Hurwitz Stable Reduced Modeling
and
D2M Delay Calculation Metric

Zhong Wang

Electrical and Computer Engineering
University of Toronto

Feb 13, 2002

`zwang@eecg.toronto.edu`
`http://www.eecg.toronto.edu/~zwang`

!!!Happy Chinese Lunar New Year!!!

Part I

- *Motivation*
- Review: Moments
- Hurwitz Stable Reduced Order Modeling
- D2M
- Future Work

Problem

- Hierarchical timing analysis, need from placing and routing tools
  - Full extraction is not available
  - Do not need to do extraction every time: Incremental
  - Help to form the idea which part is timing critical
- Stability
  - Hierwitz polynomial can reserve stability when doing hierarchical reduction
- Given moments, how to derive the delay information without reconstruct waveform.

Moments: Review

- Moments is generally defined as the coefficient of the Maclaurin series of the system transfer fuction, or say, transient response. We can also use step response moments.

$$H(s) = m_0 + m_1 \cdot s + m_2 \cdot s^2 + \cdots$$

- Moments can describe characteristics of the system, i.e., can be used to recontruct output without solving differential equation
- Lots of timing analysis approaches have been proposed to match the first several moments in a feasible way.

Moments: Continue

- Moments can be iteratively derived from conductance and capacitance matrices to any order required.

$$\mathbf{m_1} = \mathbf{A} \cdot \mathbf{r}$$

$$\mathbf{m_2} = \mathbf{A} \cdot \mathbf{m_1}$$

$$\cdots$$

while A is square matrix describe the system constructed by nodal analysis and matrix manipulation

- We can observe that for each node:
  - $m_0 = 1$
  - m_i has sign $(-1)^i$
  - $|m_i| > |m_{i+1}|$

How Many Moments Are Enough

- First moment's absolute value is actually Elmore Delay

$$-m_1 = TD_e$$

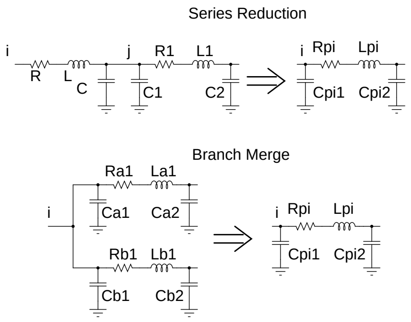
Which is detailed later

- First moment doesn't consider downstream resistance and has problem with shief effect
- First two moments can capture R and C in the whole network
- Frist three moments can reflect inductances in the circuit. So the three moments with π model are used in hierarchical reduction

---

Hierarchical Reduction

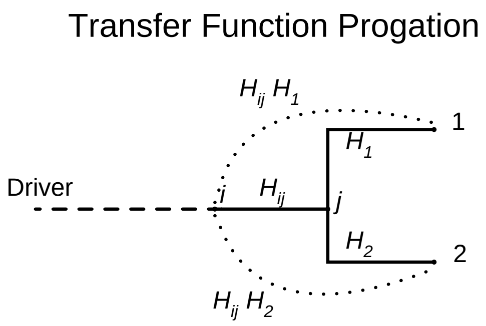
- Series reduction and Branch Merge



- What if there is Briding Capacitance?

Hurwitz Stable Reduced Order Modeling

- Transfer Fuction Propagation



Transfer Fucntion Progation

For $H_i(s)$ and $H_j(s)$, if:

$$H_i(s) = \frac{1 + a_{i1}s + a_{i2}s^2}{1 + b_{i1}s + b_{i2}s^2 + b_{i3}s^3}, \quad H_j(s) = \frac{1 + a_{j1}s + a_{j2}s^2}{1 + b_{j1}s + b_{j2}s^2 + b_{j3}s^3}$$

Do some approximation, there is $H_{ij}(s) = H_i(s)H_j(s)$, with

$$\begin{cases} a_1 = a_{i1} + a_{j1}, \qquad a_2 = a_{i2} + a_{j2} + a_{i1}a_{j1} \\ b_1 = b_{i1} + b_{j1}, \qquad b_2 = b_{i2} + b_{j2} + b_{i1}b_{j1} \\ b_3 = b_{i1}b_{j2} + b_{i2}b_{j1} + b_{i3} + b_{j3} - a_{i1}a_{j2} - a_{i2}a_{j1} \end{cases}$$

$$H_{ij}(s) = \frac{1 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2 + b_3 s^3} \tag{1}$$

Why Stability a Problem

- When using tradition method to compute poles and residues from moments, there will be positive poles after several iterations.
- Positive poles have to be discarded. So when too many positive poles show up, more iterations won't improve accuracy too much.
- Explicitly moments matching must have this problem. Reserve Stability is the main problem with common AWE method.

Hurwitz Polynomial and Stabiltiy

- Hurwitz Polynomial: A polynomial with real positive coefficients and roots which are either negative or pairwise conjugate with negative real parts.
- So if we can keep the denominator of the system transfer function's property as Hurwitz polynomial,
- It is proved in the paper using a RLC π model and Fuction 1, we can reserve this property

---

Test Result

- The author implemented their ideas, which shows close match with Spice simulation result for distributed RC line
- The program cannot parse netlist of RC trees with branches. But the idea should be able to handle branches
- As stated before, the hierarchical reduction cannot take care of briding capacitance, which is the most obvious deficiency.

D2M

- D2M is actually from moments to delay. It provides a **simple, accurate, closed form** formula to compute delay:

$$D2M = ln2\frac{m_1^2}{sqrtm_2}$$

- D2M is empirically derived: Try and Error. The author started by trying to scale Elmore delay to match real delay number since Elmore delay tends to over estimate the delay. They found $ln2\sqrt{m_1^2/m_2}$ is a good scaling factor
- The author cannot find relations between m_3 and delay

D2M Is Better Than Elmore Delay

- Elmore delay is the upper bound of D2M. For stable system, there is $m_1^2/m_2 \le 2$, so

$$D2M = -m_1 ln2\sqrt{\frac{m_1^2}{m_2}} \le -m_1 ln2\sqrt{2} \cong -0.9802m_1$$

- After a more careful analysis, there is:

$$D2M \cong 0.8003ED$$

- Both Elmore and D2M, as well as most other moments matching approaches tend to pessimistic, i.e., try to overestimate the delay. But D2M is much closer.

Dominant Pole

- Some other closed form metrics use dominant pole method
- If a system has only positive poles and one pole is much closer to zero than other ones, this is said to be the dominant pole
- A system's behavior is mainly determined by the dominant pole, since

$$f(t) = \sum_{i=1}^{N} k_i e^{p_i t}$$

and $e^{p_i t}$ dominates others exponents

- Based on this observation, there is single pole delay approximation:

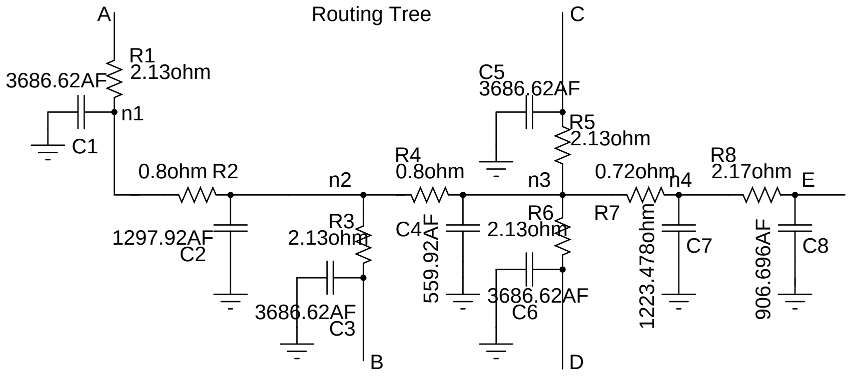
$$t_D = -\frac{1}{p}ln(2k)$$

Using More Dominant Poles

- Theoretically using more dominant poles will generate more accuarate result
- Some people have derived delay metric using 2, 3 and 4 poles, which did not show great improvement. Some results are worse probably because of the formula is not well devised.
- **No method upto now is perfect** . All of them have resistance shieding problem, i.e., overestimate delay for near end nodes. The error could be quite large, sometimes 10 times.

Test Bench

Source node is A, there are several branches and sinks. This is directly extracted from a interconnect



Simulation Result

Table 1: Different Delay Metric

Method	Poles	Delay(ns)	Error
HSPICE	N/A	4.633e-08	
D2M	N/A	4.4965e-08	2.9%
DM1	1	4.4932e-08	3.0%
DM2	2	4.2150e-08	9.0%
DM3	3	4.4849e-08	3.2%
DM4	4	4.4849e-08	3.2%

Conclsion

- Both Hurwitz polynomial and D2M method provides possible ways to solve problems in STA
- Hurwitz polynomial method is not ready to used right away. But the idea of incremental and hierarchical STA is useful
- D2M is directly applicable with possible improvement