ECE 331 — Nov 19, 2024 — 50 min

Term Test 2

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Notation: 15e3 is equivalent to 15×10^3
- Non-programmable calculator is allowed; No other aids are allowed
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

[5] **Q1.** Consider a first-order lowpass filter with a dc gain of 10 and a 3dB frequency of 100Mrad/s.

a) Estimate ω_t assuming the dc gain is much greater than 1.

b) Find the exact value of ω_t as well as the exact phase (in degrees) at ω_t (both values to at least 3 significant digits).

Solution

Defining the dc gain to be $K_{dc} = 10$ and the 3dB frequency to be $\omega_{p1} = 100\text{Mrad/s}$, we have the following transfer function

$$H(s) = \frac{K_{dc}}{1 + s/\omega_{p1}}$$

a) Assuming the dc gain is much greater than 1, we have (see equation page)

$$\omega_t = K_{dc} * \omega_{p1} = (10) * (100e6) = 1\text{Grad/s}$$

b) For exact ω_t, we have

$$1 = \left| \frac{K_{dc}}{1 + j\omega_t/\omega_{p1}} \right|$$

$$1^2 = \frac{K_{dc}^2}{1^2 + \omega_t^2/\omega_{p1}^2}$$

$$\omega_t = \omega_{p1}\sqrt{K_{dc}^2 - 1}$$

$$\omega_t = \omega_{p1} * sqrt({K_{dc}}^2 - 1) = (100e6) * sqrt((10)^2 - 1) = 995\text{Mrad/s}$$

and the phase at ω_t, ϕ_t, is given by

$$\phi_t = 0 - atand(\omega_t/\omega_{p1}) = 0 - atand((995e6)/(100e6)) = -84.26^\circ$$

[5] **Q2.** Assume the following technology parameters for NMOS transistors

$V_{tn} = 0.4\text{V}$; $\mu_n C_{ox} = 240\mu\text{A/V}^2$; $\lambda_n' = 50\text{nmV}^{-1}$; $C_{ox} = 8.5\text{fF}/\mu\text{m}^2$;

$t_{ox} = 4\text{nm}$; $L_{ov} = 40\text{nm}$; $C_{db0}/W = 0.3\text{fF}/\mu\text{m}$;

a) Given a transistor of size $W = 3\mu\text{m}$ and $L = 200\text{nm}$, find the values of C_{gs}, C_{gd} and C_{db} for the transistor (all in fF). Assume the transistor is in the active region and that $V_{db} = 0$.

b) If $V_{ov} = 0.15\text{V}$, find the unity gain frequency of the transistor in Hz (include the effect of C_{gd})

c) Describe in words what is the unity gain frequency for a transistor?

Solution

a)

$C_{ox} = 8.5e{-}15\text{F}/\mu\text{m}^2 = 8.5e{-}3\text{F}/\text{m}^2$

$C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox} = ((2/3)L + L_{ov})(WC_{ox})$

$C_{gs} = ((2/3)(200e{-}9) + (40e{-}9))(3e{-}6)(8.5e{-}3)$

$C_{gs} = 4.42\text{fF}$

$C_{gd} = W * L_{ov} * C_{ox} = (3e{-}6) * (40e{-}9) * (8.5e{-}3) = 1.02\text{fF}$

$C_{db0}/W = 300e{-}18\text{F}/\mu\text{m} = 300e{-}12\text{F}/\text{m}$

$C_{db} = C_{db0}/W * W = (300e{-}12) * (3e{-}6) = 900\text{aF}$

b)

$g_m = \mu_n C_{ox} * (W/L) * V_{ov} = (240e{-}6) * ((3e{-}6)/(200e{-}9)) * (0.15) = 540\mu\text{A/V}$

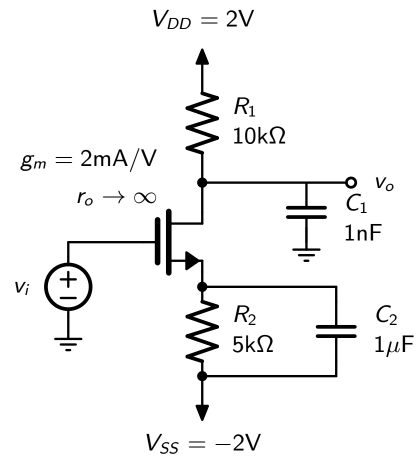
$\omega_t = g_m/(C_{gs} + C_{gd}) = (540e{-}6)/((4.42e{-}15) + (1.02e{-}15)) = 99.26\text{Grad/s}$

$f_t = \omega_t/(2 * pi) = (99.26e9)/(2 * (3.142)) = 15.8\text{GHz}$

c) The unity gain frequency of a transistor is the frequency at which the magnitude of the drain current equals the magnitude of the gate current assuming the transistor is in the active region.

[5] **Q3.** Consider the circuit shown below where the only capacitors in the circuit are C_1 and C_2.

Sketch the small-signal freq response v_o/v_i showing all values (gains in $\mathrm{V/V}$ and freq in $\mathrm{rad/s}$).



Solution

First we find the dc gain

$(G_m)_{dc} = -1/((1/g_m) + R_2) = -1/((1/(2e{-}3)) + (5e3)) = -181.8\mu\text{A/V}$

$R_{out} = R_1 = (10e3) = 10\text{k}\Omega$

$(v_o/v_i)_{dc} = (G_m)_{dc} * R_{out} = (-181.8e{-}6) * (10e3) = -1.818\text{V/V}$

Next, we find the high freq gain

$(G_m)_{hf} = -1/((1/g_m)) = -1/((1/(2e{-}3))) = -2\text{mA/V}$

$R_{out} = R_1 = (10e3) = 10\text{k}\Omega$

$(v_o/v_i)_{hf} = (G_m)_{hf} * R_{out} = (-2e{-}3) * (10e3) = -20\text{V/V}$

For the freq response, we see that a zero occurs when the parallel combination of R_2 and C_2 go to infinity (the short circuit current will go to zero at this freq). So we have

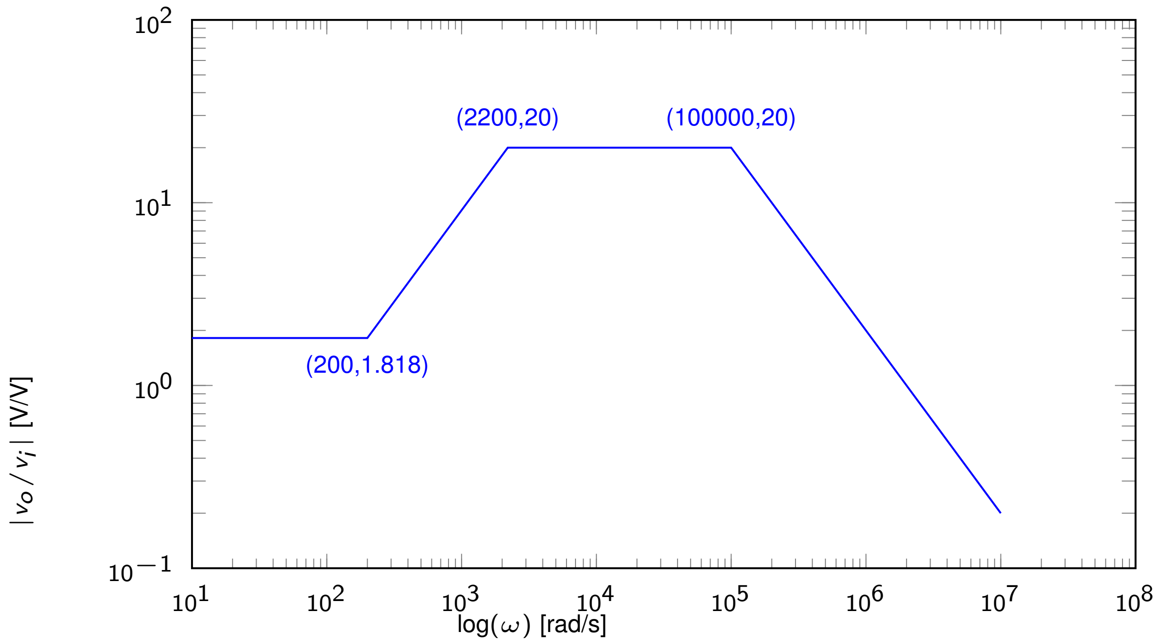
$\omega_z = 1/(R_2 * C_2) = 1/((5e3) * (1e{-}6)) = 200\text{rad/s}$

and then for C_2 a pole occurs at

$\omega_{p1} = 1/(C_2 * (R_2||1/g_m)) = 1/((1e{-}6) * ((5e3)||1/(2e{-}3))) = 2.2\text{krad/s}$

Finally, for C_1, a pole occurs at

$\omega_{p2} = 1/(R_1 * C_1) = 1/((10e3) * (1e{-}9)) = 100\text{krad/s}$



$|v_o / v_i|$ [V/V]
10^2
10^1
10^0
10^{-1}
(2200,20)
(100000,20)
(200,1.818)
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
log(ω) [rad/s]

Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$

(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$

(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;

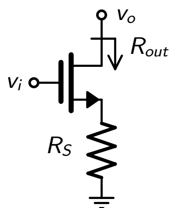
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n{}'|I_D)$

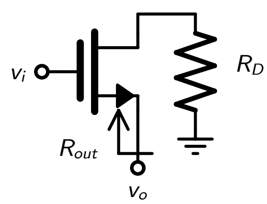
PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$

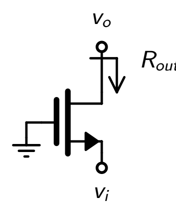
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$

(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p|v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$

$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p{}'|I_D)$







Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = ((1 + g_m r_o)/r_o)v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$	$R_{out} = r_o$
	$i_{sc} = -v_i/((1/g_m) + R_S)$	$i_{sc} = (g_m r_o v_i)/(r_o + R_D)$	$i_{sc} = g_m v_i$
	$v_{oc} = -g_m r_o v_i$	$v_{oc} = v_i$	$v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;

$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$

Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;

unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$

OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$

STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;

$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

SEE NEXT PAGE ...

Feedback: $A_f = A/(1+A\beta)$; $x_i = (1/(1+A\beta))x_s$; $dA_f/A_f = (1/(1+A\beta))dA/A$; $\omega_{Hf} = \omega_H(1+A\beta)$; $\omega_{Lf} = \omega_L/(1+A\beta)$;

Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1+L)) + d/(1+L)$; $Z_{port} = Z_{p^o}((1+L_S)/(1+L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;

Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \le 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;

Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;

$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$