University of Toronto

Final Exam

Date - Dec 13, 2012 (9:30am to 12pm)

Duration: 2.5 hrs

ECE331 — Analog Electronics
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

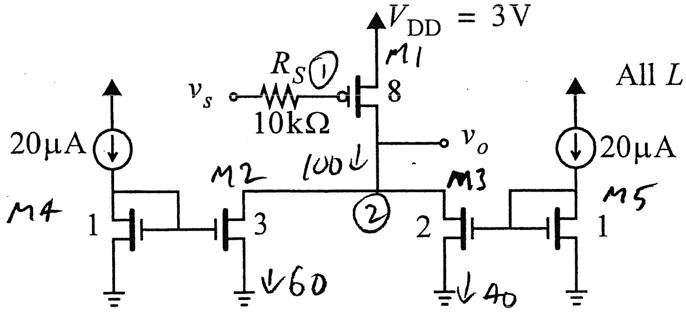
(max grade = 36)

Last Name: SOLUTIONS

First Name: ______________

Student #: ______________

[6] Question 1: Consider the circuit below where all transistors are in the active region. The numbers beside the transistors indicate the transistor width (in μm).



All $L = 0.18\mu m$

$\lambda' = 0.05 \mu m/V$

a) Find the small-signal gain v_o/v_s

$r_o = \frac{L}{|\lambda'| I_D} = \frac{3.6}{I_D}$ $\quad r_{o1} = 36k \quad r_{o2} = 60k$

$r_{o3} = 90k \quad r_{o1} \| r_{o2} \| r_{o3} = 18k$

v_o/v_s = -13.1 V/V

$g_{m1} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right) I_D} = \sqrt{2(60e{-6})\left(\frac{8}{0.18}\right)(100e{-6})} = 0.730 \, mA/V$

$\frac{v_o}{v_s} = -g_{m1}(r_{o1} \| r_{o2} \| r_{o3}) = -(0.73e{-3})(18K) = -13.14 \, V/V$

b) Estimate the 3db frequency cutoff, f_{3dB}. For C_{db} values assume $V_{db} = 0$.

f_{3dB} = 340 MHz

NODE ①

$C_{gs1} = \frac{2}{3} WL C_{ox} + W L_{ov} C_{ox}$

$= \left[\left(\frac{2}{3}\right)(8)(0.18) + (8)(0.04)\right](8.5e{-15})$

$= 10.9 \, fF$

$C_{gd1} = W L_{ov} C_{ox} = (8)(0.04)(8.5e{-15}) = 2.72 \, fF$

$\omega_{p1} = \frac{1}{R_S\left[C_{gs1} + (13.1) C_{gd1}\right]} = \frac{1}{(10k)(46.5 \, fF)} = 2.15e9 \, RAD/S$

NODE ②

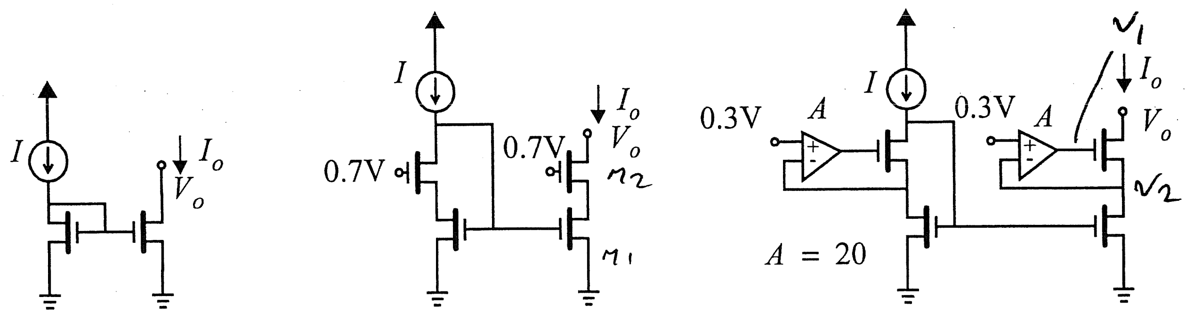
$C_d = C_{d1} + C_{d2} + C_{d3} = (W_1 + W_2 + W_3)\left(\frac{C_{db0}}{W}\right)$

$= (8+3+2)(0.3 \, fF/\mu m) = 3.9 \, fF$

$\omega_{p2} = \frac{1}{(r_{o1} \| r_{o2} \| r_{o3})(C_d)} = \frac{1}{(18k)(3.9 \, fF)} = 1.42e10 \, RAD/S$

SO $f_{p1} \approx \frac{\omega_{p1}}{2\pi} = 342 \, MHz$

[6] **Question 2:** Consider the 3 current mirrors shown below. For all transistors, $L = 0.18\mu m$, $W = 3\mu m$. Also, $I = 100\mu A$.



(I) (II) (III)

a) For the current mirror in (I), estimate the **change** in the output current for an output voltage **change** of $0.5V$. Also, what is the minimum output voltage, $V_{o(\min)}$, while keeping transistors in the active region.

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L}(V_{ov})^2 \Rightarrow V_{ov} = \sqrt{2I_D / \mu_n C_{ox}(W/L)}$$

$$V_{ov} = \sqrt{2 \times 100e\text{-}6 / (240e\text{-}6)(3/0.18)} = 0.224\ V$$

$$r_o = \frac{3.6}{I_D} = 36k \Rightarrow \Delta I = \frac{\Delta V}{r_o} = \frac{0.5}{36k} = 13.9\ \mu A$$

$$V_{o\,min} = V_{ov} = 0.224\ V$$

ΔI = 13.9 μA
$V_{o(\min)}$ = 0.224 V

$$r_{o1} = r_{o2} = 36k$$

b) Repeat question a) for the mirror in (II)

$$V_{ov} = 0.224\ V \quad g_{m2} = \frac{2I_D}{V_{ov}} = 0.893\ mA/V$$

$$r_{out} = r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1} \approx g_{m2} r_{o2} r_{o1}$$

$$r_{out} = 11.6\ M\Omega$$

$$\Delta I = \frac{\Delta V}{r_{out}} = \frac{0.5}{11.6M} = 0.43\ \mu A$$

$$V_{o(min)} = 0.7V - V_{T1} = 0.7 - 0.4 = 0.3\ V$$

ΔI = 0.43 μA
$V_{o(\min)}$ = 0.3 V

c) Repeat question a) for the mirror in (III)

$\Delta I =$ 2	mA
$V_{o(min)} =$ 0.52	V

LOOP GAIN $L \cong 20$

SINCE $\frac{v_2}{v_1} \simeq 1$ (SOURCE FOLLOWER WITH $R_L = r_o$)

SO $R_{OUT} = (g_{m2} r_o^2)(1 + L) = (11.6M)(21)$

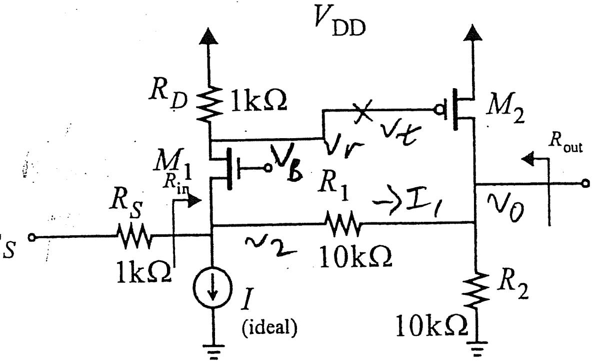
$= 244 M\Omega$

$\Delta I = \frac{\Delta V}{R_{OUT}} = \frac{0.5}{244M} = 2 nA$

$V_{o(min)} = V_2 + V_{OV} = 0.3 + 0.224$

$= 0.524$

[6] **Question 3:** Consider the circuit below.



$g_{m1} = g_{m2} = 2\text{mA/V}$

$r_{o1} = r_{o2} \rightarrow \infty$

a) Find L, A_∞ and d.

$L = 0.657$ V/V
$A_\infty = -10$ V/V
$d = 0.164$ V/V

$L = -\left(\frac{v_o}{v_t}\right)\left(\frac{v_2}{v_o}\right)\left(\frac{v_r}{v_2}\right)$ 333 Ω

$\frac{v_o}{v_t} = -g_{m2}\left(R_2 \| \left[R_1 + \left(R_S \| \frac{1}{g_{m1}}\right)\right]\right)$

$= -(2e-3)(10k \| 10.33k) = -10.2$ V/V

$\frac{v_2}{v_o} = \frac{333}{333+10k} = 0.0322$ V/V $\frac{v_r}{v_2} = g_{m1} R_D = 2$

$L = (10.2)(0.0322)(2) = 0.657$ V/V

$A_\infty \Rightarrow$ SHUNT FEEDBACK SO $v_2 \to 0$ & $\frac{v_s}{R_S} = I_1 \Rightarrow v_o = -\frac{R_1}{R_S} v_s$

$\frac{v_o}{v_s} = -10$ V/V

$A_\infty = -10$

$d = \left(\frac{v_2}{v_s}\right)\left(\frac{v_o}{v_2}\right) = \left(\frac{20k \| \frac{1}{g_{m1}}}{20k \| \frac{1}{g_{m1}} + R_S}\right)\left(\frac{R_2}{R_1 + R_2}\right) = \left(\frac{487}{487+1k}\right)(0.5)$

$(R_1 + R_2)$

$= 0.164$

b) Find v_o/v_s, R_{in} and R_{out}.

$v_o/v_s = -3.9$ V/V
$R_{in} =$ ~~198~~ 247 Ω
$R_{out} = 3.07$ kΩ

$\frac{v_o}{v_s} = A_\infty\left(\frac{L}{1+L}\right) + \frac{d}{1+L} = -3.87$ V/V

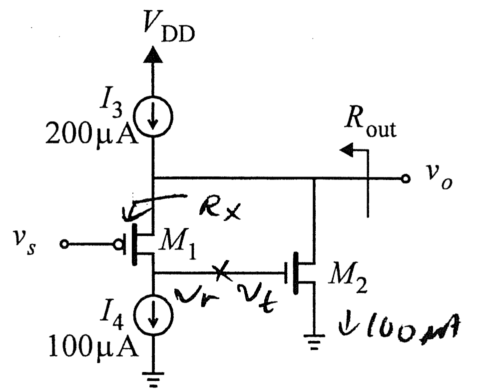
$R_{IN}' = \left[\frac{1}{g_{m1}} \| R_S \| (R_1 + R_2)\right]\left(\frac{1}{1+L}\right)$ $L_S = 0$, $L_O = L$

$R_{IN}' = 328/(1+0.657) = 198\ \Omega$

$R_{OUT} = \left[R_2 \| \left[R_1 + \left(R_S \| \frac{1}{g_{m1}}\right)\right]\right]\left(\frac{1}{1+L}\right) = \frac{5.08k}{(1+0.657)} = 3.07k$

BUT $R_{IN}' = R_{IN} \| R_S \Rightarrow R_{IN} = \left(\frac{1}{R_{IN}'} - \frac{1}{R_S}\right)^{-1} = 247\ \Omega$

[6] Question 4: The circuit below is called a "super source follower". Assume the current sources are realized using single transistors with $L = 0.18\mu m$ (include their output resistances r_{o3} and r_{o4}).



All $L = 0.18\mu m$
All $|V_{ov}| = 200mV$

$r_o = \frac{3.6}{I_D}$

$r_{o1} = r_{o2} = r_{o4} = \frac{3.6}{100\mu A} = 36k$

$r_{o3} = 18k$

$g_{m1} = g_{m2} = \frac{2(100e\text{-}6)}{200e\text{-}3} = 1mA/V$

Find the value for the output resistance, R_{out}, assuming $g_m r_o \gg 1$.

$R_{out} = 55\Omega$

$R_x = \frac{r_{o4} + r_{o1}}{1 + g_{m1} r_{o1}} \cong \frac{2}{g_{m1}} = 2k\Omega$

$R_{OUT}(k=0) = r_{o2} \| r_{o3} \| R_x$
$= 36k \| 18k \| 2k = 1.71k$

$L = -\left(\frac{v_o}{v_t}\right)\left(\frac{v_r}{v_o}\right)$

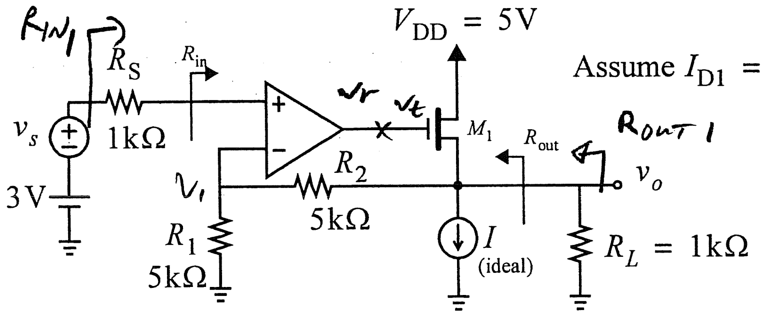
$\frac{v_o}{v_t} = -g_{m2}(r_{o2} \| r_{o3} \| R_x) = -1.71\ V/V$

$\frac{v_r}{v_o} \cong g_{m1}(r_{o4} \| r_{o1}) = 18$

$L = 1.71 \times 18 = 30.9\ V/V$

$R_{OUT} = R_{OUT}(k=0)\left(\frac{1}{1+L}\right) = 55\Omega$

[6] Question 5: Consider the circuit shown below. For the opamp, $A_o = 40$, $R_{id} = 10\text{k}\Omega$, $R_o = 1\text{k}\Omega$. For M_1, $V_{ov} = 200\text{mV}$ and $L = 0.18um$ (include r_o).



$r_{o1} = \frac{3.6}{I_{D1}} = 18\text{K}$

$g_{m1} = \frac{2I_D}{V_{ov}} = 2\text{mA/V}$

a) Find L, A_∞ and d.

$L = -\left(\frac{v_o}{v_t}\right)\left(\frac{v_1}{v_o}\right)\left(\frac{v_r}{v_1}\right)$

LET $R_x = R_L \| r_{o1} \| \left(R_2 + \left[R_1 \| (R_{id} + R_S)\right]\right)$

$= 852\,\Omega$

$\frac{v_o}{v_t} = \frac{R_x}{R_x + r_{s1}} = 0.63 \text{ V/V}$

LET $R_Z = R_1 \| (R_{id} + R_S) = 3.44\text{K}$

$\frac{v_1}{v_o} = \frac{R_Z}{R_Z + R_2} = 0.407 \text{ V/V}$

$\frac{v_r}{v_1} = -\left(\frac{R_{id}}{R_{id} + R_S}\right) A = 36.4 \text{ V/V}$

$L = 0.63 \times 0.407 \times 36.4 = 9.32 \text{ V/V}$

$A_\infty = 1 + \frac{R_2}{R_1} = 2 \text{ V/V}$

$d \approx 0$

$L = 9.32$	V/V
$A_\infty = 2$	V/V
$d = 0$	

b) Find v_o/v_s, R_{in} and R_{out}.

v_o/v_s = 1.81	V/V
R_{in} = 139	kΩ
R_{out} = 31.5	Ω

$$\frac{v_o}{v_s} = A_\infty\left(\frac{L}{1+L}\right) + \frac{d}{1+L} \quad (d=0)$$

$$= 2\left(\frac{9.32}{1+9.32}\right) = 1.81 \text{ V/V}$$

$$R_{IN_1}\,(k=0) = R_S + R_{id} + \underbrace{\left[R_1 \| \left[R_2 + \left(R_L \| \tfrac{1}{g_{m1}} \| r_{o1}\right)\right]\right]}_{2.58k}$$

$$= 13.58\,k\Omega$$

$$R_{IN_1} = R_{IN_1(k=0)}(1+L) = 140.1\,k\Omega$$

$$R_{IN} = R_{IN_1} - R_S = \underline{139.1\,k\Omega}$$

$$R_{OUT_1}\,(k=0) = R_L \| \tfrac{1}{g_{m1}} \| r_{o1} \| \underbrace{\left(R_2 + \left[R_1 \| (R_{id}+R_S)\right]\right)}_{8.44k}$$

$$= 315\,\Omega$$

$$R_{OUT_1} = R_{OUT_1(k=0)} / (1+L) = 30.5\,\Omega$$

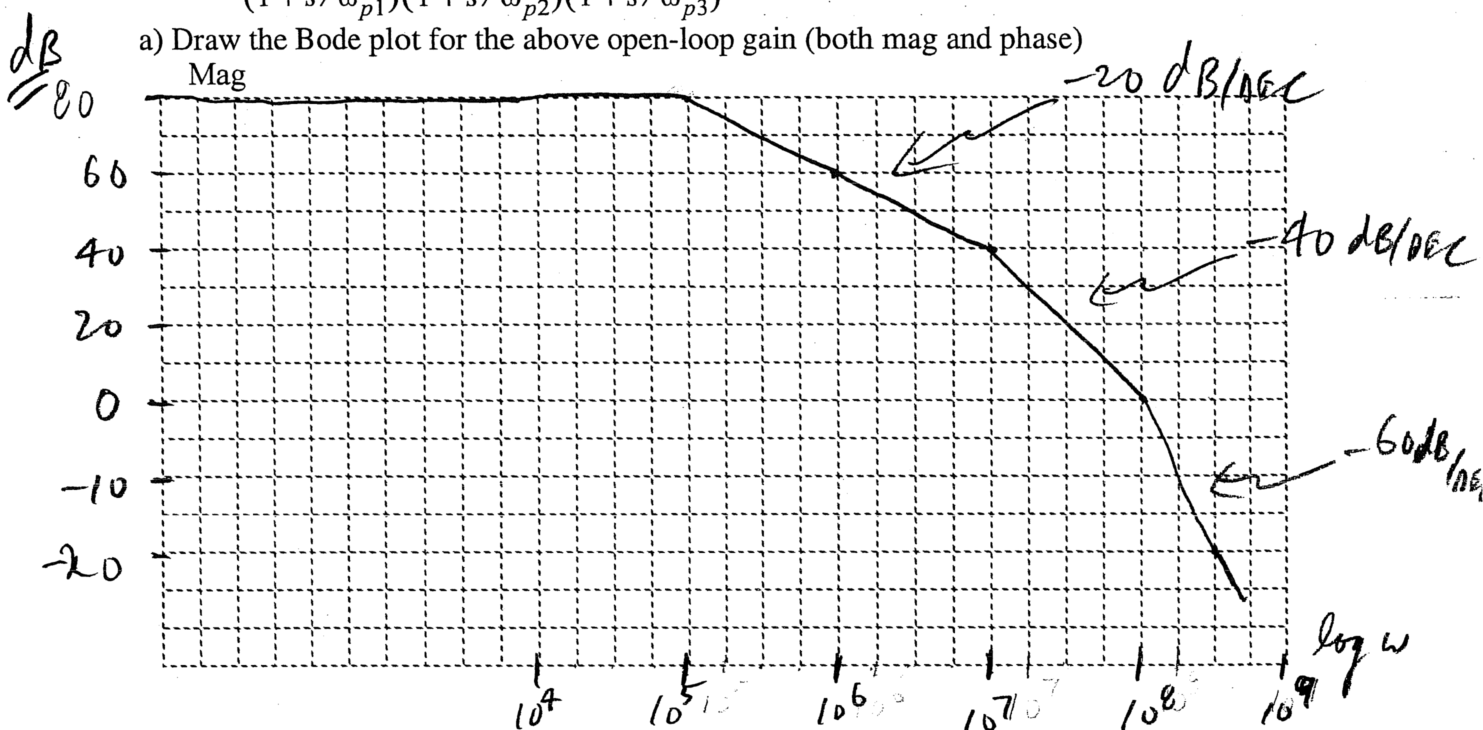
$$R_{OUT_1} = R_{OUT} \| R_L \Rightarrow \frac{1}{R_{OUT}} = \frac{1}{R_{OUT_1}} - \frac{1}{R_L}$$

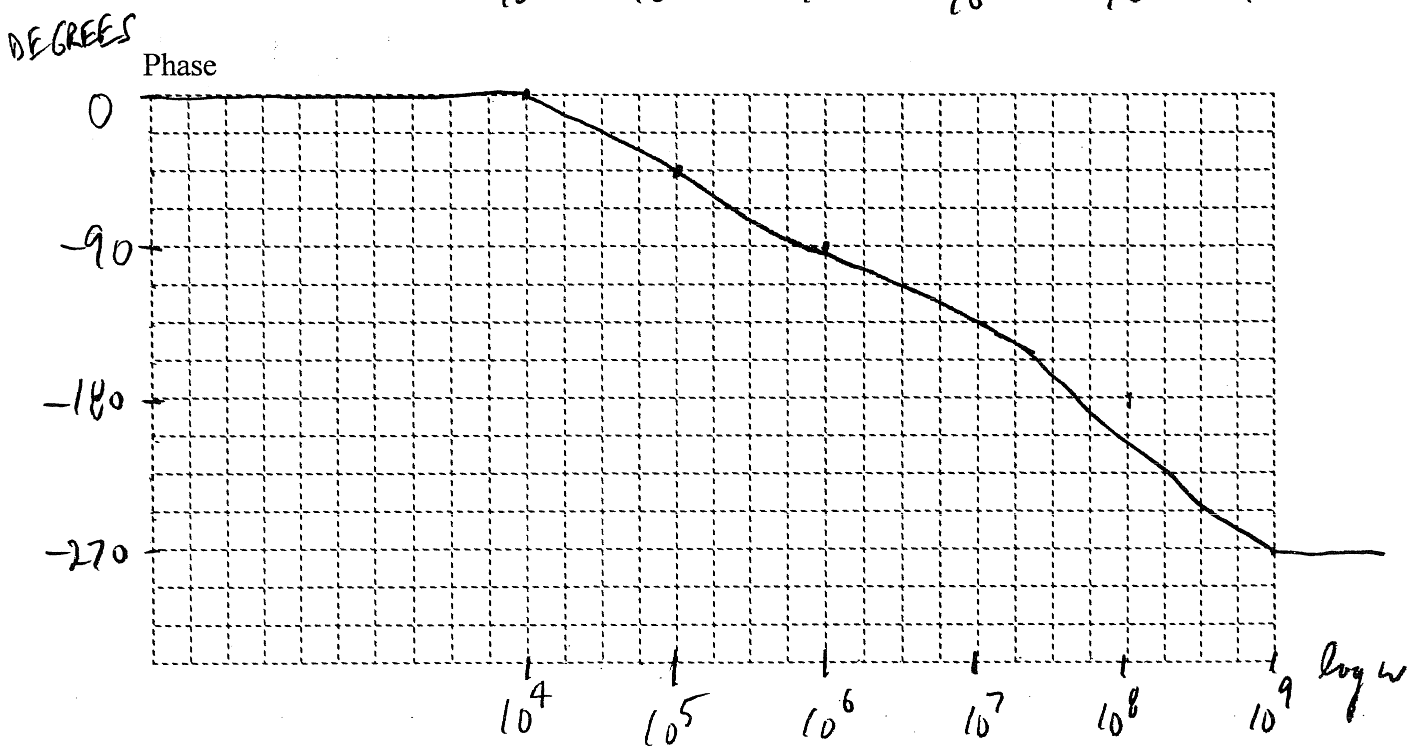
$$R_{OUT} = 31.5\,\Omega$$

[6] Question 6: Assume an opamp is ideal but has the following open-loop gain and will be used in a non-inverting configuration with 2 resistors.

$$A(s) = \frac{10^4}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})(1 + s/\omega_{p3})} \quad \text{where } \omega_{p1} = 10^5,\ \omega_{p2} = 10^7,\ \omega_{p3} = 10^8.$$

a) Draw the Bode plot for the above open-loop gain (both mag and phase)





b) Estimate the minimum closed-loop gain, A_{min}, that can be realized while having a phase margin about 45°

A_{min} = 100 V/V

$\angle A(j\omega_1) \cong -135°$ WHEN $\omega_1 \cong 10^7$ RAD/S

& $|A(j\omega_1)| \approx 40$ dB (OR 100 V/V)

SO FOR LOOP GAIN $\approx 1 \Rightarrow \beta \approx 0.01$

& FOR NON-INVERTING AMP, $A_{CL} \approx \frac{1}{\beta}$

$\therefore$ $A_{MIN} = 100$ V/V (OR 40 dB)

c) If it is desired to have a closed-loop gain of 10 estimate the new value of ω_{p1} if it is moved to a lower freq and a phase margin of about 45° is desired.

$\omega_{p1}' = 10^4$ RAD/S

$A_{CL} = 10 \Rightarrow \beta = 0.1$

LOOP GAIN UNITY GAIN FREQ SHOULD OCCUR AT ω_{p2}

$|L(j\omega_{p2})| \cong 1$

$\beta|A(j\omega_{p2})| \approx 1 \Rightarrow |A(j\omega_{p2})| \approx 10$ FOR $\omega \approx \omega_{p2}$

$$|A(j\omega)| = \left|\frac{10^4}{\left(1+\frac{j\omega}{\omega_{p1}'}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)}\right| \approx \frac{10^4}{\left|\frac{j\omega}{\omega_{p1}'}\right|} \approx \frac{\omega_{p1}' 10^4}{\omega}$$

$$\omega_{p1}' \cong \frac{10\,\omega_{p2}}{10^4} = \frac{\omega_{p2}}{10^3} = \underline{10^4}$$

ECE331 — Analog Electronics — Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\varepsilon_0)/t_{ox}$

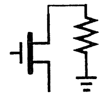
NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \ge 0$; $v_{ov} = v_{GS} - V_{tn}$
(triode) $v_{DS} \le v_{ov}$ (or $v_D < v_G - V_{tn}$) ; $i_D = k_n((v_{ov})v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5 k_n v_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'| I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \ge 0$; $v_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \le v_{ov}$ (or ($v_D > v_G + |V_{tp}|$)) ; $i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5 k_p v_{ov}^2(1 + |\lambda| v_{SD})$; $g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'| I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$
$i_C = \beta i_B$; $i_E = (\beta + 1)i_B$; $\alpha = \beta/(\beta + 1)$; $i_C = \alpha i_E$; $R_b = (\beta + 1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta + 1)$

Cascode:


$R_{out} = R_S + r_o + g_m r_o R_S \approx (1 + g_m R_S) r_o$



$R_{in} = (R_D + r_o)/(1 + g_m r_o) \approx R_D/(g_m r_o) + 1/g_m$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$
$V_{os} = \Delta V_t$; $V_{os} = (V_{ov}/2)((\Delta R_D)/R_D)$; $V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$ unity gain freq for $T(s) = \dfrac{A_M}{1 + s/\omega_{3dB}}$ $f_t \approx |A_M|\omega_{3dB}$ when $A_M \gg 1$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $f_H = 1/(2\pi\sum\tau_i)$; dominant pole estimate $f_H = 1/(2\pi\tau_{max})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/(\sqrt{1 + V_{db}/V_0})$
$f_t = g_m/(2\pi(C_{gs} + C_{gd}))$ assuming $C_{gd} \ll C_{gs}$ $f_t = (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{port(k=0)}((1 + L_S)/(1 + L_O))$
PM $= \angle L(j\omega_1) + 180$; GM $= -|L(j\omega_{180})|_{dB}$
Pole Splitting $\omega_{p1}' \cong 1/(g_m R_2 C_f R_1)$; $\omega_{p2}' \cong (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair $s^2 + (\omega_o/Q)s + \omega_o^2 = 0$; $Q \le 0.5 \Rightarrow$ real poles ; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

MOS Transistor; CMOS basic parameters. Channel length = $0.18\mu m$

	V_t (V)	μC_{ox} $(\mu A/V^2)$	λ' (µm/V)	C_{ox} (fF/µm²)	t_{ox} (nm)	L_{ov} (µm)	$\frac{C_{db0}}{W}$ $\left(\frac{fF}{\mu m}\right)$
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.04	0.3