

Motivation

- Hierarchical timing analysis: need from placing and routing tools
 - Full extraction is not available
 - Do not need to do extraction every time: Incremental
 - Help to form the idea which part is timing critical
- Stability
 - Hurwitz polynomial can reserve stability when doing hierarchical reduction
- Given moments, how to derive the delay information without reconstruct waveform.

Moments: Review

Moment is generally defined as the coefficient of the Maclaurin series of the system transfer function, or say, transient response. We can also use step response moments.

$H(s) = m_0 + m_1 \cdot s + m_2 \cdot s^2 + \cdots$

- Moments can describe characteristics of the system, i.e., can be used to reconstruct output without solving differential equation
- Lots of timing analysis approaches have been proposed to match the first several moments in a feasible way.

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Moments: Continue

Moments can be iteratively derived from conductance and capacitance matrices. A is square matrix describe the system constructed by nodal analysis and matrix manipulation

 $\mathbf{m_1} = \mathbf{A} \cdot \mathbf{r}$

 $m_2 = A \cdot m_1$. . .

- We can observe that for each node:
 - $m_0 = 1$
 - $\blacksquare m_i \text{ has sign } (-1)^i$
 - $|m_i| > |m_{i+1}|$

Hierarchical Reduction

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Series reduction and Branch Merge

Series Reduction





■ What if there is Bridging Capacitance?

How Many Moments Are Enough

First moment's absolute value is actually Elmore Delay

 $-m_1 = TD_e$

- First moment doesn't consider downstream resistance and has problem with resistance shieding effect
- First two moments can capture R and C in the whole network
- First three moments can reflect inductances in the circuit. So the three moments with π model are used in hierarchical reduction

Hurwitz Stable Reduced Order Modeling

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Transfer Function Propagation

Transfer Function Progation



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Transfer Fucntion Progation

For $H_i(s)$ and $H_i(s)$, if: When using tradition method to compute poles and residues from moments, there will be positive poles after several iterations. $H_i(s) = \frac{1 + a_{i1}s + a_{i2}s^2}{1 + b_{i1}s + b_{i2}s^2 + b_{i3}s^3}, \quad H_j(s) = \frac{1 + a_{j1}s + a_{j2}s^2}{1 + b_{i1}s + b_{i2}s^2 + b_{i3}s^3}$ Positive poles have to be discarded. So when too many positive poles show up, more iterations won't improve accuracy too much. Do some approximation, there is $H_{ij}(s) = H_i(s)H_j(s)$, with Explicitly moments matching must have stability problem. Reserve $\begin{cases} a_1 = a_{i1} + a_{j1}, & a_2 = a_{i2} + a_{j2} + a_{i1}a_{j1} \\ b_1 = b_{i1} + b_{j1}, & b_2 = b_{i2} + b_{j2} + b_{i1}b_{j1} \\ b_3 = b_{i1}b_{j2} + b_{i2}b_{j1} + b_{i3} + b_{j3} - a_{i1}a_{j2} - a_{i2}a_{j1} \end{cases}$ Stability is the main problem with common AWE method. $H_{ij}(s) = \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2 + b_3 s^3}$ (1)Copyright © Zhong Wang, Feb 2002, ECE, Univ. of Toronto Copyright © Zhong Wang, Feb 2002, ECE, Univ. of Toronto Synthesis Reading Group Synthesis Reading Group 10 Hurwitz Polynomial and Stabiltiv **Test Result** Hurwitz Polynomial: A polynomial with real positive coefficients The author implemented their ideas, which shows close match with Spice simulation result for distributed RC line and roots which are either negative or pairwise conjugate with

- The program cannot parse netlist of RC trees with branches. But the idea should be able to handle branches
- As stated before, the hierarchical reduction cannot take care of briding capacitance, which is the most obvious deficiency

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negative real parts.

$$A(x) = 1 + a_1 x + a_2 x^2 + \dots + a_n x^n, \qquad a_i > 0$$

- So we should keep the denominator of the system transfer function's property as Hurwitz polynomial during the transformatioin
- It is proved in the paper using a RLC π model and Fuction 1, we can reserve this property

D2M

D2M is actually from moments to delay. It provides a simple, accurate, closed form formula to compute delay:

$$D2M = ln2\frac{m_1^2}{\sqrt{m_2}}$$

- D2M is empirically derived using try and error. The author started by trying to scale Elmore delay to match real delay number since Elmore delay tends to over estimate the delay. They found ln2\sqrt{m_1^2/m_2} is a good scaling factor
- **I** The author cannot find relations between m_3 and delay

D2M Is Better Than Elmore Delay

Elmore delay is the upper bound of D2M. For stable system, there is $m_1^2/m_2 \leq 2$, so

$$D2M = -m_1 ln 2\sqrt{\frac{m_1^2}{m_2}} \le -m_1 ln 2\sqrt{2} \cong -0.9802m_1$$

After a more careful analysis, there is:

$D2M\cong 0.8003ED$

Both Elmore and D2M, as well as most other moment matching approaches tend to pessimistic, i.e., try to overestimate the delay. But D2M is much closer.

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Dominant Pole

- Some other closed form metrics use dominant pole method
- If a system has only positive poles and one pole is much closer to zero than the other ones, it is said to be a dominant pole
- A system's behavior is mainly determined by the dominant pole, since e^{p_it} dominates others exponents in:

$$f(t) = \sum_{i=1}^{N} k_i e^{p_i t}$$

Based on this observation, there is single pole delay approximation:

$$t_D = -\frac{1}{p} ln(2k)$$

Using More Dominant Poles

- Theoretically using more dominant poles will generate more accuarate result
- Some people have derived delay metric using 2, 3 and 4 poles, which did not show great improvement. Some results are worse probably because of the formula is not well devised.
- NO METHOD UP TO NOW IS PERFECT. All of them have resistance shieding problem, i.e., overestimate delay for near end nodes. The error could be quite large, sometimes 10 times.

Simulation Result

Test Bench

Source node is A, there are several branches and sinks. We choose the capacitance value large enough to make the error significant. Delay is measured from A to E.



Table 1: Different Delay Metric

Method	Poles	Delay(ns)	Error	
HSPICE	N/A	4.633e-08		
D2M	N/A	4.4965e-08	2.9%	
DM1	1	4.4932e-08	3.0%	
DM2	2	4.2150e-08	9.0%	
DM3	3	4.4849e-08	3.2%	
DM4	4	4.4849e-08	3.2%	

Conclsion

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- Both Hurwitz polynomial and D2M method provides possible ways to solve problems in STA
- Hurwitz polynomial method is not ready to be used right away. But the idea of incremental and hierarchical STA is useful
- D2M is directly applicable with possible improvement

Future Work

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- Method to deal with bridging capacitance in hierarchicall reduction
- A mathematical derivation of D2M method
- A better delay metric for near end nodes

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