University of Toronto

Term Test 1

Date — Oct 16, 2023: 4:10pm

Duration — 50 min

ECE 331 — Analog Electronics

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Notation: 15e3 is equivalent to 15×10^3
- Non-programmable calculator is allowed; No other aids are allowed
- Write using a non-erasable ink.
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: ______________________________

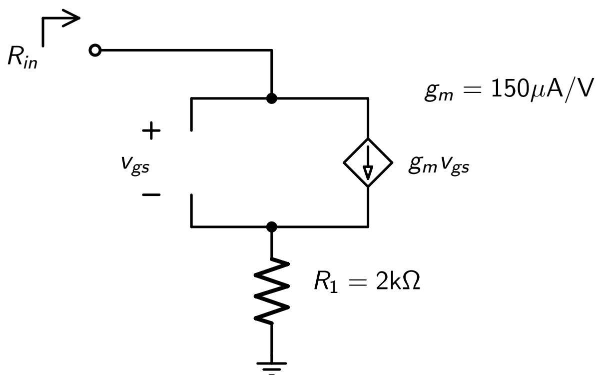
First Name: ______________________________

Student #: ______________________________

Question	1	2	3	Total
Points:	5	5	5	15
Score:				

Grading Table

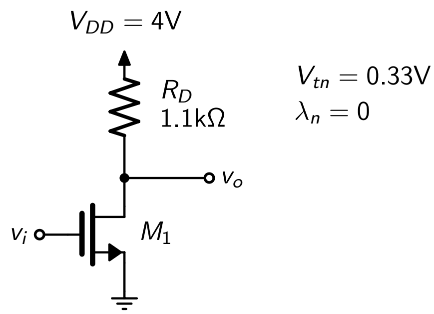
[5] **Q1.** Derive the input impedance, R_{in} for the circuit below.



Solution

At the R_{in} node, apply a voltage v_x and determine i_x going into that node and by definition, $R_{in} = v_x / i_x$
$i_x = g_m v_{gs}$
$i_{R1} = g_m v_{gs} = i_x$
$v_{gs} = v_x - i_{R1} R_1 = v_x - i_x R_1$
Subtituting in for v_{gs} in the first equation...
$i_x = g_m(v_x - i_x R_1) \Rightarrow i_x((1/g_m) + R_1) = v_x$
$R_{in} = v_x / i_x = (1/g_m) + R_1$
$R_{in} = 1/g_m + R_1 = 1/(150e{-}6) + (2e3) = 8.667k\Omega$

[5] **Q2.**



For the circuit above, when the dc bias voltage for v_i is 0.6V, then the measured dc voltage at v_o is 1.8V. Using the small-signal model, find the change in drain current, ΔI_D when v_i goes from 0.6V to 0.63V

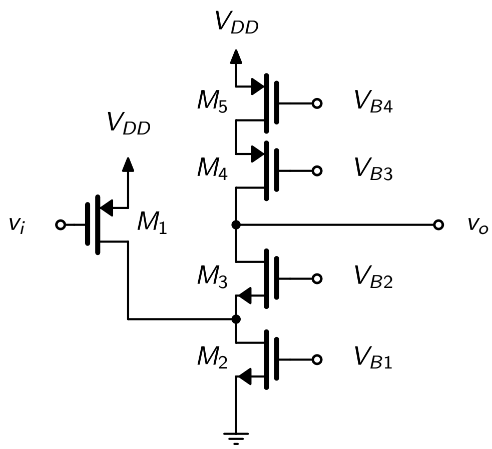
Solution

For $v_{I1} = 0.6\text{V}$, it is given that $v_{O1} = 1.8\text{V}$. So we have

$I_D = (V_{DD} - v_{O1})/R_D = ((4) - (1.8))/(1.1e3) = 2\text{mA}$
$V_{ov} = v_{I1} - V_{tn} = (0.6) - (0.33) = 0.27\text{V}$
$g_m = (2 * I_D)/V_{ov} = (2 * (2e{-}3))/(0.27) = 14.81\text{mA/V}$
$\Delta v_i = v_{I2} - v_{I1} = (0.63) - (0.6) = 30\text{mV}$
$\Delta I_D = g_m * \Delta v_i = (14.81e{-}3) * (30e{-}3) = 444.4\mu\text{A}$

$\Delta I_D = 444.4\mu\text{A}$

[5] **Q3.** Consider the circuit shown below.



Find the small-signal output impedance, R_{out} and small-signal gain, v_o/v_i.
For R_{out}, do NOT assume $g_m r_o \gg 1$
For i_{sc}, assume $\lambda = 0$ (in other words, $r_o \to \infty$)

Solution

Define R_{op} to be the impedance looking up into the drain of M_4
Since r_{o4} is the source impedance attached to the source of M_4

$R_{op} = r_{o4} + (1 + g_{m4} * r_{o4}) * r_{o5} = (20e3) + (1 + (1e{-}3) * (20e3)) * (20e3) = 440\text{k}\Omega$
Define R_{on} to be the impedance looking down into the drain of M_3
The source impedance attached to M_3 is
$R_x = r_{o1}||r_{o2} = (20e3)||(20e3) = 10\text{k}\Omega$ leading to
$R_{on} = r_{o3} + (1 + g_{m3} * r_{o3}) * R_x = (20e3) + (1 + (1e{-}3) * (20e3)) * (10e3) = 230\text{k}\Omega$
and so the output impedance is given by
$R_{out} = R_{op}||R_{on} = (440e3)||(230e3) = 151\text{k}\Omega$

$R_{out} = 151\text{k}\Omega$

For i_{sc}, we assume all $r_o \to \infty$ resulting in all of the drain current of M_1 going straight to the short circuit output. As a result,

$i_{sc} = G_m v_i$ where $G_m = -g_{m1} = -(1e{-}3) = -1\text{mA/V}$

and we have

$v_o/v_i = G_m * R_{out} = (-1e{-}3) * (151e3) = -151\text{V/V}$

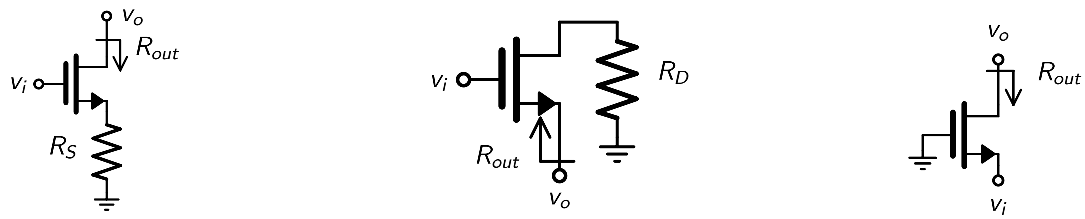
$v_o/v_i = -151\text{V/V}$

Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n{}'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p| v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p{}'|I_D)$



Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = ((1 + g_m r_o)/r_o)v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = -v_i/((1/g_m) + R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = g_m v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = v_i$	$v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^\circ}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;
Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$