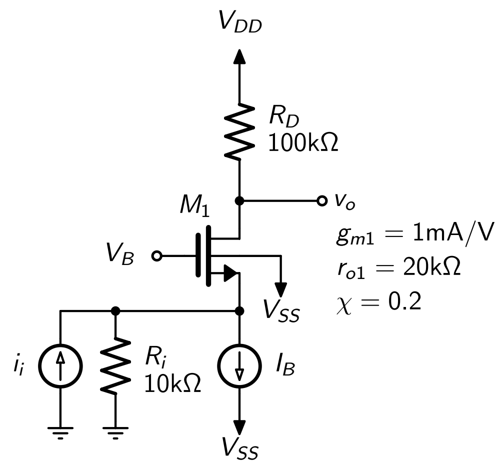
Problem Set 3C - Body Effect

Question 1

It is desired to create a voltage output from a small current source input (say from a photodetector). Shown below, the small current source input and its output impedance is shown as i_i and R_i, respectively. V_B is a dc bias voltage and assume the current source I_B is ideal.



(a) Find the small-signal gain, v_o/i_i assuming no body effect (in other words, $\chi = 0$.

(b) Find the small-signal gain, v_o/i_i assuming $\chi = 0.2$.

Solution

(a) We can start by finding the output impedance, R_o

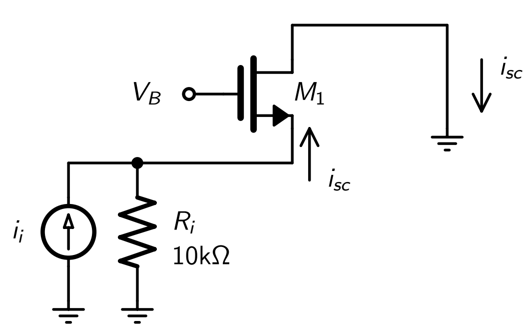
Define R_{dx} to be the small signal resistance looking into the drain of M_1

$R_{dx} = r_{o1} + (1 + g_{m1} * r_{o1}) * R_i = (20e3) + (1 + (1e{-}3) * (20e3)) * (10e3) = 230\text{k}\Omega$

$R_o = R_{dx}||R_D = (230e3)||(100e3) = 69.7\text{k}\Omega$

Next, we find the short circuit current, i_{sc}

We have the following small circuit circuit



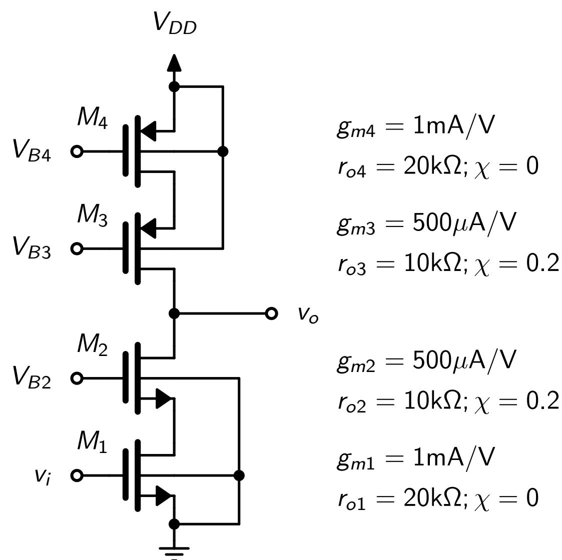
Defining R_{sx} to be the impedance looking in to the source of M_1 we have

$R_{sx} = (1/g_{m1})||r_{o1} = (1/(1e{-}3))||(20e3) = 952.4\Omega$

and we see a current divider, so we have
$i_{sc}/i_i = R_i/(R_i + R_{sx}) = (10e3)/((10e3) + (952.4)) = 0.913\text{A/A}$ leading to
$(v_o/i_i)_a = i_{sc}/i_i * R_o = (0.913) * (69.7e3) = 63.64\text{k}\Omega$

(b) We go through the same analysis as in (a) except that we have
$g'_{m1} = g_{m1} * (1 + \chi) = (1e{-}3) * (1 + (0.2)) = 1.2e{-}3$
Define R_{dx} to be the small signal resistance looking into the drain of M_1
$R_{dx} = r_{o1} + (1 + g'_{m1} * r_{o1}) * R_i = (20e3) + (1 + (1.2e{-}3) * (20e3)) * (10e3) = 270\text{k}\Omega$
$R_o = R_{dx}||R_D = (270e3)||(100e3) = 72.97\text{k}\Omega$
Next, we find the short circuit current, i_{sc}
Defining R_{sx} to be the impedance looking in to the source of M_1 we have
$R_{sx} = (1/g'_{m1})||r_{o1} = (1/(1.2e{-}3))||(20e3) = 800\Omega$
and we see a current divider, so we have
$i_{sc}/i_i = R_i/(R_i + R_{sx}) = (10e3)/((10e3) + (800)) = 0.9259\text{A/A}$ leading to
$(v_o/i_i)_b = i_{sc}/i_i * R_o = (0.9259) * (72.97e3) = 67.57\text{k}\Omega$

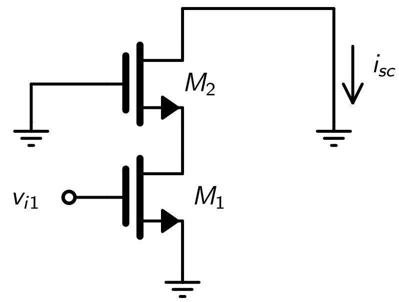
Question 2



For the circuit above
(a) Find v_o/v_i ignoring body effect (all $\chi = 0$).
(b) Find v_o/v_i including body effect where for $M2$, $M3$, $\chi = 0.2$.

Solution

(a) Define R_{op} to be the impedance looking up into the drain of M_3 and define R_{on} to be the impedance looking down into the drain of M_2
$R_{op} = r_{o3} + (1 + g_{m3} * r_{o3}) * r_{o4} = (10e3) + (1 + (500e{-}6) * (10e3)) * (20e3) = 130\text{k}\Omega$
$R_{on} = r_{o2} + (1 + g_{m2} * r_{o2}) * r_{o1} = (10e3) + (1 + (500e{-}6) * (10e3)) * (20e3) = 130\text{k}\Omega$
Define R_o to be the impedance to ground at node v_o
$R_o = R_{op}||R_{on} = (130e3)||(130e3) = 65\text{k}\Omega$
For i_{sc}, we have the following circuit



Define R_{S2} to be the impedance looking up into the source of M_2

$R_{S2} = (1/g_{m2})||r_{o2} = (1/(500e{-}6))||(10e3) = 1.667\text{k}\Omega$

The drain current of M_1 current divides between R_{S2} and r_{o1} resulting in

$G_{Ma} = -g_{m1} * (r_{o1})/(r_{o1} + R_{S2}) = -(1e{-}3) * ((20e3))/((20e3) + (1.667e3)) = -923.1e{-}6$

and $i_{sc} = G_{Ma} * v_i$. The resulting gain is

$(v_o/v_i)_a = G_{Ma} * R_o = (-923.1e{-}6) * (65e3) = -60\text{V/V}$

(b) To include the body effect, we have

$g'_m2 = g_{m2} * (1 + \chi) = (500e{-}6) * (1 + (0.2)) = 600e{-}6$

$g'_m3 = g_{m3} * (1 + \chi) = (500e{-}6) * (1 + (0.2)) = 600e{-}6$

Define R_{op} to be the impedance looking up into the drain of M_3 and define R_{on} to be the impedance looking down into the drain of M_2

$R_{op} = r_{o3} + (1 + g'_m3 * r_{o3}) * r_{o4} = (10e3) + (1 + (600e{-}6) * (10e3)) * (20e3) = 150\text{k}\Omega$

$R_{on} = r_{o2} + (1 + g'_m2 * r_{o2}) * r_{o1} = (10e3) + (1 + (600e{-}6) * (10e3)) * (20e3) = 150\text{k}\Omega$

Define R_o to be the impedance to ground at node v_o

$R_o = R_{op}||R_{on} = (150e3)||(150e3) = 75\text{k}\Omega$

For i_{sc}, we define R_{S2} to be the impedance looking up into the source of M_2 when the drain of M_2 is grounded

$R_{S2} = (1/g'_m2)||r_{o2} = (1/(600e{-}6))||(10e3) = 1.429\text{k}\Omega$

The drain current of M_1 current divides between R_{S2} and r_{o1} resulting in

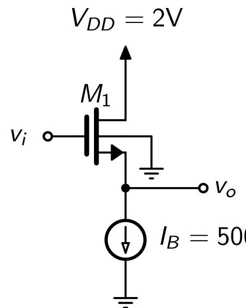
$G_{Ma} = -g_{m1} * (r_{o1})/(r_{o1} + R_{S2}) = -(1e{-}3) * ((20e3))/((20e3) + (1.429e3)) = -933.3e{-}6$

and $i_{sc} = G_{Ma} * v_i$. The resulting gain is

$(v_o/v_i)_b = G_{Ma} * R_o = (-933.3e{-}6) * (75e3) = -70\text{V/V}$

Question 3

Consider the common-drain (or source follower) shown below.



$\mu_n C_{ox} = 200\mu\text{A/V}^2$
$W = 5\mu\text{m}; L = 100\text{nm}$
$V_{tn} = 0.25\text{V}$
$\gamma = 0.4\text{V}^{1/2}$
$\phi_f = 0.3\text{V}$

(a) Ignoring the body effect, find the voltage at v_o when $v_i = V_{DD}$
(b) Repeat (a) but include the body effect and find the output voltage (an iterative approach is needed here).

Solution

(a) First find V_{ov} using the equation
$I_D = 0.5\mu_n C_{ox}(W/L)V_{ov}^2$
and we see that $I_D = I_B = (500e{-}6) = 500\mu\text{A}$
$V_{ov} = sqrt(2 * I_D/(\mu_n C_{ox} * (W/L))) = sqrt(2 * (500e{-}6)/((200e{-}6) * ((5e{-}6)/(100e{-}9)))) = 0.3162\text{V}$
So we have
$V_{GS} = V_{ov} + V_{tn} = (0.3162) + (0.25) = 0.5662\text{V}$
and when $v_i = V_{DD}$, we have
$v_{o,a} = V_{DD} - V_{GS} = (2) - (0.5662) = 1.434\text{V}$
Note that $v_o = V_{SB}$ since v_o is at the source voltage and $V_B = 0$

(b) When including the body effect, V_{tn} will change. So we define
$V_{tn0} = 0.25\text{V}$
V_{tn0} is the threshold voltage with $V_{SB} = 0$
Also, the value for V_{ov} is the same we found in (a)
$V_{ov} = 0.3162\text{V}$
Now, we make use of the body equation for the threshold voltage
$V_{tn} = V_{tn0} + \gamma[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}]$
as well as the equation
$V_{SB} = v_o = V_{DD} - (V_{ov} + V_{tn})$
Now we use an iterative approach to find the value of V_{tn} and therefore the value for $v_o = V_{SB}$
Our first guess for V_{SB} can be any value but lets start with the value we found when we ignored the body effect.
$V_{SB0} = 1.434\text{V}$
$V_{tn1} = V_{tn0} + \gamma * (sqrt(2 * \phi_f + V_{SB0}) - sqrt(2 * \phi_f)) = (0.25) + (0.4) * (sqrt(2 * (0.3) + (1.434)) - sqrt(2 * (0.3))) = 0.5106\text{V}$
$V_{SB1} = V_{DD} - (V_{ov} + V_{tn1}) = (2) - ((0.3162) + (0.5106)) = 1.173\text{V}$
$V_{tn2} = V_{tn0} + \gamma * (sqrt(2 * \phi_f + V_{SB1}) - sqrt(2 * \phi_f)) = (0.25) + (0.4) * (sqrt(2 * (0.3) + (1.173)) - sqrt(2 * (0.3))) = 0.4728\text{V}$
$V_{SB2} = V_{DD} - (V_{ov} + V_{tn2}) = (2) - ((0.3162) + (0.4728)) = 1.211\text{V}$
$V_{tn3} = V_{tn0} + \gamma * (sqrt(2 * \phi_f + V_{SB2}) - sqrt(2 * \phi_f)) = (0.25) + (0.4) * (sqrt(2 * (0.3) + (1.211)) - sqrt(2 * (0.3))) = 0.4785\text{V}$

$V_{SB3} = V_{DD} - (V_{ov} + V_{tn3}) = (2) - ((0.3162) + (0.4785)) = 1.205\text{V}$

Since V_{SB} is now changing very little, we can say our answer for v_o when the body effect is taken into account is

$v_{o,b} = V_{SB3} = (1.205) = 1.205\text{V}$