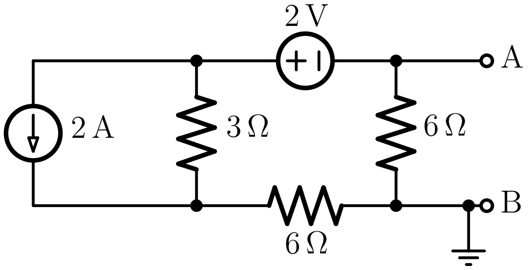
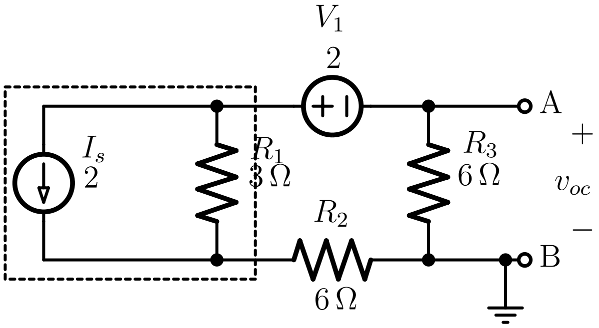
Problem Set 1

Q1. Find the voltage source equivalent and current source equivalent circuits for the following circuits between the nodes A and B. Use I_{sc} for the short circuit output current, V_{oc} for the open circuit output voltage and R_{th} for the output resistance.

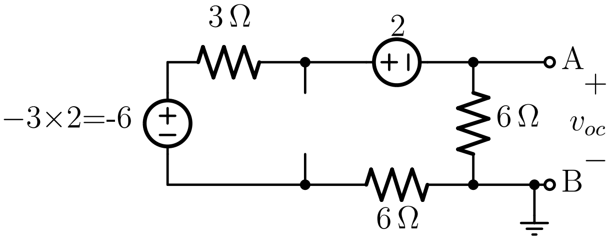
(a)



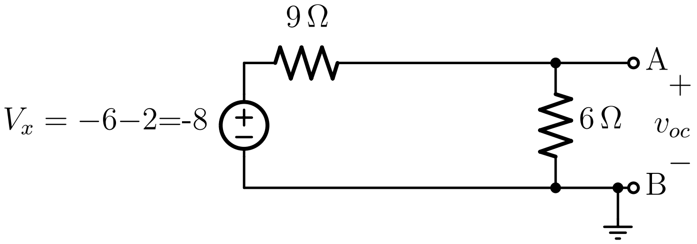
Solution



Using source transformation:



Simplifying Voltage source and resistor in series:



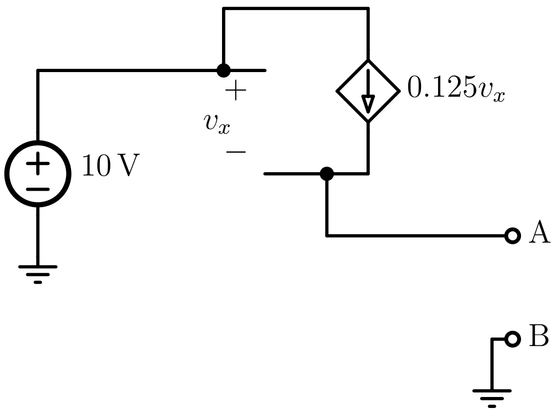
Then, we calculate: $V_x = -(R_1 * I_s) - V_1 = -((3) * (2)) - (2) = -8\,\text{V}$

$V_{oc} = V_x * (R_3)/(R_3 + R_1 + R_2) = (-8) * ((6))/((6) + (3) + (6)) = -3.2\,\text{V}$

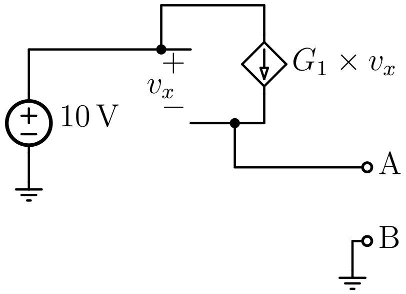
$R_{th} = R_3||R_1 + R_2 = (6)||(3) + (6) = 3.6\,\Omega$

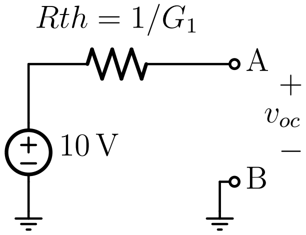
$I_{sc} = V_{oc}/R_{th} = (-3.2)/(3.6) = -0.8889\,\text{A}$

(b)



Solution



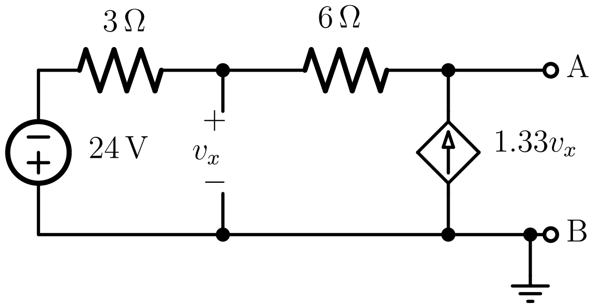


We calculate: $V_{oc} = V_2 = (10) = 10\,\text{V}$

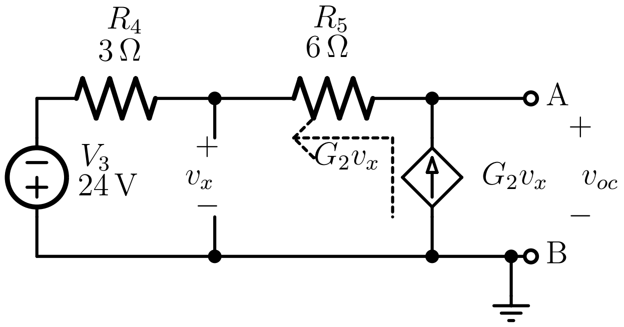
$R_{th} = 1/G_1 = 1/(0.125) = 8\,\Omega$

$I_{sc} = V_{oc}/R_{th} = (10)/(8) = 1.25\,\text{A}$

(c)



Solution

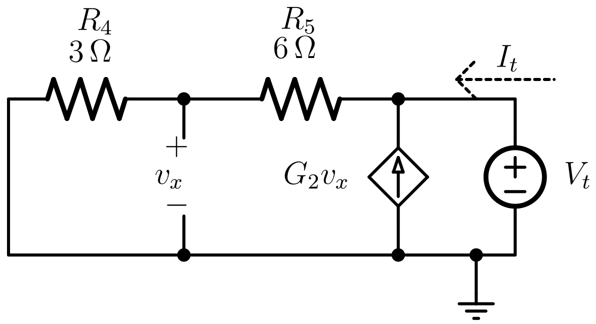


KVL:

$-V_3 + v_x + G_2 v_x R_4 = 0$

$v_x = V_3/(G_2 * R_4 - 1) = (24)/((1.33)*(3) - 1) = 8.027\,\text{V}$

$v_{oc} = -V_3 + (R_4 + R_5) * G_2 * v_x = -(24) + ((3) + (6)) * (1.33) * (8.027) = 72.08\,\text{V}$



$v_x = V_t * R_4/(R_4 + R_5) = (1) * (3)/((3) + (6)) = 0.3333\,\text{V}$

$I_{sc} = v_x/R_4 - G_2 * v_x = (0.3333)/(3) - (1.33) * (0.3333) = -0.3322\,\text{A}$

$R_{th} = V_t/I_{sc} = (1)/(-0.3322) = -3.01\,\Omega$

Q2. An NMOS transistor that is operated with a small v_{DS} is found to exhibit a resistance r_{DS}. By what factor will r_{DS} change in each of the following situations?

(a) Overdrive is multiplied by a factor of 2.
(b) The device is replaced with another fabricated in the same technology but width multiplied by a factor of 2.
(c) The device is replaced with another fabricated in the same technology but with both the width and length are multiplied by a factor of 2.
(d) The device is replaced with another fabricated in a more advanced technology for which the oxide thickness is multiplied by a factor of 0.5 and both W and L are each multiplied by a factor of 0.5 (assume μ_n remains unchanged).

Solution

With V_{DS} small compared to V_{OV}, The drain to source resistance r_{DS} can be approximated with

$$r_{DS} = \frac{1}{\mu_n C_{ox}(W/L)V_{OV}}$$

(a) If V'_{OV} =2V_{OV}, then $r'_{DS} = r_{DS}/2$.

(b) If W' =2W, then $r'_{DS} = r_{DS}/2$.

(c) If W' =2W and L' =2L, then $r'_{DS} = r_{DS}$.

(d) If oxide thickness t'_{ox} =0.5t_{ox}, then $C_{ox} = \epsilon_{ox}/t_{ox}$ is multiplied by a factor of 2. If W and L are scaled by the same factor, then $r'_{DS} = r_{DS}/0.5$.

Q3. An NMOS transistor with $k_n = 1\,\text{mA/V}^2$ and $V_t = 1\,\text{V}$ is operated with V_{GS} =2.5 V. At what value of V_{DS} does the transistor enter the saturation region? What value of I_D is obtained in saturation?

Solution

$V_{OV} = V_{GS} - V_t = (2.5) - (1) = 1.5\,\text{V}$
The minimum drain to source voltage to keep the device in saturation is:
$V_{DSAT} = V_{OV} = (1.5) = 1.5\,\text{V}$
In Saturation, the Drain current is:
$I_D = 0.5 * k_n * V_{OV}^2 = 0.5 * (1e-3) * (1.5)^2 = 1.125\,\text{mA}$

Q4. With the knowledge that $\mu_p \approx 0.4\,\mu_n$, what must be the relative width of n-channel and p-channel devices if they are to have equal drain currents when operated in the saturation mode with overdrive voltages of the same magnitude?

Solution
$i_D = \frac{1}{2}k'_n\frac{W}{L}(V_{GS} - V_{th})^2$, where $k'_n = \mu_n Cox$.
for equal drain currents:
$\mu_n Cox\frac{W_n}{L} = \mu_p Cox\frac{W_p}{L} \Rightarrow \mu_n W_n = \mu_p W_p \Rightarrow W_p/W_n = \mu_n/\mu_p = 2.5$

Q5. An NMOS transistor is fabricated in a 0.8-μm process having $k'_n = 130\,\mu\text{A/V}^2$ and $V'_A = 20\,\text{V}/\mu\text{m}$ of channel length. If $L = 1.6\,\mu$m and $W = 16\,\mu$m, find V_A and λ. Find the value of I_D that results when the device is operated with an overdrive voltage of 0.5 V and $V_{DS} = 2$ V. Also, find the value of r_o at this operating point. If V_{DS} is increased by $1V$, what is the corresponding change in I_D?

Solution

$V_A = V_A' L = 32\,\mathrm{V}$

$\lambda = (31.25e-3)V^{-1}$

$V_{DS} = 2\,\mathrm{V} > V_{ov} = 0.5\,\mathrm{V} \Rightarrow \mathit{Saturation}$

$i_D = 0.5 * k_n' * W/L * V_{ov}^2 = 162.5\,\mu\mathrm{A}$

$r_o = V_A/i_D = (32)/(162.5e-6) = 196.9\,\mathrm{k}\Omega = \frac{\Delta V_{DS}}{\Delta i_D} = \frac{1}{\Delta i_D} \Rightarrow \Delta i_D = 5.078\,\mu\mathrm{A}$

Q6. A p-channel transistor for which $|V_t| = 1\,\mathrm{V}$ and $|V_A| = 50\,\mathrm{V}$ operates in saturation with $|V_{GS}| = 3\,\mathrm{V}$, $|V_{DS}| = 4\,\mathrm{V}$, $i_D = 3\,\mathrm{mA}$. Find the corresponding signed values for V_{GS}, V_{SG}, V_{DS}, V_{SD}, V_t, λ, and $k_p'(W/L)$.

Solution

$V_{SG} = 3\,\mathrm{V}$

$V_{GS} = -3\,\mathrm{V}$

$V_{SD} = 4\,\mathrm{V}$

$V_{DS} = -4\,\mathrm{V}$

$V_t = -1\,\mathrm{V}$

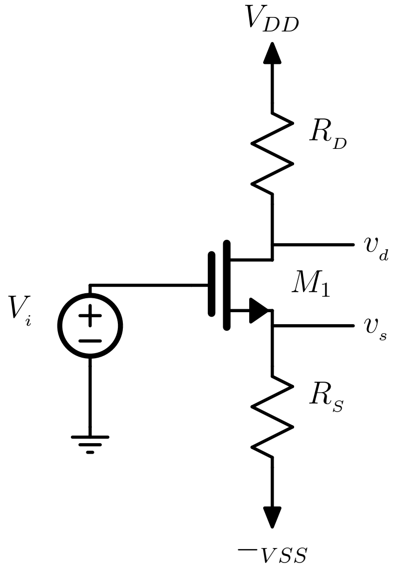
$\lambda = 1/V_A = 1/(50) = 20\,\mathrm{mV}^{-1}$

$i_D = 1/2 k_p' \frac{W}{L}(V_{GS} - V_t)^2(1 + \lambda V_{DS})$

$\Rightarrow 3e-3 = 1/2 k_p' \frac{W}{L}(-3-(-1))^2(1 + 20e-3(-4))$

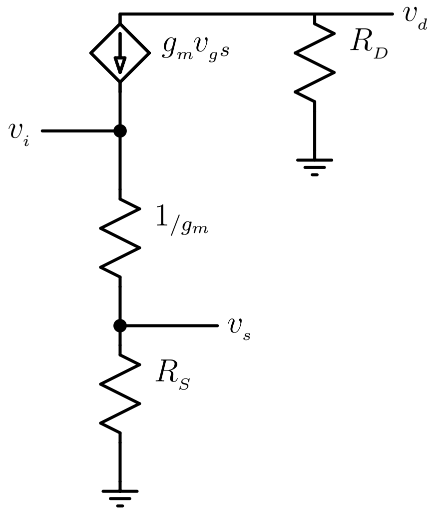
$\Rightarrow k_p' \frac{W}{L} = 1.389\,\mathrm{m}A/V^2$

Q7. For the NMOS amplifier below, replace the transistor with its T equivalent circuit, assuming $\lambda = 0$. Derive the expressions for the voltage gains v_s/v_i and v_d/v_i.



Solution

The equivalent T model is:



$\lambda = 0 \Rightarrow r_o = \infty$
$v_i = g_m v_{gs} \left(\frac{1}{g_m} + R_S\right)$
$v_d = -g_m v_{gs} R_D$
$v_s = g_m v_{gs} R_S$

Note the voltage divider:
$\frac{v_s}{v_i} = \frac{R_s}{1/g_m + R_S} = \frac{g_m R_S}{1 + g_m R_S} \approx 1$
$\frac{v_d}{v_i} = -\frac{R_D}{1/g_m + R_S} = \frac{-g_m R_D}{1 + g_m R_S}$