University of Toronto

Faculty of Applied Science & Engineering

Final Exam

Date — Dec 19, 2022: 2pm

Duration — 2hr 30min

ECE 331 — Analog Electronics

Exam Type: A

Non-programmable calculator is allowed

Lecturer — D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

- Equation sheet is on the last page of this test.
- Unless otherwise stated, assume $g_m r_o \gg 1$
- Notation: 15e3 is equivalent to 15×10^3
- Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.
- If you need more space, write on the back of pages.

Last Name: ______________________________

First Name: ______________________________

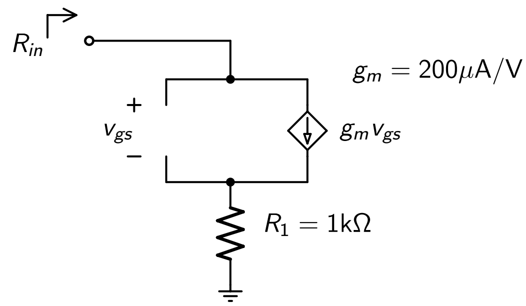
Student #: ______________________________

Question	1	2	3	4	5	6	Total
Points:	6	6	6	6	6	6	36
Score:							

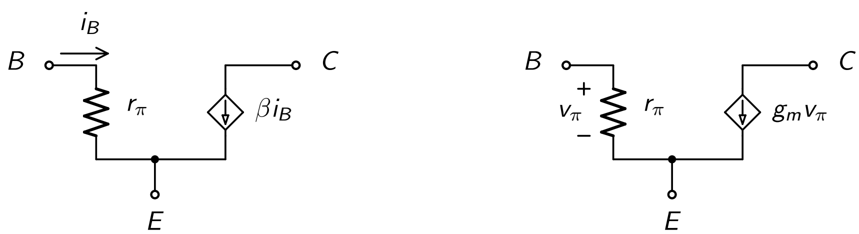
Grading Table

Q1.

[3] (a) Find the input impedance for the circuit model shown below.



[3] (b) Below is shown two different models for the same transistor WITH input resistance r_π. The model in (i) makes use of a CCCS while the model in (ii) makes use of a VCCS. Derive the relationship between g_m, β and r_π such that the 2 models are equivalent.



Solution

(a) At the R_{in} node, apply a voltage v_x and determine i_x going into that node and by definition, $R_{in} = v_x/i_x$

$$i_x = g_m v_{gs}$$

$$i_{R1} = g_m v_{gs} = i_x$$

$$v_{gs} = v_x - i_{R1}R_1 = v_x - i_x R_1$$

Subtituting in for v_{gs} in the first equation...

$$i_x = g_m(v_x - i_x R_1) \Rightarrow i_x((1/g_m) + R_1) = v_x$$

$$R_{in} = v_x/i_x = (1/g_m) + R_1$$

$$R_{in} = 1/g_m + R_1 = 1/(200e{-}6) + (1e3) = 6k\Omega$$

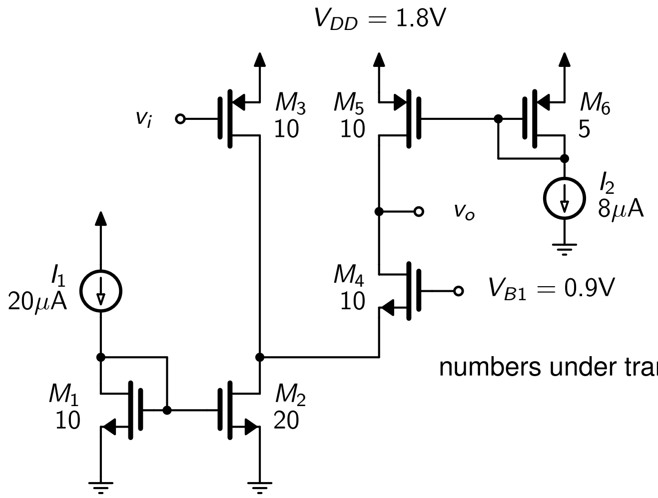
(b)

$$g_m v_\pi = \beta i_B$$

$$i_B = v_\pi/r_\pi \Rightarrow g_m v_\pi = \beta(v_\pi/r_\pi)$$

$$g_m = \beta/r_\pi$$

Q2. Consider the amplifier shown below where all the transistor lengths are 180nm. Assume the current sources are ideal and all $r_o \rightarrow \infty$.



$V_{tn} = 0.4$V
$\mu_n C_{ox} = 240\mu$A/V^2

$V_{tp} = 0.3$V
$\mu_p C_{ox} = 80\mu$A/V^2

numbers under transistor names indicate W/L values

[3] (a) Calculate the drain currents and overdrive voltages for transistors $M_2/M_3/M_4/M_5$. (assume the bias voltage of v_i is set so that all transistors remain in the active region)

[3] (b) Find the maximum $V_{o,max}$ and minimum $V_{o,min}$ voltage at the output, v_o, while keeping all transistors in the active region.

Solution

(a) $I_{D2} = I_1 * ((W/L)_2/(W/L)_1) = (20e{-}6) * ((20)/(10)) = 40\mu$A

$I_{D5} = I_2 * ((W/L)_5/(W/L)_6) = (8e{-}6) * ((10)/(5)) = 16\mu\text{A} = I_{D4}$

$I_{D3} = I_{D2} - I_{D4} = (40e{-}6) - (16e{-}6) = 24\mu$A

$I_{D2} = 40\mu$A; $I_{D3} = 24\mu$A; $I_{D4} = 16\mu$A; $I_{D5} = 16\mu$A

$$V_{ov} = \sqrt{2I_D/(\mu_n C_{ox}(W/L))}$$

$V_{OV2} = sqrt(2 * I_{D2}/(\mu_n C_{ox} * (W/L)_2)) = sqrt(2 * (40e{-}6)/((240e{-}6) * (20))) = 0.1291$V

$V_{OV4} = sqrt(2 * I_{D4}/(\mu_n C_{ox} * (W/L)_4)) = sqrt(2 * (16e{-}6)/((240e{-}6) * (10))) = 0.1155$V

$V_{OV3} = sqrt(2 * I_{D3}/(\mu_p C_{ox} * (W/L)_3)) = sqrt(2 * (24e{-}6)/((80e{-}6) * (10))) = 0.2449$V

$V_{OV5} = sqrt(2 * I_{D5}/(\mu_p C_{ox} * (W/L)_5)) = sqrt(2 * (16e{-}6)/((80e{-}6) * (10))) = 0.2$V

$V_{OV2} = 0.1291$V; $V_{OV3} = 0.2449$V; $V_{OV4} = 0.1155$V; $V_{OV5} = 0.2$V

(b) For $V_{o,max}$

$V_{o,max} = V_{DD} - V_{OV5} = (1.8) - (0.2) = 1.6$V

For $V_{o,min}$

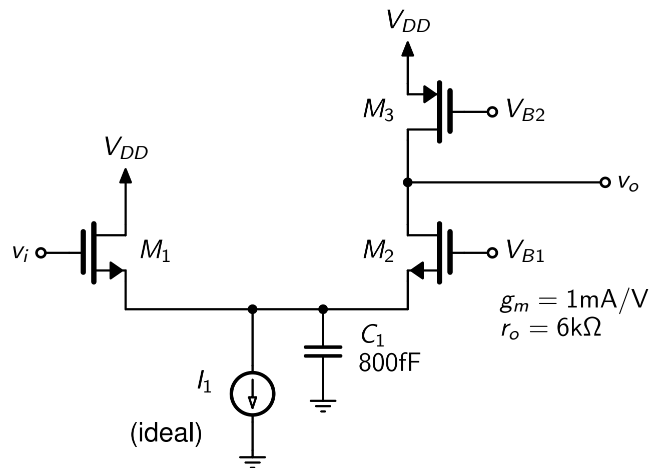
$V_{S1} = V_{B1} - (V_{tn} + V_{OV4}) = (0.9) - ((0.4) + (0.1155)) = 0.3845$V

$V_{o,min} = V_{S1} + V_{OV4} = (0.3845) + (0.1155) = 0.5$V

Note that we could have found this more directly from the gate voltage of M_4 as

$V_{o,min} = V_{B1} - V_{tn} = (0.9) - (0.4) = 0.5$V

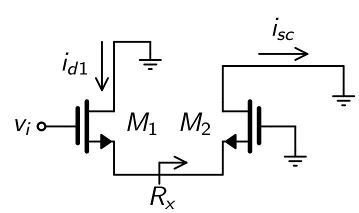
Q3. Consider the amplifier stage shown below and only consider the shown capacitors. All the transistors have the same g_m and r_o.



[3] (a) Find the small-signal dc gain, v_o/v_i.
(For the short circuit output current, assume all $r_o \to \infty$ while for the output impedance, use the accurate formula on the equation sheet - in other words, do not assume $g_m r_o \gg 1$ when finding the output impedance)

Solution

(a) For the short circuit current, i_{sc} at node v_o, we have the following small-signal circuit



Assuming $r_o \to \infty$, we have

$R_x = 1/g_m2$

$i_{sc} = i_{d1} = v_i/(1/g_{m1} + R_x) = v_i/(1/g_{m1} + 1/g_{m2})$

$i_{sc} = 500e{-}6v_i$

For R_{out}, we use to original figure and define R_z to be the impedance looking into the source of M_1 and R_y is the impedance looking into the drain of M_2

$R_z = r_{o1}/(1 + g_{m1} * r_{o1}) = (6e3)/(1 + (1e{-}3) * (6e3)) = 857.1\Omega$

$R_y = r_{o2} + (1 + g_{m2} * r_{o2}) * R_z = (6e3) + (1 + (1e{-}3) * (6e3)) * (857.1) = 12\text{k}\Omega$

$R_{out} = r_{o2}||R_y = (6e3)||(12e3) = 4\text{k}\Omega$

Finally, we have

$v_o = i_{sc}R_{out} = (500e{-}6v_i)(4\text{k}\Omega) = 2v_i$

$v_o/v_i = 2\text{V/V}$

[3] (b) Find the pole frequency due to C_1 in rad/s.
(Do not assume $g_m r_o \gg 1$)

Solution

(b) For finding the pole frequency due to C_1, we use the original circuit.

The impedance looking into the source of M_2 is

$R_w = (r_{o2} + r_{o3})/(1 + g_{m2} * r_{o2}) = ((6e3) + (6e3))/(1 + (1e{-}3) * (6e3)) = 1.714\text{k}\Omega$

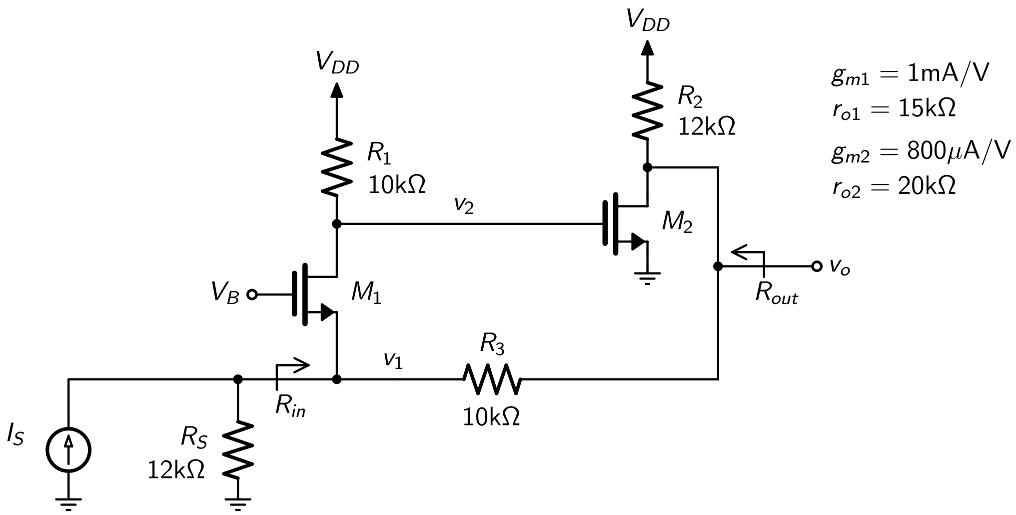
The impedance seen by the capacitor C_1 is (making use of R_z found in (a) above)

$R_{C1} = R_w || R_z = (1.714e3)||(857.1) = 571.4\Omega$

The pole frequency is

$\omega_{p1} = 1/(R_{C1} * C_1) = 1/((571.4) * (800e{-}15)) = 2.188\text{Grad/s}$

Q4. Consider the feedback amp shown below where the input is a current source, I_S with a parallel resistance of R_S.



[3] (a) Find L, $A_{L\infty}$ and A_{CL}. (Assume $A_{L0} = 0$)

Solution

Define R_x to be the impedance looking into the source of M_1

$R_x = 1/g_{m1} + R_1/(g_{m1} * r_{o1}) = 1/(1e{-}3) + (10e3)/((1e{-}3) * (15e3)) = 1.667k\Omega$

Define R_y to be the impedance at the v_o node to ground when the loop is broken at v_2

$R_y = r_{o2}||R_2||(R_3 + R_S||R_x) = (20e3)||(12e3)||((10e3) + (12e3)||(1.667e3)) = 4.534k\Omega$

Breaking the loop at v_2, we have

$v_o/v_2 = -g_{m2} * R_y = -(800e{-}6) * (4.534e3) = -3.627V/V$

$v_1/v_o = (R_S||R_x)/(R_S||R_x + R_3) = ((12e3)||(1.667e3))/((12e3)||(1.667e3) + (10e3)) = 0.1277V/V$

$v_2/v_1 = g_{m1} * (r_{o1}||R_1) = (1e{-}3) * ((15e3)||(10e3)) = 6V/V$

$L = -v_o/v_2 * v_1/v_o * v_2/v_1 = -(-3.627) * (0.1277) * (6) = 2.778$

If the loop is broken at v_2 and an infinite gain amplifier is inserted, then the small-signal drain voltage of M_1 is zero, so $i_{D1} = 0$ so $v_{gs1} = 0$ which means $v_1 = 0$ (all small-signal values) so we have

$A_{L\infty} = -R_3 = -(10e3) = -10k\Omega$

$A_{CL} = A_{L\infty} * (L/(1 + L)) = (-10e3) * ((2.778)/(1 + (2.778))) = -7.353k\Omega$

[3] (b) Find R_{in} and R_{out}

Solution

For R_{out},

$R_{out,0} = R_y = (4.534e3) = 4.534\text{k}\Omega$

where R_y was found in part (a) and is the output impedance with the loop broken.

Also, for this port, $L_S = 0$ and $L_O = L$

$R_{out} = R_{out,0} * (1 + L_S)/(1 + L_O) = (4.534e3) * (1 + (0))/(1 + (2.778)) = 1.2\text{k}\Omega$

For the R_{in} port, we have the port impedance when the loop is broken

$R_{in,0} = R_x||(R_3 + r_{o2}||R_2) = (1.667e3)||((10e3) + (20e3)||(12e3)) = 1.522\text{k}\Omega$

We also have for this port, $L_S = 0$ while we need to find the new value of L_O since R_S is no longer attached to the circuit when the port is open.

We now have

$R'_y = r_{o2}||R_2||(R_3 + R_x) = (20e3)||(12e3)||((10e3) + (1.667e3)) = 4.565\text{k}\Omega$

$v_o/v_2 = -g_{m2} * R'_y = -(800e{-6}) * (4.565e3) = -3.652\text{V/V}$

$v_1/v_o = R_x/(R_x + R_3) = (1.667e3)/((1.667e3) + (10e3)) = 0.1429\text{V/V}$

$v_2/v_1 = g_{m1} * (r_{o1}||R_1) = (1e{-3}) * ((15e3)||(10e3)) = 6\text{V/V}$

$L_O = -v_o/v_2 * v_1/v_o * v_2/v_1 = -(-3.652) * (0.1429) * (6) = 3.13$

resulting in

$R_{in} = R_{in,0} * (1 + L_S)/(1 + L_O) = (1.522e3) * (1 + (0))/(1 + (3.13)) = 368.4\Omega$

Q5. A multipole amplifier has a dc gain of 55dB and poles at $f_{p1} = 1\text{MHz}$, $f_{p2} = 20\text{MHz}$ and $f_{p3} = 400\text{MHz}$.

[3] (a) If an extra dominant pole is added to the amplifer, at what frequency (in Hz) should it be added to obtain a phase-margin of roughly 45 degrees?

[3] (b) If the pole at $f_{p1} = 1\text{MHz}$ is located in the circuit and extra capacitance is added at that node (other poles are unaffected) to move f_{p1} to become the dominant pole, where should f'_{p1} be located to obtain a phase-margin of roughly 45 degrees?

Solution

(a) The relationship between $A_{o,dB}$ and A_o is

$A_{o,dB} = 20log_{10}(A_o)$

$A_o = 10^{(A_{o,dB}/20)} = 10^{(55\text{dB}/20)}562.3$

So we have a dominant transfer-function of

$A(s) = \frac{A_o}{1+s/\omega_{p,dom}}$

and for roughly a 45 degree phase margin, we want the unity gain freq of $A(s)$ to equal the first non-dominant pole which in this case would be f_{p1}.

We also know the unity gain freq of $A(s)$ is approx equal to $A_o\omega_{p,dom}$, so we have

$A_o\omega_{p,dom} = \omega_{p1}$

or equivalently (in Hz)

$A_o f_{p,dom} = f_{p1}$

$f_{p,dom} = f_{p1}/A_o = (1e6)/(562.3) = 1.778\text{kHz}$

(b) We now have the non-dominant pole is f_{p2} (since f_{p1} is moved to become the dominant pole) and we have

$f'_{p1} = f_{p2}/A_o = (20e6)/(562.3) = 35.57\text{kHz}$

Q6. Consider a feedback amplifier that has a low frequency gain of 100kV/V, a dominant pole at 10rad/s and a non-dominant pole at 1Mrad/s

[3] (a) Assuming the feedback factor, β, is independent of frequency, find the value of β that will result in a 70° phase margin.

[3] (b) For the β found above, if a step voltage is applied to the input of the closed loop amplifier, estimate the time it takes to settle to 90% of the final value.

Solution

(a) The loop gain given by

$$L(s) = \frac{A_o \beta}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

where $A_o = 100\text{kV/V}, \omega_{p1} = 10\text{rad/s}, \omega_{p2} = 1\text{Mrad/s}$

We want to find ω_x where $\angle L(j\omega_x) = -110°$ since that would result in a phase-margin of 70°.

Since $\omega_{p2} \gg \omega_{p1}$ and $A_o \gg 1$, we can assume $\omega_x \gg \omega_{p1}$ so we can write

$$\angle L(j\omega_x) = -90° - tan^{-1}(\omega_x/\omega_{p2}) = -110°$$

$$tan^{-1}(\omega_x/\omega_{p2}) = 20° \Rightarrow \omega_x = 0.364\omega_{p2} = 364\text{krad/s}$$

The loop gain unity gain frequency should occur at ω_x implying that $|L(j\omega_x)| = 1$

$$|L(j\omega_x)| = \frac{A_o \beta}{(\omega_x/\omega_{p1})\sqrt{(1 + (\omega_x/\omega_{p2})^2)}} = 1$$

$$\beta = \frac{(\omega_x/\omega_{p1}) * sqrt(1 + (\omega_x/\omega_{p2})^2)}{A_o} = 0.3873$$

(b) The open loop gain looks mostly like a first-order system with the loop gain

$$L(s) \approx \frac{A_o \beta}{1 + s/\omega_{p1}}$$

When this loop gain is closed, its 3dB freq is given by

$$\omega_{3dB} \approx \omega_{p1} A_o \beta$$

$$\omega_{3dB} = \omega_{p1} * A_o * \beta = (10) * (100e3) * (0.3873) = 387.3\text{krad/s}$$

$$\tau = 1/\omega_{3dB} = 1/(387.3e3) = 2.582\mu\text{s}$$

To settle to 90% of the final value

$$0.9 = (1 - e^{(-t/\tau)}) \Rightarrow e^{(-t/\tau)} = 0.1 \Rightarrow t/\tau = 2.3$$

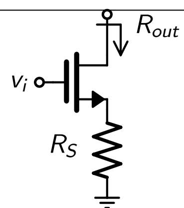
$$t = 2.3 * \tau = 2.3 * (2.582e{-6}) = 5.938\mu\text{s}$$

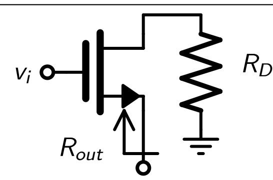
Equation Sheet

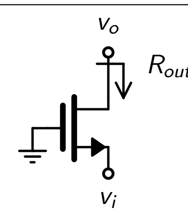
Constants: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$; $q = 1.602 \times 10^{-19}\,\mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at $300\,\mathrm{K}$; $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$; $\omega = 2\pi f$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $V_{ov} = V_{GS} - V_{tn}$
(triode) $v_{DS} \leq V_{ov}$; $v_D < v_G - V_{tn}$; $i_D = k_n(V_{ov}v_{DS} - (v_{DS}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{DS} \geq V_{ov}$; $i_D = 0.5k_n V_{ov}^2(1 + \lambda_n v'_{DS})$; $v'_{DS} = v_{DS} - V_{ov}$;
$g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_n'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $V_{ov} = V_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq V_{ov}$; $v_D > v_G + |V_{tp}|$; $i_D = k_p(V_{ov}v_{SD} - (v_{SD}^2/2))$; $r_{ds} = 1/(\mu_p C_{ox}(W/L)V_{ov})$
(active) $v_{SD} \geq V_{ov}$; $i_D = 0.5k_p V_{ov}^2(1 + |\lambda_p|v'_{SD})$; $v'_{SD} = v_{SD} - V_{ov}$
$g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda_p'|I_D)$







Accurate:	$R_{out} = r_o + (1 + g_m r_o)R_S$ $i_{sc} = (-g_m r_o v_i)/(r_o + (1 + g_m r_o)R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (r_o + R_D)/(1 + g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = (g_m r_o v_i)/(1 + g_m r_o)$	$R_{out} = r_o$ $i_{sc} = ((1 + g_m r_o)/r_o)v_i$ $v_{oc} = (1 + g_m r_o)v_i$
$g_m r_o \gg 1$	$R_{out} = (1 + g_m R_S)r_o$ $i_{sc} = -v_i/((1/g_m) + R_S)$ $v_{oc} = -g_m r_o v_i$	$R_{out} = (1/g_m) + (R_D/g_m r_o)$ $i_{sc} = (g_m r_o v_i)/(r_o + R_D)$ $v_{oc} = v_i$	$R_{out} = r_o$ $i_{sc} = g_m v_i$ $v_{oc} = g_m r_o v_i$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta R_D/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))(\Delta g_m/g_m)$;
$V_{OS} = \Delta V_t$; $V_{OS} = (V_{OV}/2)(\Delta R_D/R_D)$; $V_{OS} = (V_{OV}/2)(\Delta(W/L)/(W/L))$
Large signal: $i_{D1} = (I/2) + (I/V_{ov})(v_{id}/2)(1 - (v_{id}/2V_{ov})^2)^{1/2}$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$;
unity gain freq for $T(s) = A_M/(1 + (s/\omega_{3dB}))$ for $A_M \gg 1 \Rightarrow \omega_t \simeq |A_M|\omega_{3dB}$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $\omega_H \simeq 1/(\sum \tau_i)$; dominant pole estimate $\omega_H \simeq 1/(\tau_{max})$
STC estimate $\omega_L \simeq \sum 1/\tau_i$; dominant pole estimate $\omega_L \simeq 1/(\tau_{min})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/\sqrt{1 + V_{db}/V_0}$;
$\omega_t = g_m/(C_{gs} + C_{gd})$; for $C_{gs} \gg C_{gd} \Rightarrow f_t \simeq (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$;
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p^\circ}((1 + L_S)/(1 + L_O))$: $PM = \angle L(j\omega_t) + 180$; $GM = -|L(j\omega_{180})|_{db}$;
Pole splitting $\omega'_{p1} \simeq 1/(g_m R_2 C_f R_1)$; $\omega'_{p2} \simeq (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A : $\eta = (1/4)(\hat{V}_O/IR_L)(\hat{V}_O/V_{CC})$; Class B : $\eta = (\pi/4)(\hat{V}_O/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$;
Class AB : $i_n i_p = I_Q^2$; $I_Q = (I_S/\alpha)e^{V_{BB}/(2V_T)}$; $i_n^2 - i_L i_n - I_Q^2 = 0$

2-stage opamp: $\omega_{p1} \simeq (R_1 G_{m2} R_2 C_c)^{-1}$; $\omega_{p2} = G_{m2}/C_2$; $\omega_z = (C_c(1/G_{m2} - R))^{-1}$;
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_O < SR$