN}(y)$ & $\mathrm{SGN}(y)\, y = |y|$

SO $\langle v, y \rangle = E[|y|]$

& $\langle y, y \rangle = E[y^2]$

$$\boxed{k = \frac{E[|y|]}{E[y^2]}}$$

SO k IS DETERMINED BY STATISTICS OF y.

$$k = \frac{E[|y|]}{E[y^2]}$$

SANITY CHECK
<u>EXAMPLE</u>

LET k BE ASSOCIATED WITH AN INPUT y.

WHAT IS k' FOR AN INPUT $y' = 10y$.

$$k' = \frac{E[|y'|]}{E[(y')^2]} = \frac{E[|10y|]}{E[(10y)^2]} = \frac{10\,E[|y|]}{100\,E[y^2]}$$

$$= \frac{1}{10}k \quad \text{SINCE GIVEN} \quad k = \frac{E[|y|]}{E[y^2]}$$

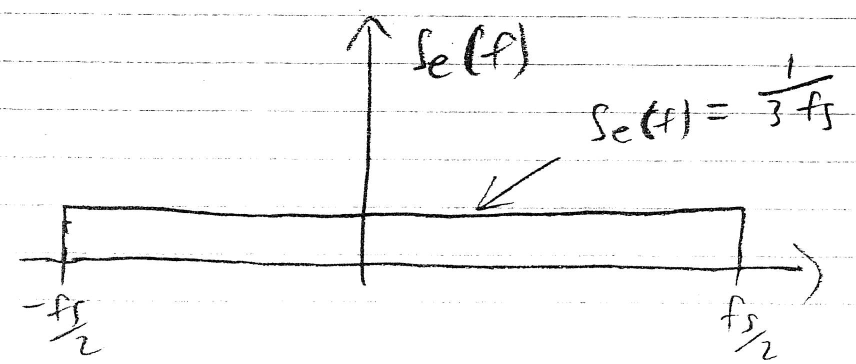
MAKES SENSE.

<u>NOTE</u> IF $\Delta = 2$ & QUANTIZATION ERROR IS ASSUMED TO BE WHITE + UNIFORMLY DISTRIBUTED BETWEEN $\pm\Delta/2 = \pm 1$

THEN $\sigma_e^2 = \frac{\Delta^2}{12} = \frac{4}{12} = \frac{1}{3}$

$\sigma_e^2 = \frac{1}{3}$ FOR $\Delta = 2$

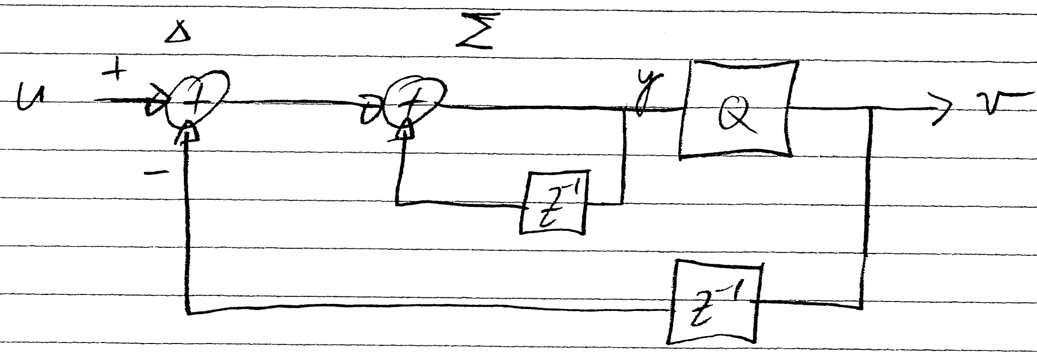
$S_e(f) = \frac{1}{3 f_s}$ & IF $f_s \equiv 1$ $\Rightarrow S_e(f) = \frac{1}{3}$



2.2 MOD 1

2 LEVEL QUANTIZER

$v(n) = \text{SGN}(y(n))$



$$v(n) = \text{SGN}[y(n)]$$

$$y(n) = y(n-1) + u(n) - v(n-1)$$

$$Y(z) = z^{-1}Y(z) + U(z) - z^{-1}V(z)$$

CAN SHOW

$$V(z) = U(z) + (1 - z^{-1})E(z)$$

$$STF(z) \equiv \frac{V(z)}{U(z)} = 1$$

$$NTF(z) \equiv \frac{V(z)}{E(z)} = (1 - z^{-1})$$

$$V(z) = STF(z)\, U(z) + NTF(z)\, E(z)$$

$$|NTF(e^{j2\pi f})|^2 = [2\sin(\pi f)]^2$$

ASSUMES $f_s \equiv 1$ & SO BAND OF INTREST $f_0 \equiv \frac{1}{2OSR}$

FOR $f \ll 1 \Rightarrow |NTF|^2 \cong (2\pi f)^2$

QUANTIZATION NOISE POWER

$$\sigma_q^2 = \int_{-\frac{1}{2OSR}}^{\frac{1}{2OSR}} (2\pi f)^2 \left(\frac{1}{3}\right) df = \frac{\pi^2}{9(OSR)^3}$$

ASSUME SIGNAL IS FULL-SCALE SINUSOID WITH PEAK AMPLITUDE M

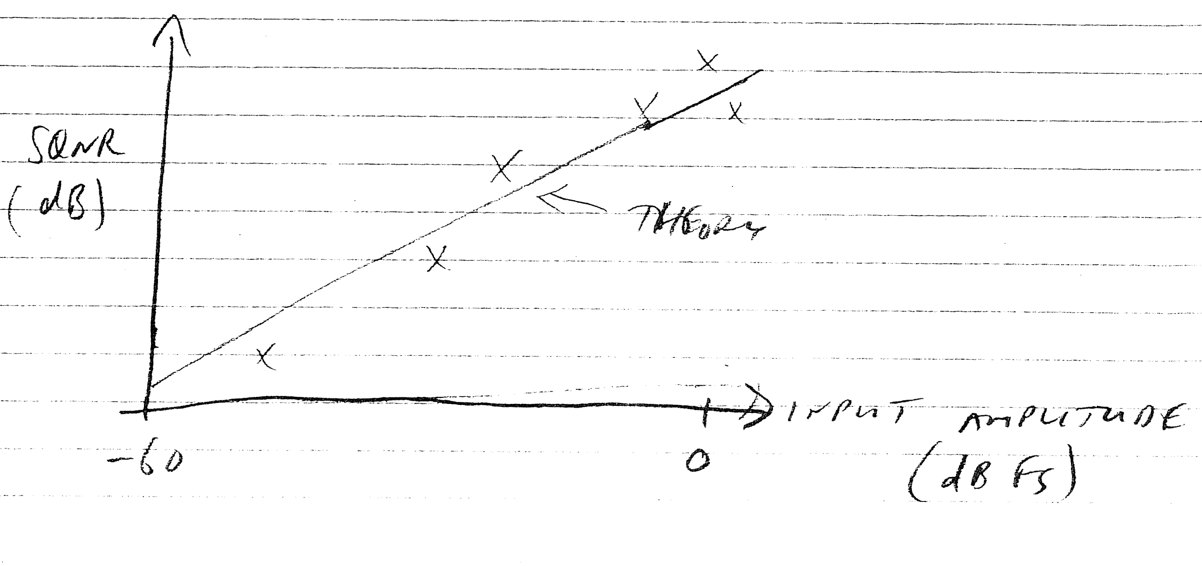
SINCE $STF = 1 \Rightarrow \sigma_u^2 = \dfrac{M^2}{2}$ ← SIGNAL POWER

$$SQNR = \frac{\sigma_u^2}{\sigma_q^2} = \frac{9M^2(OSR)^3}{2\pi^2}$$

2.5 SIMULATION OF MOD 1

ANALYSIS OF $OSR = 128$ $M = 1$
GIVES $SQNR = 60\,dB$

BUT SIMULATION SHOWS $SQNR \approx 55\,dB$
& $k \approx 0.9$
ALSO SQNR IS AN ERRATIC FUNCTION OF INPUT AMPLITUDE



2.6 MOD 1 UNDER dc INPUT SIGNALS.

IDLE TONES

$$y(n) = y(n-1) + u - v(n-1)$$

$$v(n) = \mathrm{SGN}[y(n)] \quad \& \quad \text{FOR } y(n) = 0 \;\; v(n) = 1$$

$$y(n) = y(n-1) + u - \mathrm{SGN}[y(n-1)]$$

$u = \frac{1}{2} \quad y(0) = \frac{1}{2}$

n	$y(n)$	$v(n)$
0	$\frac{1}{2}$	1
1	0	1
2	$-\frac{1}{2}$	-1
3	1	1
4	$\frac{1}{2}$	1
⋮	⋮	⋮

$E[v(n)] = \frac{1}{2}$ AS EXPECTED.

TONE AT $f_S/4$

SAME $v(n)$ SEQUENCE FOR OTHER $y(0)$

ASSUME A RATIONAL INPUT $u = \frac{a}{b}$ $\quad 0 < a < b$
& a & b ODD WITH NO COMMON FACTORS

$|y(0)| < 1$

CAN SHOW FIRST b OUTPUT SAMPLES WILL HAVE

$$\left.\begin{aligned} &\left(\frac{a+b}{2}\right) +1 \text{ SAMPLES} \\ \& &\left(\frac{b-a}{2}\right) -1 \text{ SAMPLES} \end{aligned}\right\} \text{AVERAGE } \frac{\left(\frac{a+b}{2}\right) - \left(\frac{b-a}{2}\right)}{b} = \frac{a}{b} \checkmark$$

PERIODIC WITH PERIOD $\underline{\underline{b}}$

CAN ALSO SHOW IF $u = \frac{a}{b}$ WITH ONE OF a OR b EVEN

SO THAT $a+b$ ODD $\Rightarrow$ REPEATS AFTER PERIOD $2b$

$$\left.\begin{array}{l} a+b \Rightarrow +1 \\ b-a \Rightarrow -1 \end{array}\right\} \frac{a+b-(b-a)}{2b} = \frac{a}{b}$$

CAN ALSO SHOW THAT IF MOD 1 HAS A PERIODIC OUTPUT WITH PERIOD p

CONTAINING m SAMPLES OF $+1$
& $p-m$ SAMPLES OF -1

AVERAGE IS $\frac{2m-p}{p} \Rightarrow u$ IS A RATIONAL INPUT

$$u = \frac{2m-p}{p}$$

HOWEVER IF u IS IRRATIONAL THEN OUTPUT WILL NOT BE PERIODIC.

PERIODIC SEQUENCES $\Rightarrow$ PATTERN NOISE
IDLE TONES
LIMIT CYCLES.

SOME TONES ARE IN STOPBAND & NOT IMPORTANT.

HOWEVER $\Rightarrow u = \frac{1}{100} \Rightarrow$ TONE AT $\frac{f_s}{200}$

& IF OSR < 100 THEN IN-BAND TONE

IMPORTANT IN DIGITAL AUDIO WHERE HEARING CAN DETECT TONES 20dB BELOW WHITE NOISE.

2.7 STABILITY OF MOD 1

LINEAR ANALYSIS SAY MOD 1 STABLE

BUT IGNORES NON-LINEAR QUANTIZER EFFECT.

FIRST IF $|u| > 1$ y GROWS WITHOUT BOUND.

UNSTABLE.

NOW CONSIDER $|u| \leq 1$ & $y(0) \leq 2$

$$y(n) = y(n-1) + u(n) - \mathrm{SGN}[y(n-1)]$$

$$|y(0)| \leq 2 \Rightarrow |y(0) - \mathrm{SGN}[y(0)]| \leq 1 \Rightarrow |y(1)| \leq 2$$

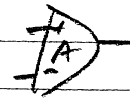
$$|y(1)| \leq 2 \Rightarrow |y(3)| \leq 2 \quad \ldots$$

SO $|y(n)| \leq 2$ FOR ALL TIME IF $|u| \leq 1$ & $|y(0)| \leq 2$

IF $|y(0)| > 2$ THEN MODULATOR WILL DECAY UNTIL $|y| \leq 2$ & THEN ABOVE HOLDS.

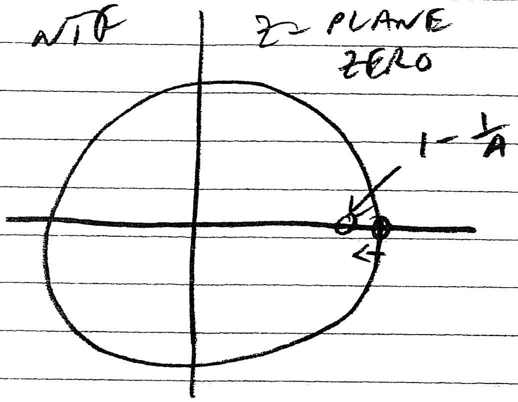
CAN RECOVER FROM ANY INITIAL CONDITION

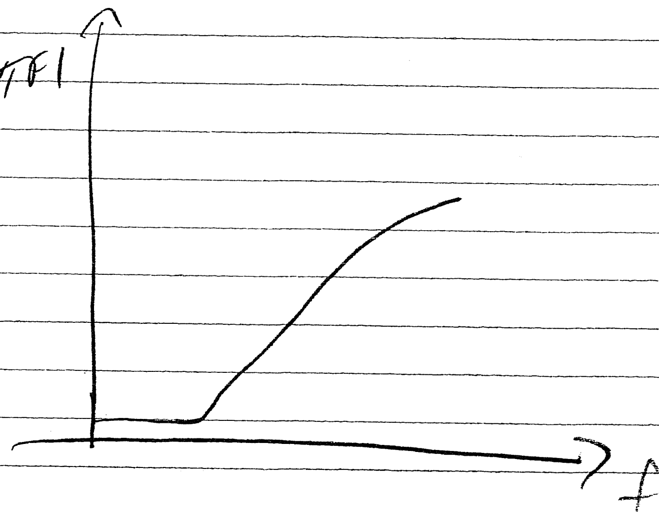
2.8 FINITE OPAMP GAIN A



RESULTS IN A "LEAKY" INTEGRATOR

$NTF(z) = 1 - pz^{-1} \qquad p \lessapprox 1 \qquad p = 1 - \frac{1}{A}$





IF $A > OSR$

ADDITIONAL NOISE $\approx 0.2dB$

BUT NON-LINEAR EFFECTS OFTEN DICTATE A MUCH HIGHER "A"

DEAD-BAND ZONES

IF $A \to \infty \qquad y(0) = 0 \qquad |u| \ll 1 \qquad u > 0$

$$y(n) = y(n-1) + u - SGN[y(n-1)]$$

$$y(1) = y(0) + u - SGN[y(0)] = u - 1 < 0$$

$$y(2) = (u-1) + u + 1 = 2u > 0$$

$$y(3) = 2u + u - 1 = 3u - 1 < 0$$

$\vdots$

$v(n) \to \quad +1 \quad -1 \quad +1 \quad -1 \quad \cdots$

$$y(k) = \begin{cases} ku - 1 & \text{IF } k \text{ ODD} \leftarrow \text{FINALLY } \geq 0 \text{ WHEN } ku \geq 1 \\ ku & \text{IF } k \text{ EVEN} \leftarrow \text{ALWAYS } > 0 \end{cases}$$

IF $A \neq \infty$ & INTEGRATOR LOSSY $\quad p \equiv 1 - \frac{1}{A}$

$$y(n) = p\, y(n-1) + u - \mathrm{SGN}[y(n-1)]$$

$|u| \ll 1$ & $u > 0 \quad y(0) = 0$

$$y(1) = y(0) + u - \mathrm{SGN}[y(0)] = u - 1 < 0$$

$$y(2) = pu - p + u + 1 = (1+p)u + (1-p) > 0$$

$$y(3) = p(1+p)u + p(1-p) + u - 1 = (1+p+p^2)u - (1-p+p^2) < 0$$

$$y(k) = \underbrace{\sum_{i=0}^{k-1} p^i u}_{} + (-1)^k \underbrace{\sum_{i=0}^{k-1} (-p)^i}_{}$$

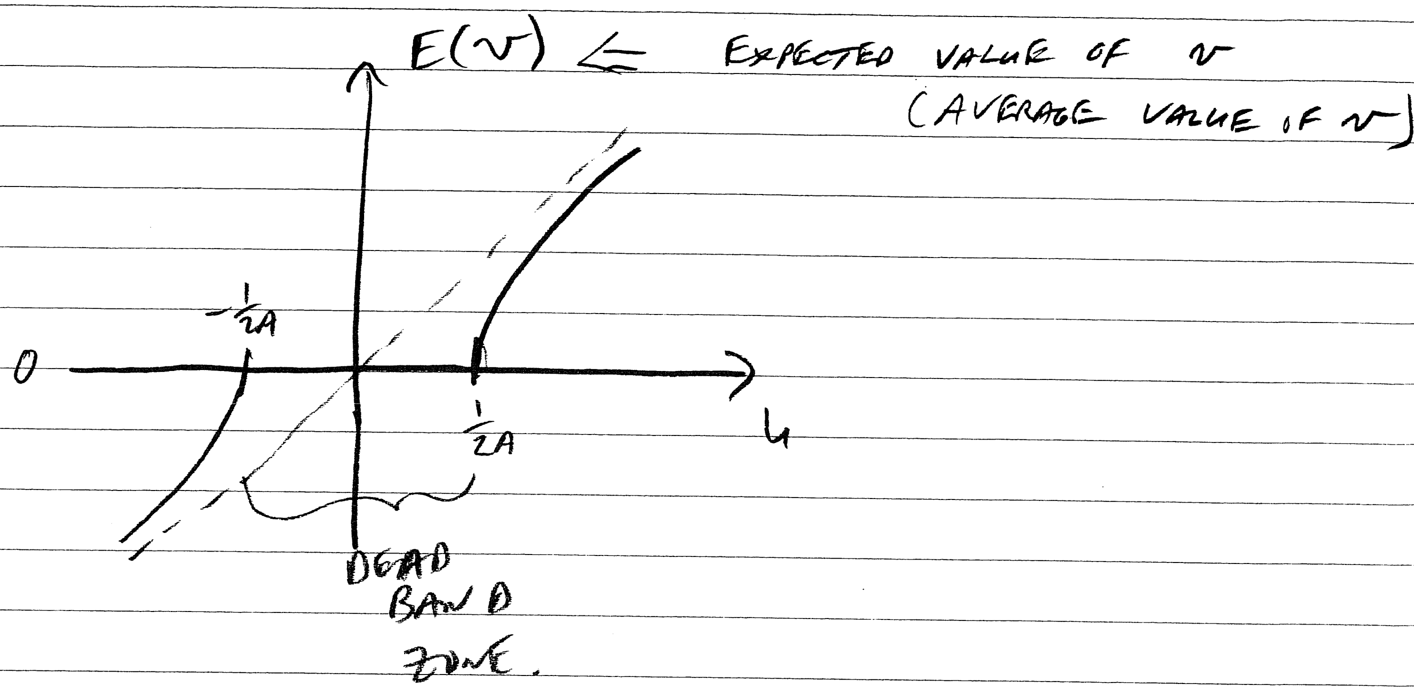
MUST BECOME LARGER THAN

FOR u TO HAVE AN EFFECT

AS $k \to \infty$

$$\frac{u}{1-p} > \frac{1}{1+p} \Rightarrow u > \frac{1-p}{1+p} = \frac{1/A}{2 - \frac{1}{A}} \simeq \frac{1}{2A}$$

WHERE $A \gg 1$


E(v) ⇐ EXPECTED VALUE OF v
(AVERAGE VALUE OF v)
$-\frac{1}{2A}$
0
$\frac{1}{2A}$
u
DEAD
BAND
ZONE.