DATE 17
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Quantifying Error: Extending Static Timing Analysis with Probabilistic Transitions

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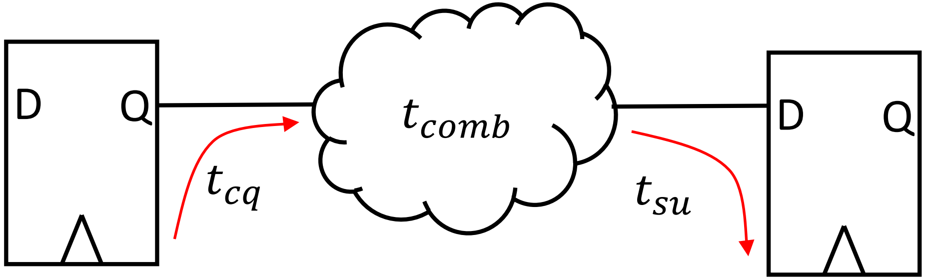
Error Tolerance

- **Error tolerant applications**
  - **Errors correctable: Iterative algorithms**
  - **Inherently noisy: Sensor readings**
  - **Errors acceptable: Image processing**

[Shi et. al, DAC 2014]

Timing Errors

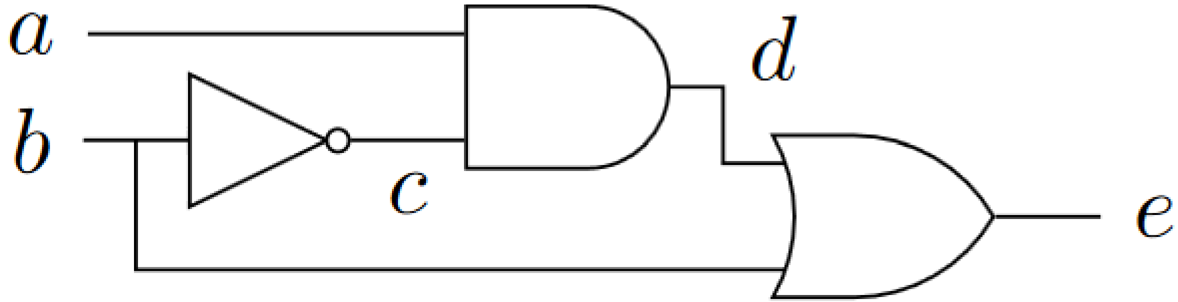


$$t_{cq} + t_{comb} + t_{su} \leq \frac{1}{f_{max}}$$

- **Running system beyond f_{max} may cause errors**
- **But critical path not always exercised**

Analysis Capability Gap

- **How often do timing errors occur?**
  - **How likely are timing paths to be exercised?**



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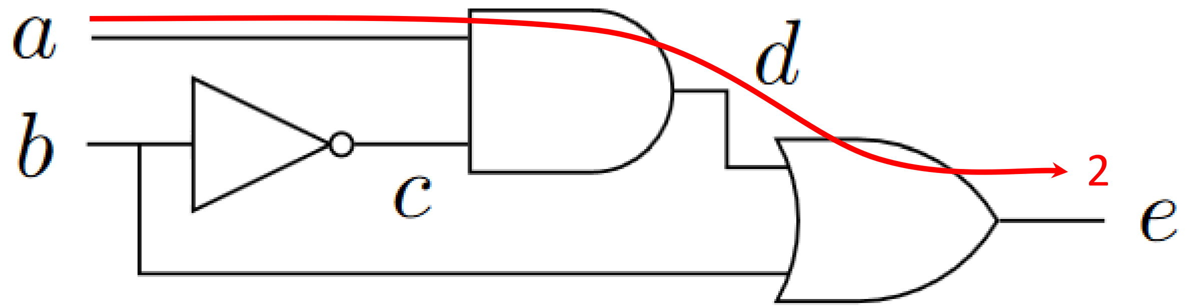

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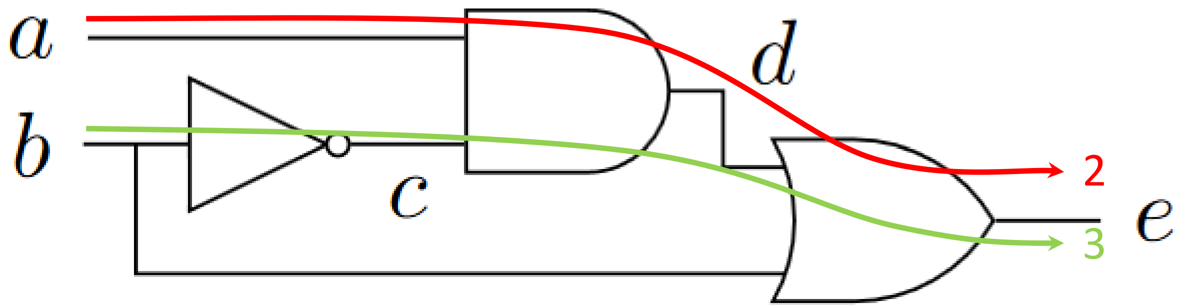

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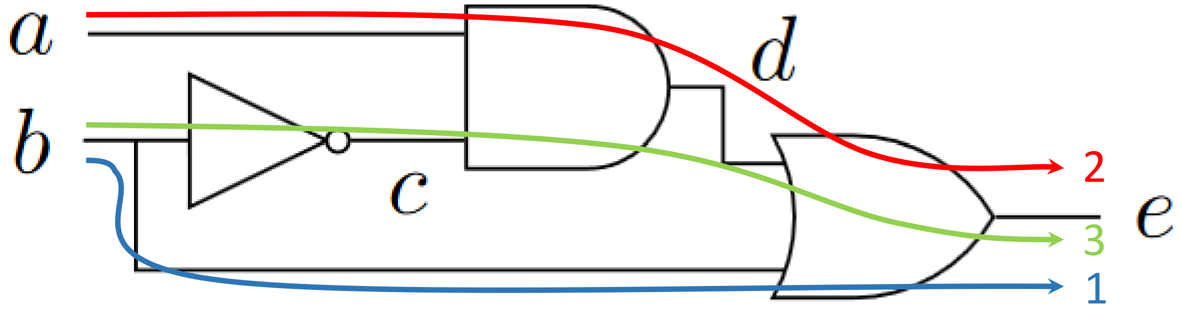

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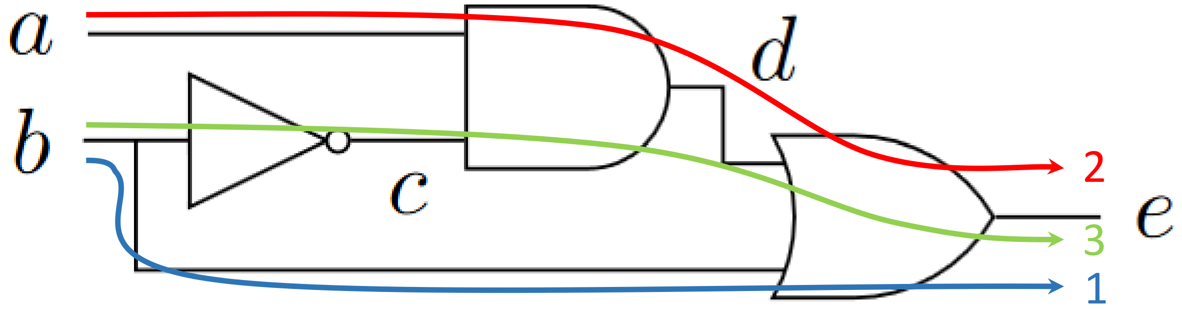
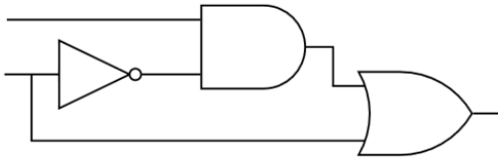


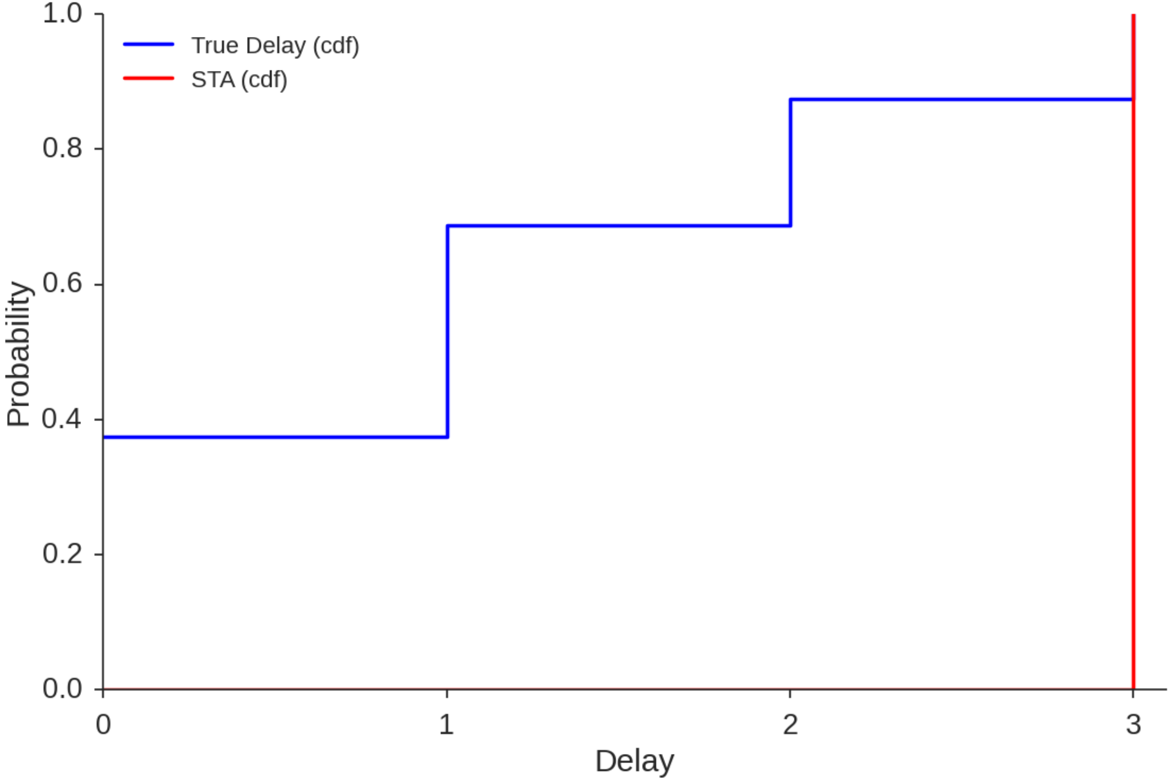
- Probability depends on:
  - Circuit logic
  - Input data

- Traditional Static Timing Analysis (STA) does not answer this!

Extended Static Timing Analysis (ESTA)

- **Goal: Calculate probability timing paths are exercised**





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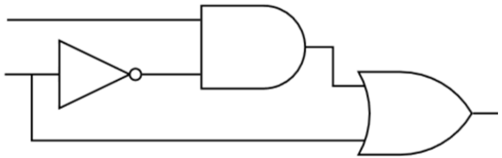

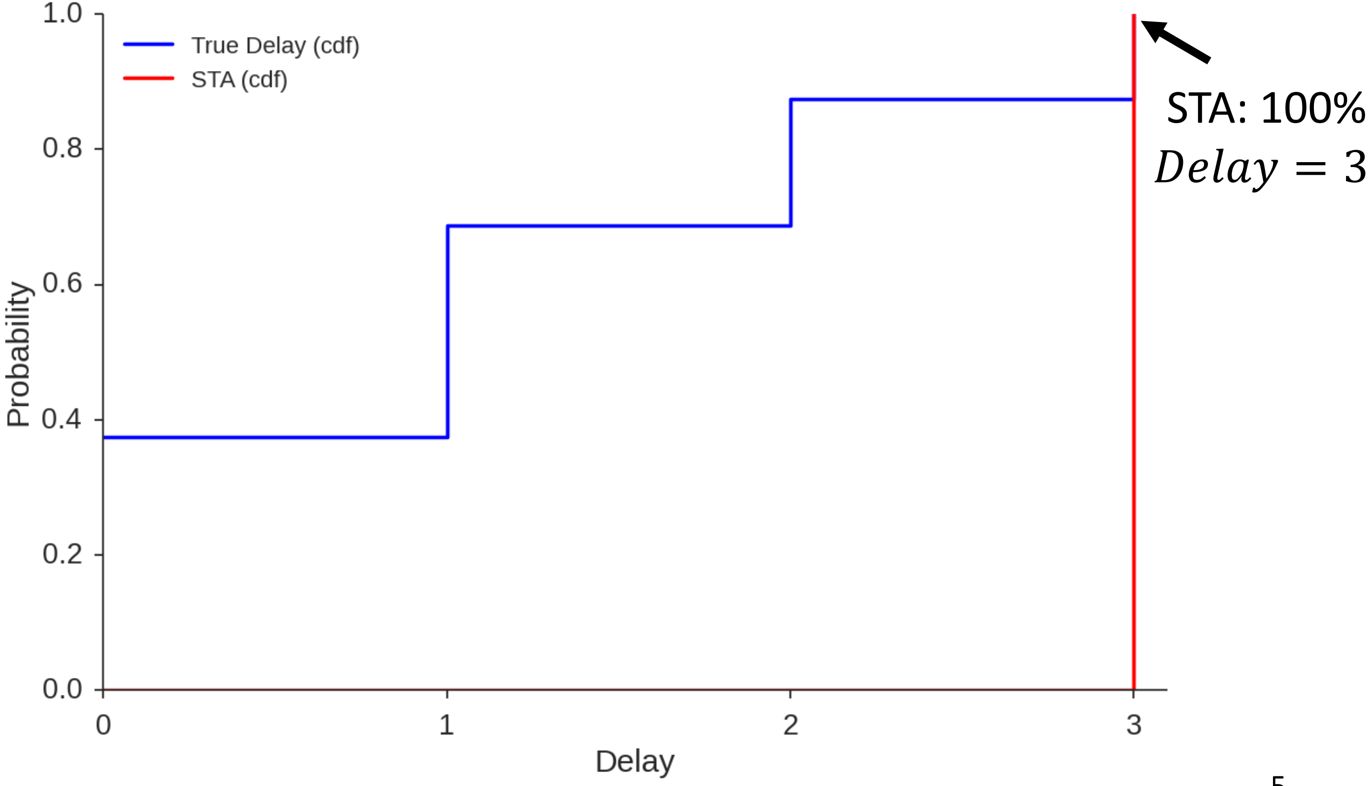

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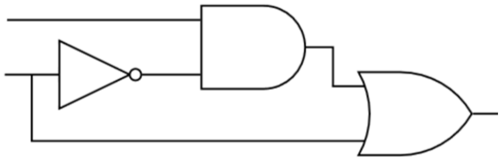
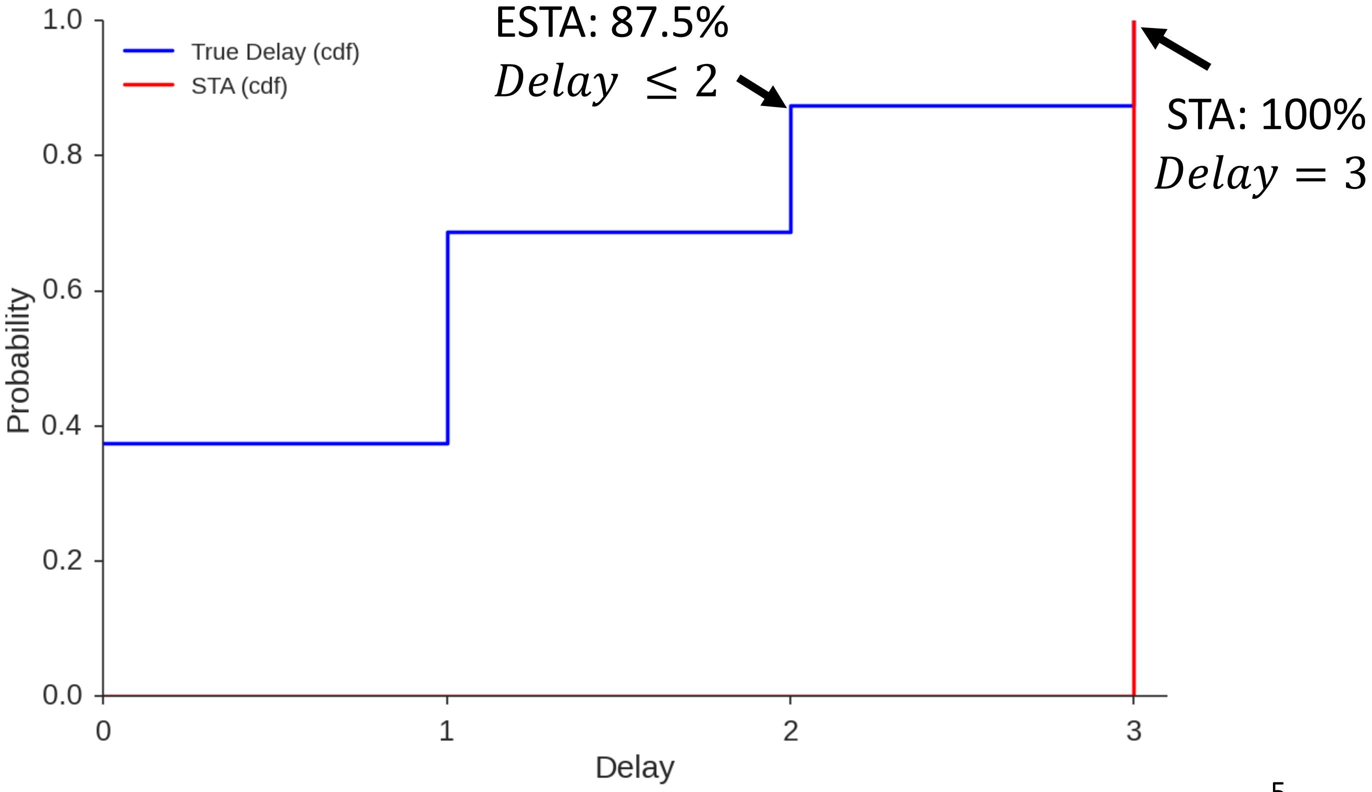
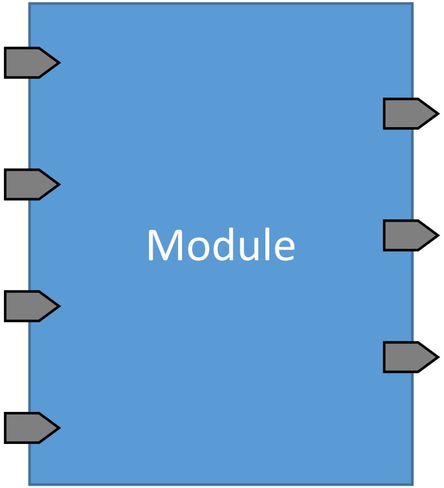




Multiple Outputs



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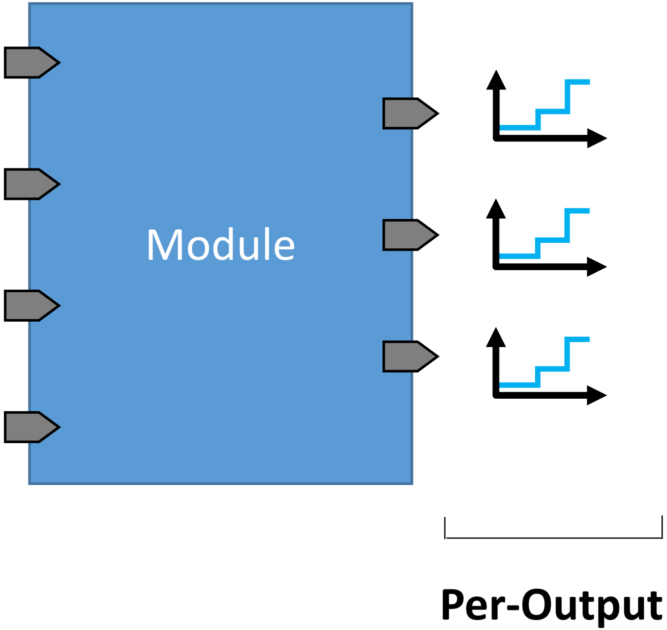

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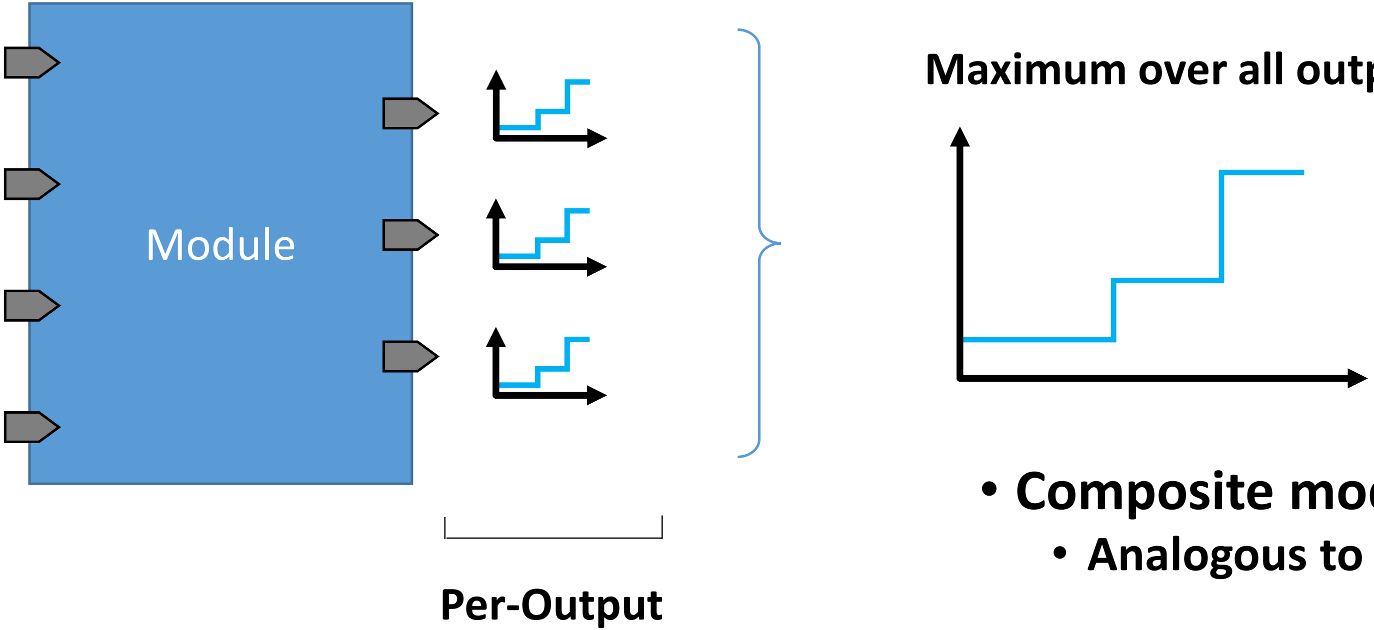
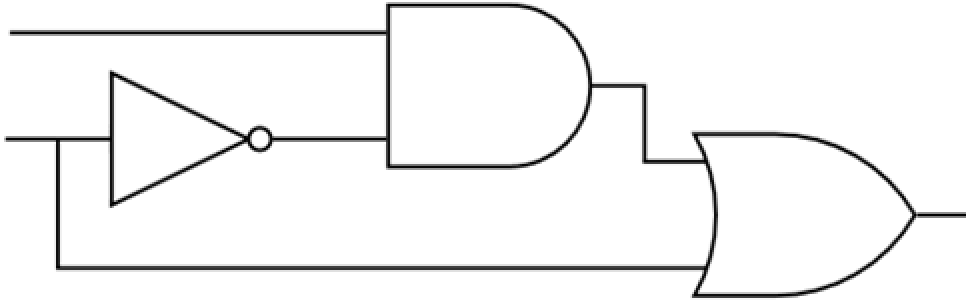


- **Composite module-level error**
  - **Analogous to critical path delay**

What about Statistical STA (SSTA)?

- **SSTA**
  - **Models *delay* uncertainty**
  - **Still assumes all paths switch**



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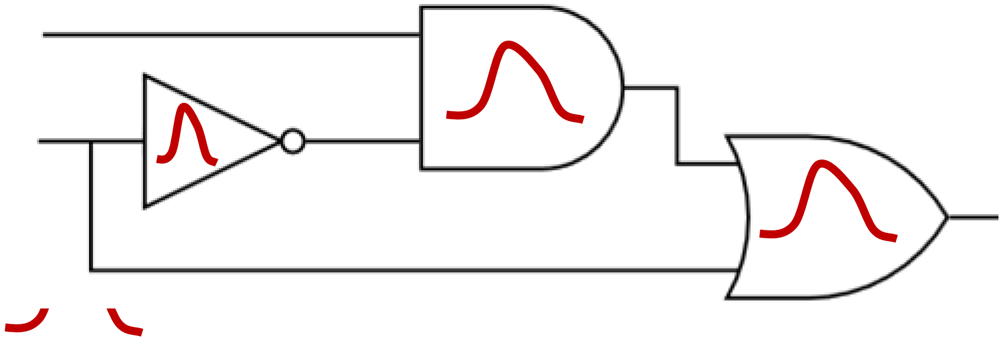


- **ESTA**
  - **Models *input* uncertainty**

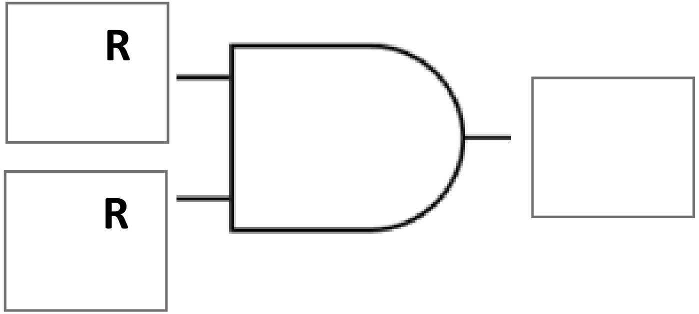
Outline

- Introduction
- Extended STA Formulation
- Evaluation Framework
- Experimental Results
- Conclusion and Future Work

ESTA Transition & Delay Modeling

- **Model four transitions:**

Symbol	Meaning
R	Rising
F	Falling
H	High (static)
L	Low (static)



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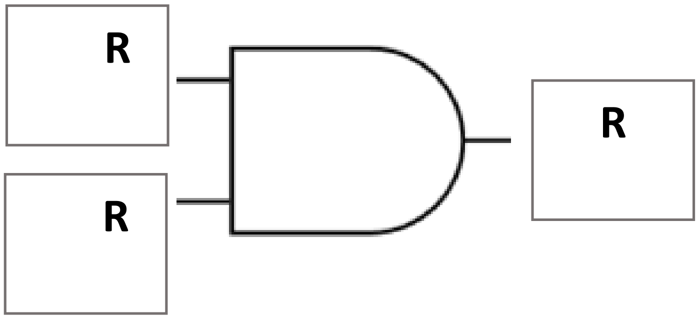

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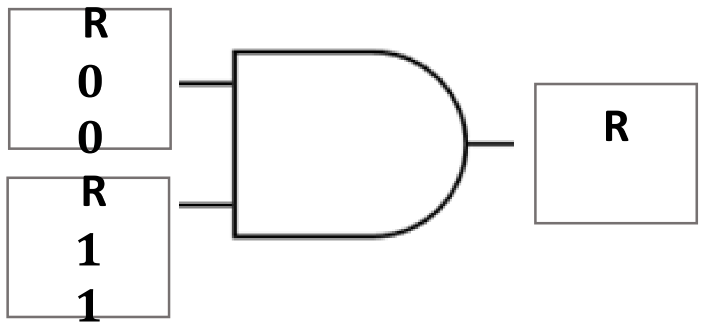

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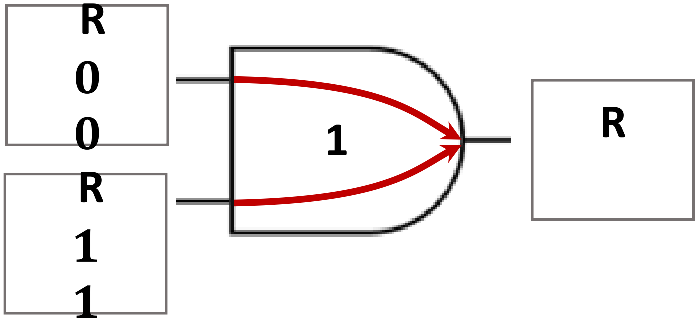

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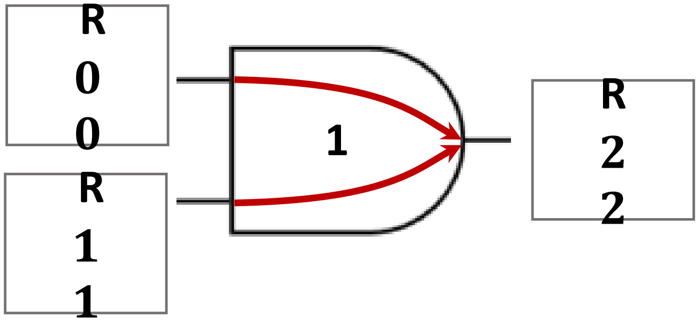


Activation Functions

- **Let $f \in \mathbb{B}^n \rightarrow \mathbb{B}$ be an *activation function:***
  - **Evaluates TRUE when an event occurs**

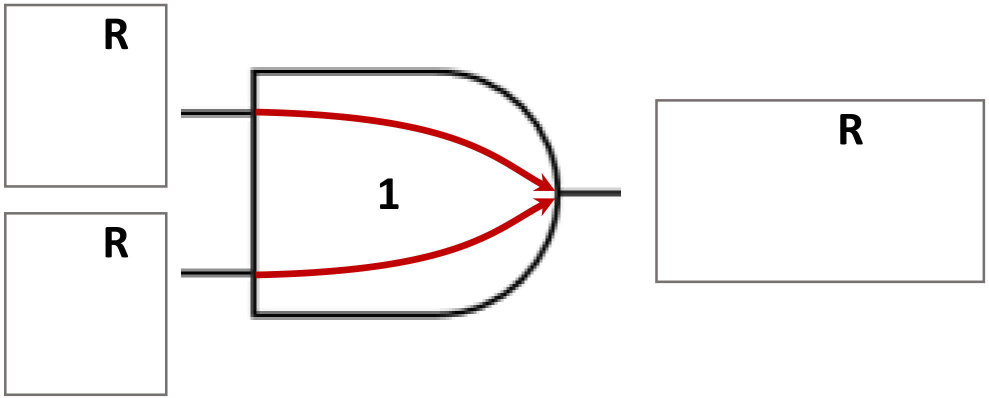
- **We can combine activation functions:**

$$\boldsymbol{A}, \boldsymbol{B} \Rightarrow \boldsymbol{C}$$

$$\boldsymbol{f_C} = \boldsymbol{f_A} \wedge \boldsymbol{f_B}$$

Timing Tags

- Define a *timing tag* tuple:
  - Transition
  - Delay
  - Activation Function



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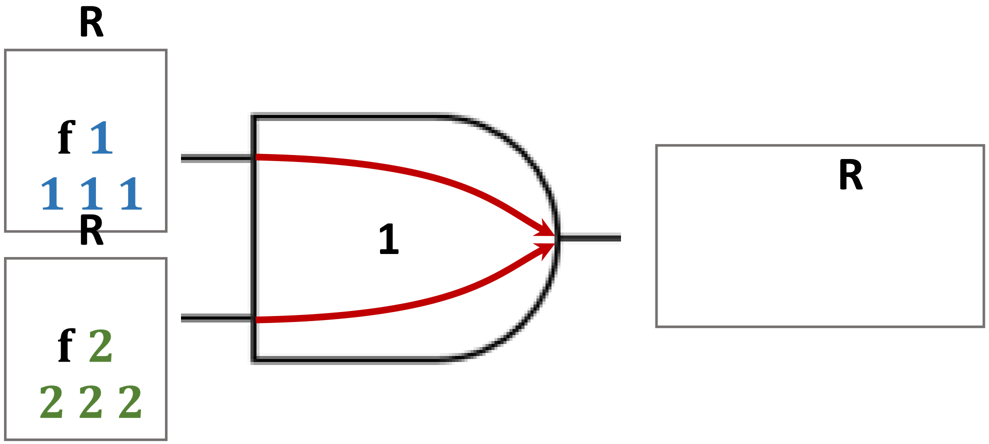

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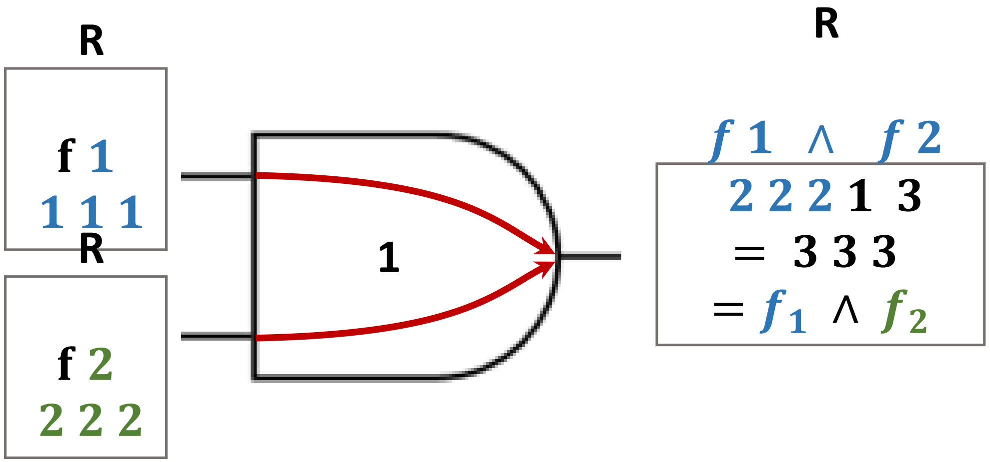

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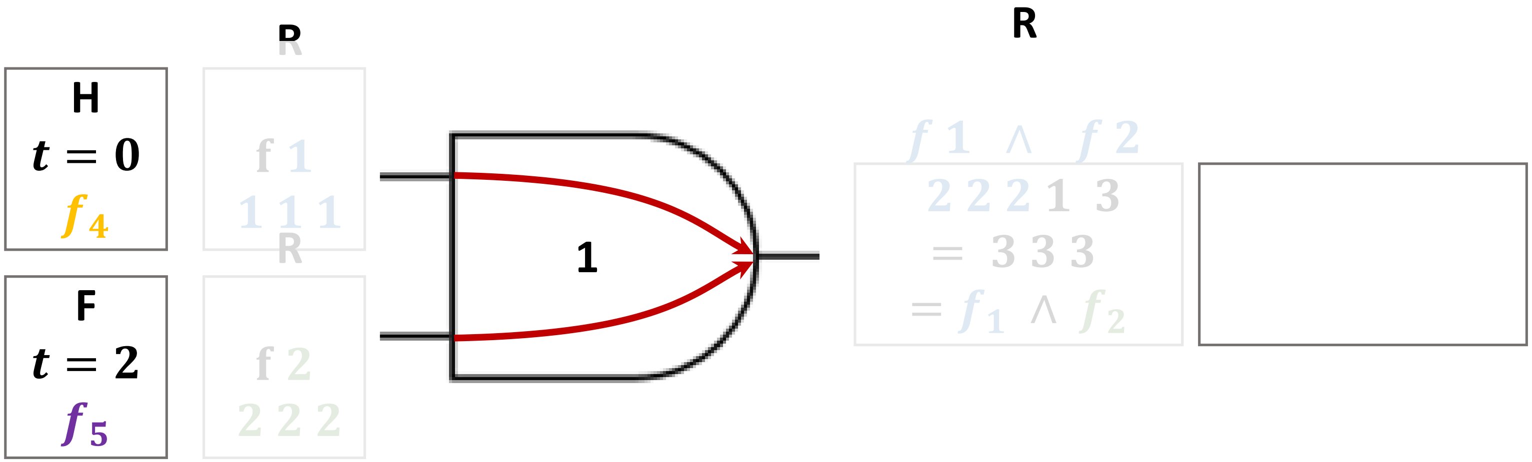

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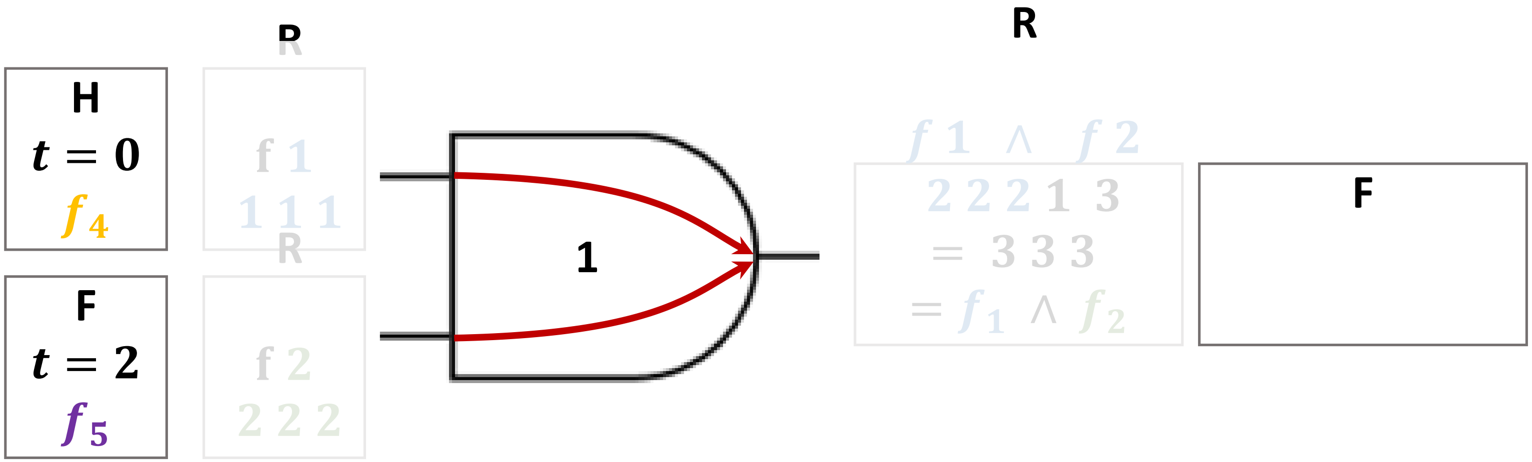

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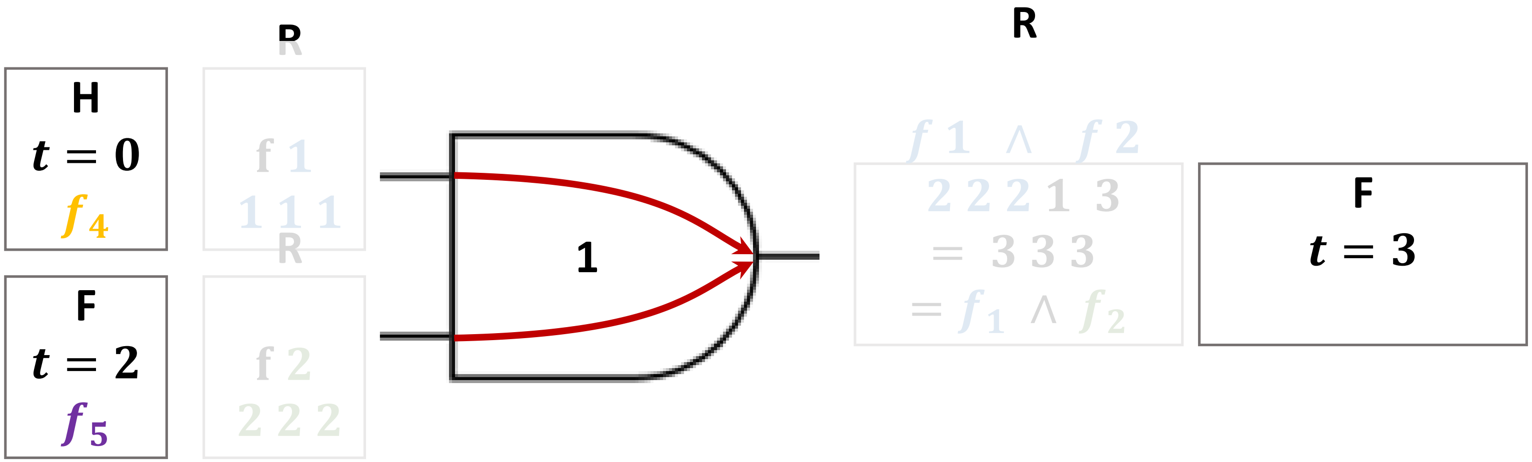

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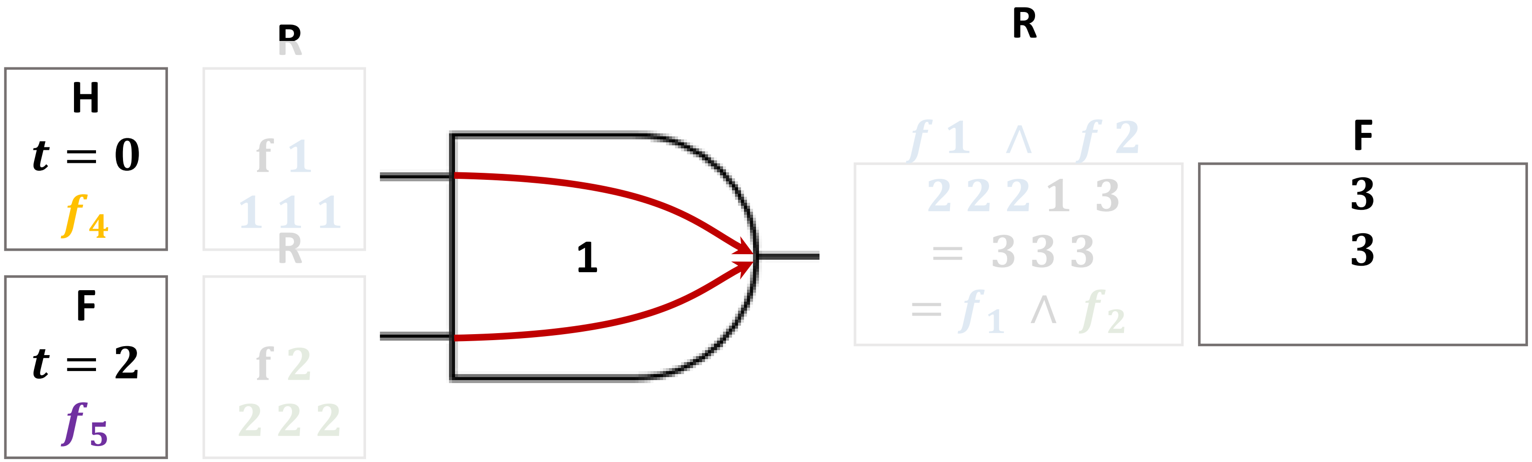

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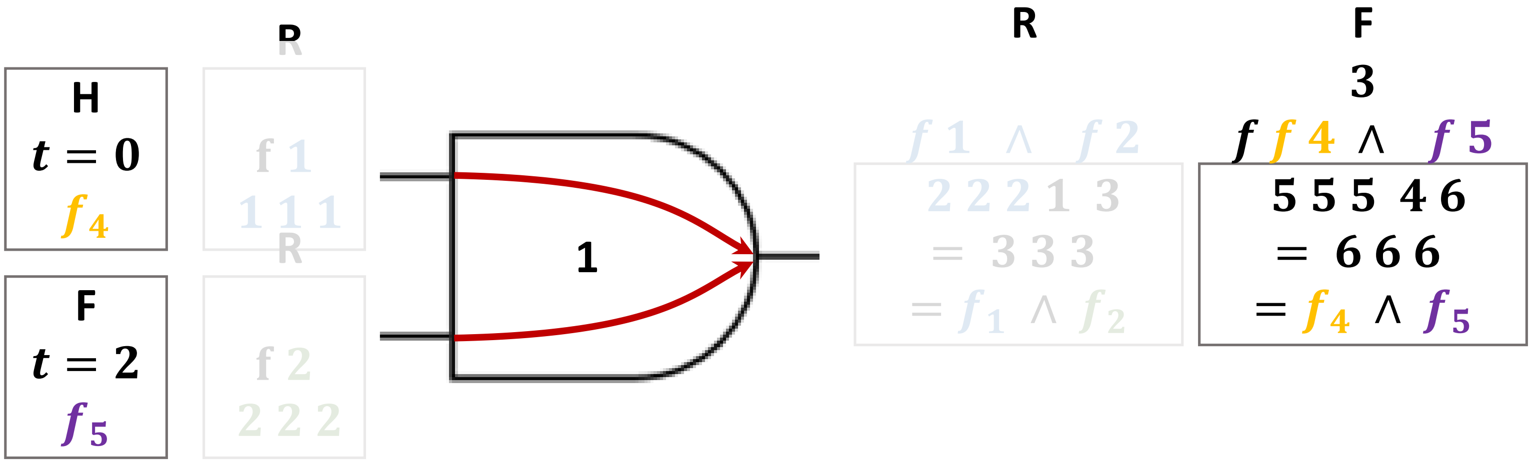

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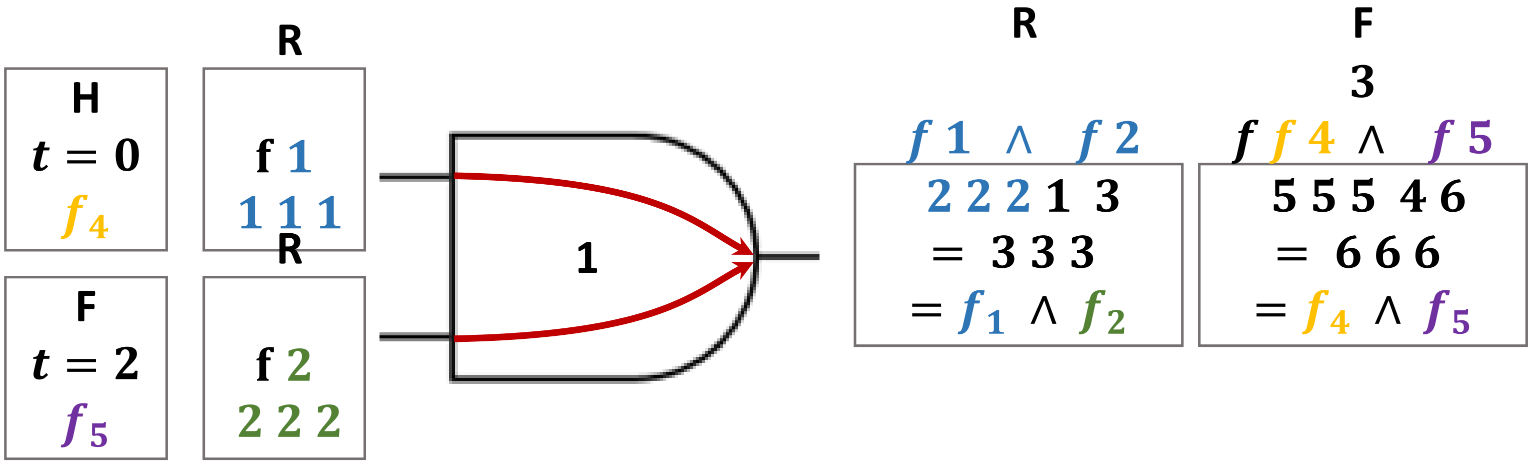
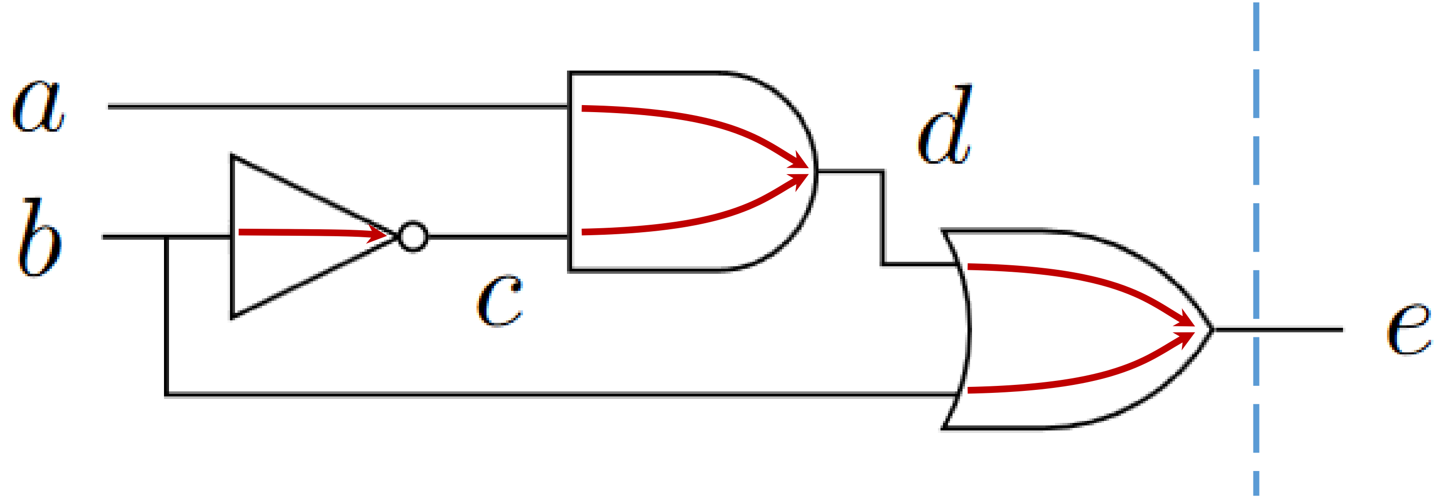


Timing Tag Propagation

- **Propagate timing tags**

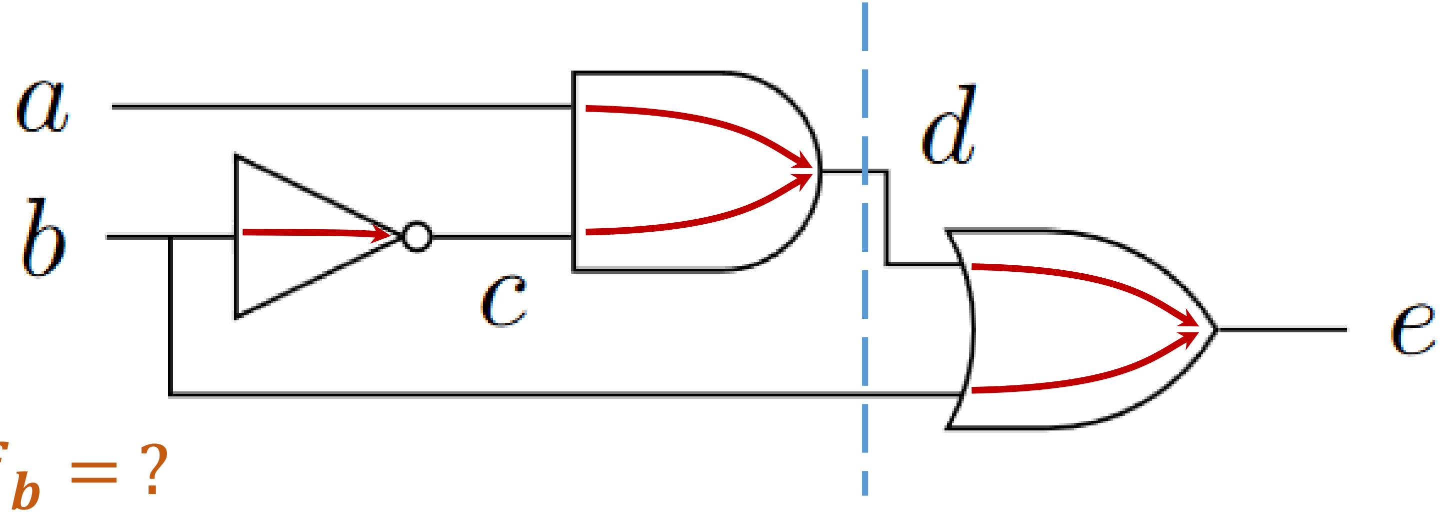


$\boldsymbol{f_e} = ?$

Timing Tag Propagation

- **Propagate timing tags**

$f_d = ?$



$f_b = ?$

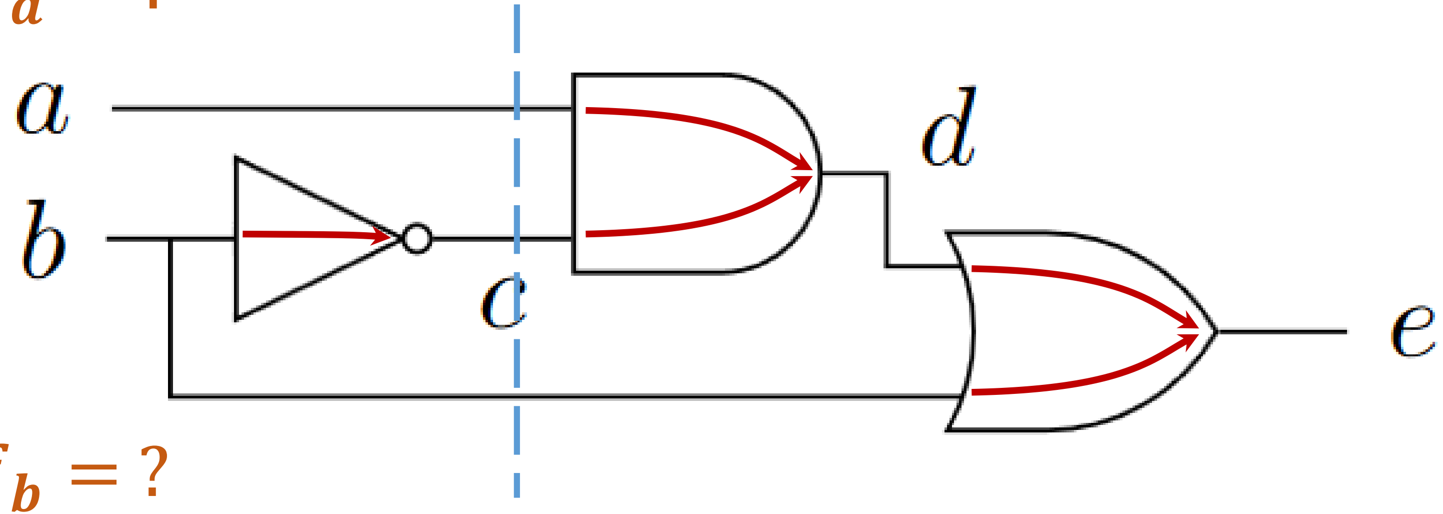
$f_e = f_d \wedge f_b$

Timing Tag Propagation

- **Propagate timing tags**

$$f_d = f_a \wedge f_c$$

$$f_a = ?$$



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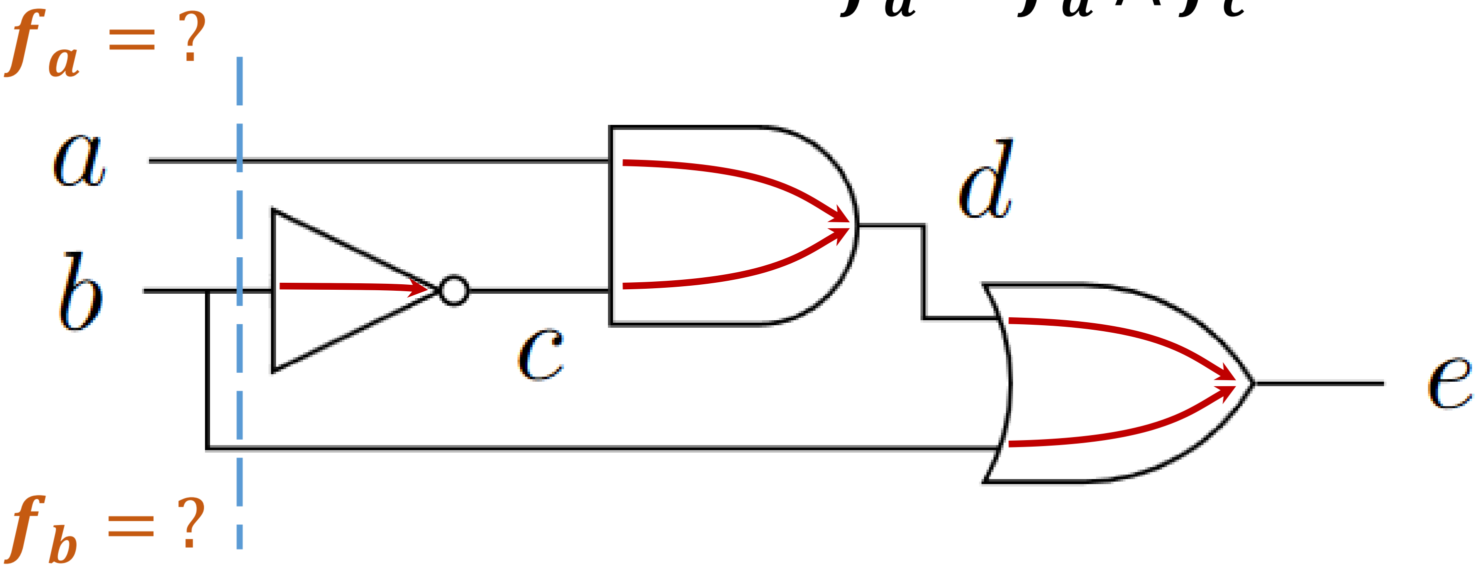

$$f_c = f_b$$

$$f_e = f_d \wedge f_b$$

Base Activation Functions

- **Base-case activation functions at Primary Inputs**
- **Two variables per input**

$$f_{rise}^{(a)} = \overline{x_a} \wedge x'_a$$

$$f_{fall}^{(a)} = x_a \wedge \overline{x'_a}$$

$$f_{high}^{(a)} = x_a \wedge x'_a$$

$$f_{low}^{(a)} = \overline{x_a} \wedge \overline{x'_a}$$

Base Activation Functions

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- **Two variables per input**

$$f_{rise}^{(a)} = \overline{x_a} \wedge x'_a$$

$$f_{fall}^{(a)} = x_a \wedge \overline{x'_a}$$

Present State

$$f_{high}^{(a)} = x_a \wedge x'_a$$

Next State

$$f_{low}^{(a)} = \overline{x_a} \wedge \overline{x'_a}$$

Calculating Probabilities with #SAT

- Let f be an activation function, and $|f|$ its support size then:

$$\mathbf{p}(f) = \frac{\#\mathbf{SAT}(f)}{2^{|f|}}$$

- Assumes uniformly distributed independent variables

Calculating Probabilities with #SAT

- **Let f be an activation function, and $|f|$ its support size then:**

$$\mathbf{p}(f) = \frac{\#\mathbf{SAT}(f)}{2^{|f|}}$$

- **Assumes uniformly distributed independent variables**
  - **Assumption can be removed with pre-processing**

Probability Example

$$\mathrm{p}(\boldsymbol{f}) = \frac{\#\mathbf{SAT}(\boldsymbol{f})}{\mathbf{2}^{|\boldsymbol{f}|}}$$

$$\boldsymbol{f}(\boldsymbol{x_a}, \boldsymbol{x'_a}, \boldsymbol{x_b}, \boldsymbol{x'_b}) = \boldsymbol{x_a} \wedge \boldsymbol{x'_a} \wedge \boldsymbol{x_b} \wedge \boldsymbol{x'_b}$$

Probability Example

$$\mathbf{p}(f) = \frac{\#\mathbf{SAT}(f)}{2^{|f|}}$$

$$f(x_a, x_a', x_b, x_b') = x_a \wedge x_a' \wedge x_b \wedge x_b'$$

$$\#\mathbf{SAT}(f) = \mathbf{1}$$

$$|f| = \mathbf{4}$$

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$$\mathbf{p}\ f = \ \mathbf{1}\ \mathbf{2}\ \mathbf{4}\ = \ \mathbf{0.0625}$$

$$\mathbf{4}\ \mathbf{4}\ \mathbf{4}\ \ \mathbf{2}\ \mathbf{4}\ \ \mathbf{2}\ \mathbf{4}\ \ \frac{\mathbf{1}}{\mathbf{2}^{\mathbf{4}}} = \mathbf{0.0625}$$

ESTA Formulation Summary

- **Propagate timing tags**
  - **Calculate STA-like path-based delay estimates**
  - **Build activation functions**

- **Probability Calculation**
  - **Use #SAT to determine path probabilities**

ESTA Implementation

- **Activation functions represented with Binary Decision Diagrams (BDDs)**
  - **Easy manipulation**
  - **Efficient #SAT**

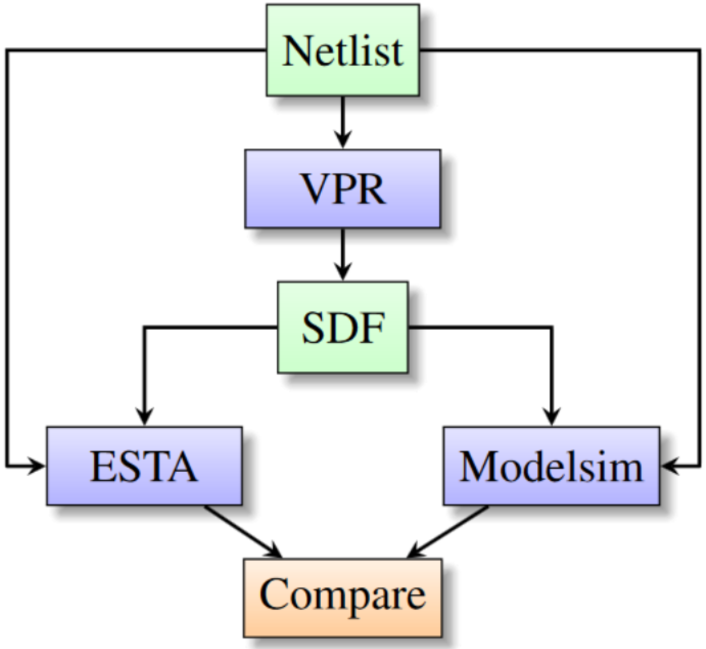
- **Bound some path-delay combinations**
  - **Improves scalability (fewer tags)**
  - **Adds some pessimism**

Evaluation Framework

- **Determine maximum delay histogram across *all* outputs**

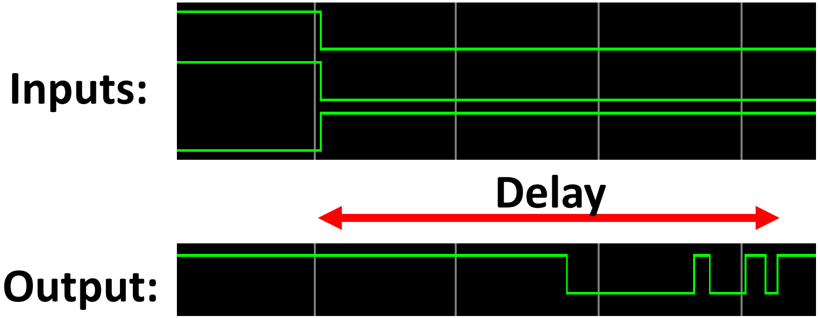
- **Benchmarks:**
  - **MCNC20:**
    - **554 to 6239 LUTs**
    - **8 to 1545 Primary Inputs**
  - **Post-place-and-route delays from 40nm FPGA**

- **Comparison:**
  - **Timing Simulation (Ground Truth)**
  - **ESTA**
  - **STA**



Timing Simulation

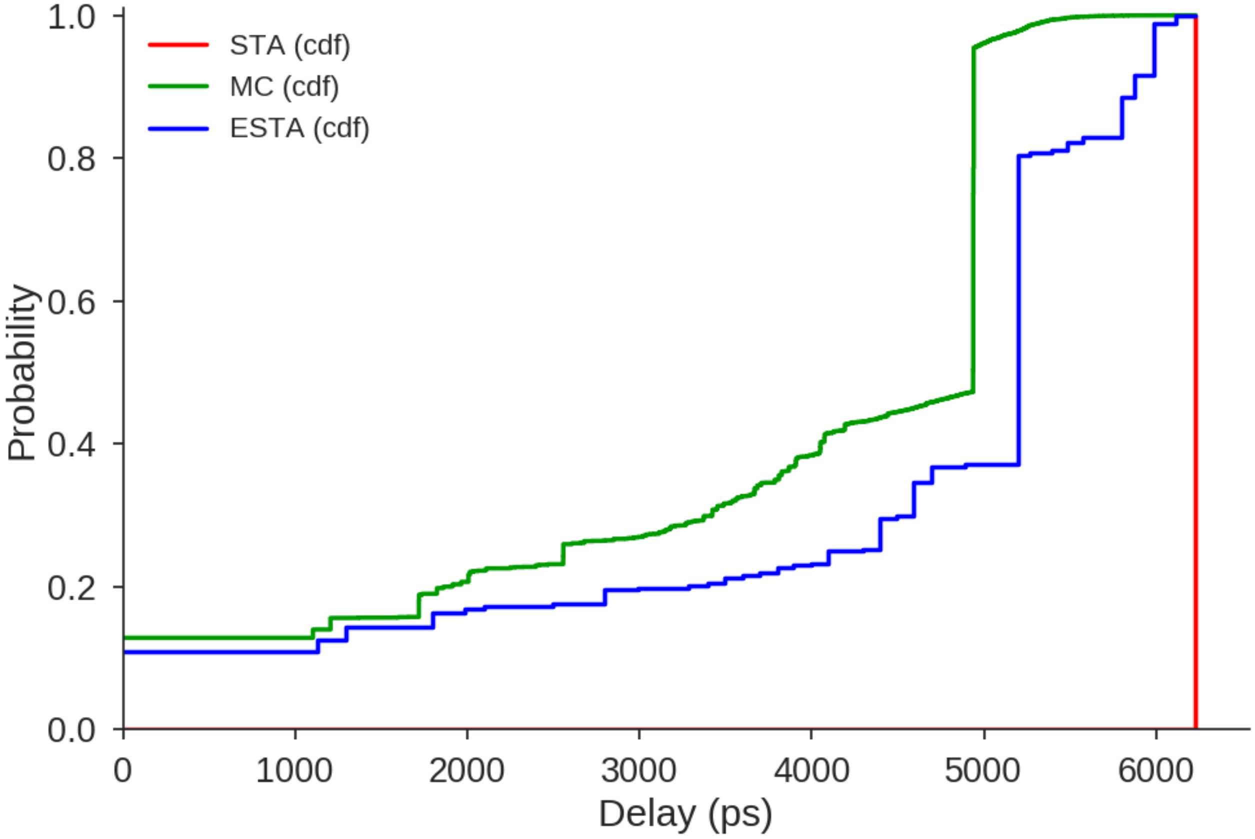
- Simulate primary input transitions
- Determine when primary outputs have stabilized



- Exhaustive Timing Simulation:
  - Test all possible input transitions
  - Impractical!
    - Input space too large

- Monte-Carlo Simulation:
  - Randomly sample input space
  - Detect when maximum delay has converged
  - Assumed ground truth

Experimental Results: Accuracy on SPLA



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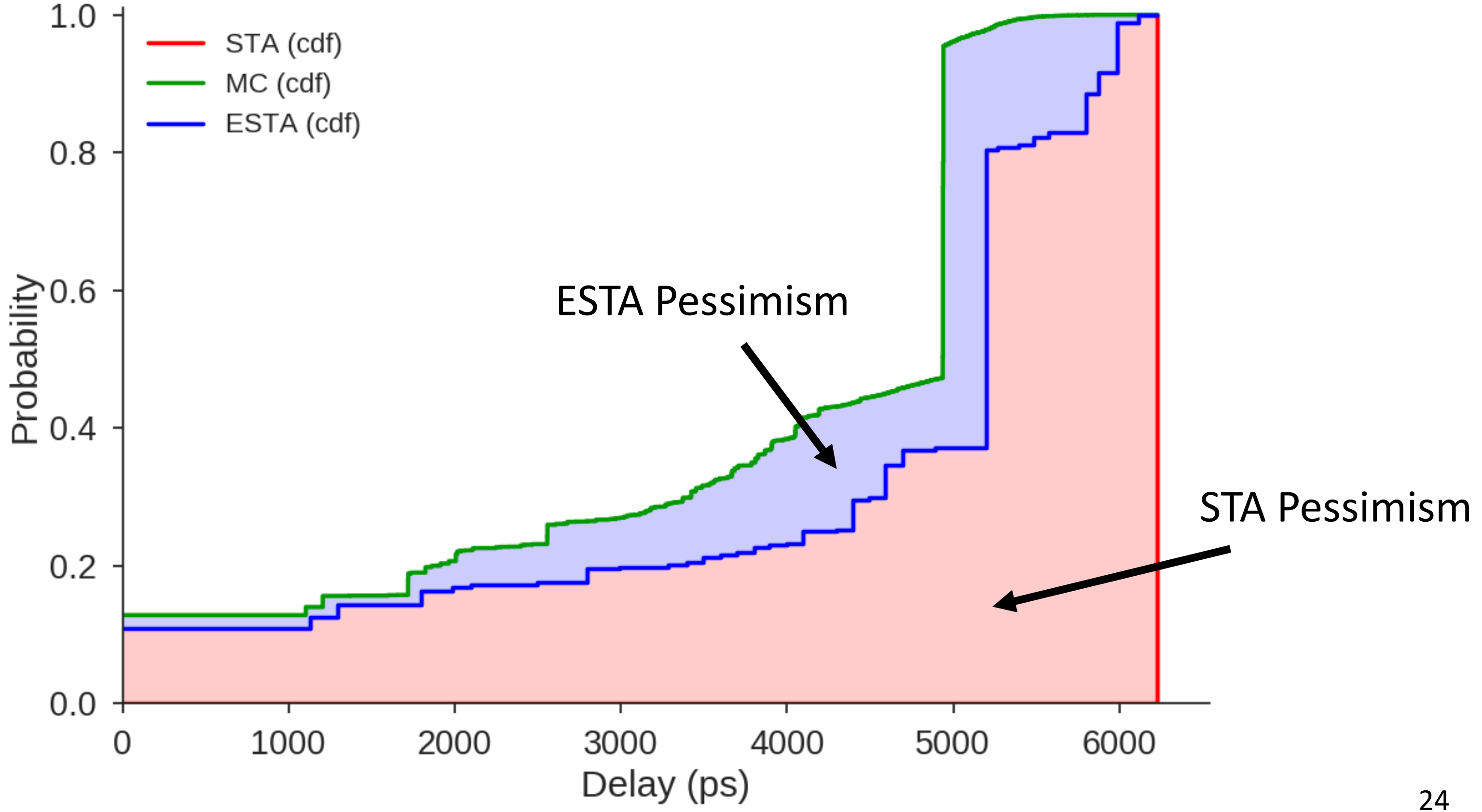
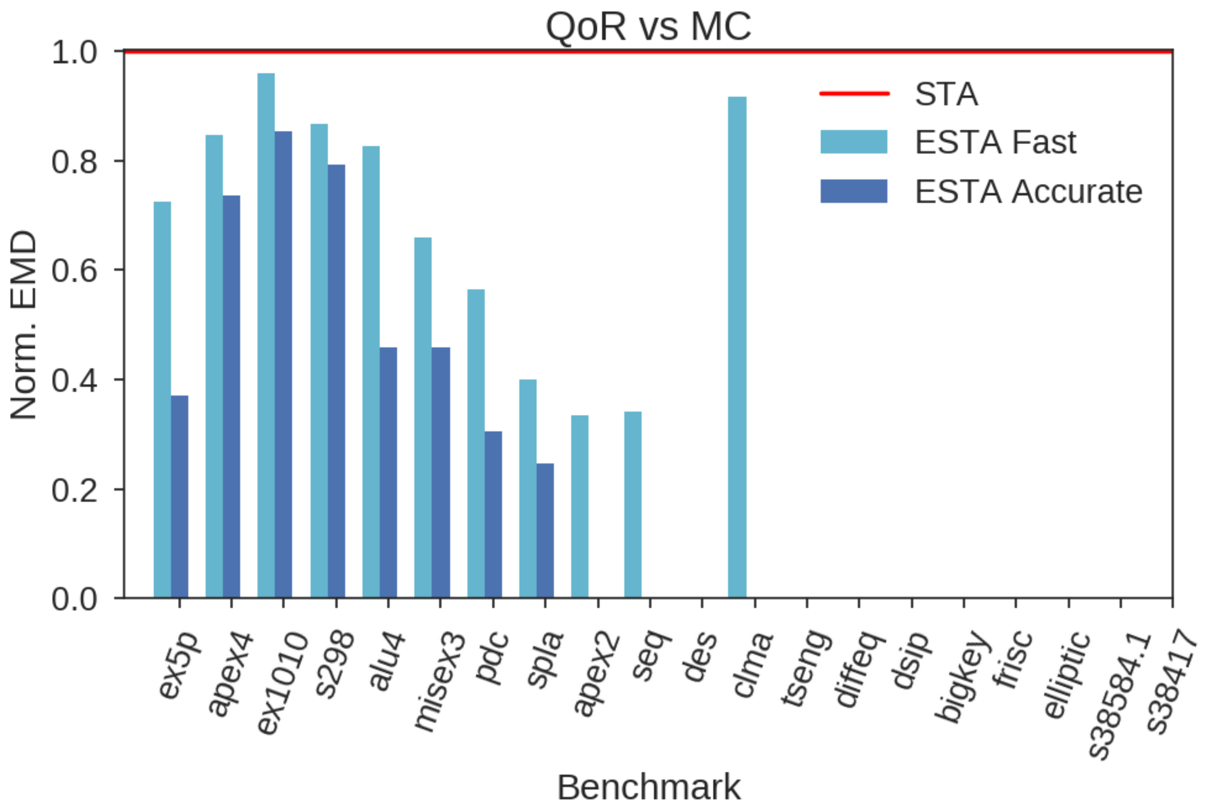
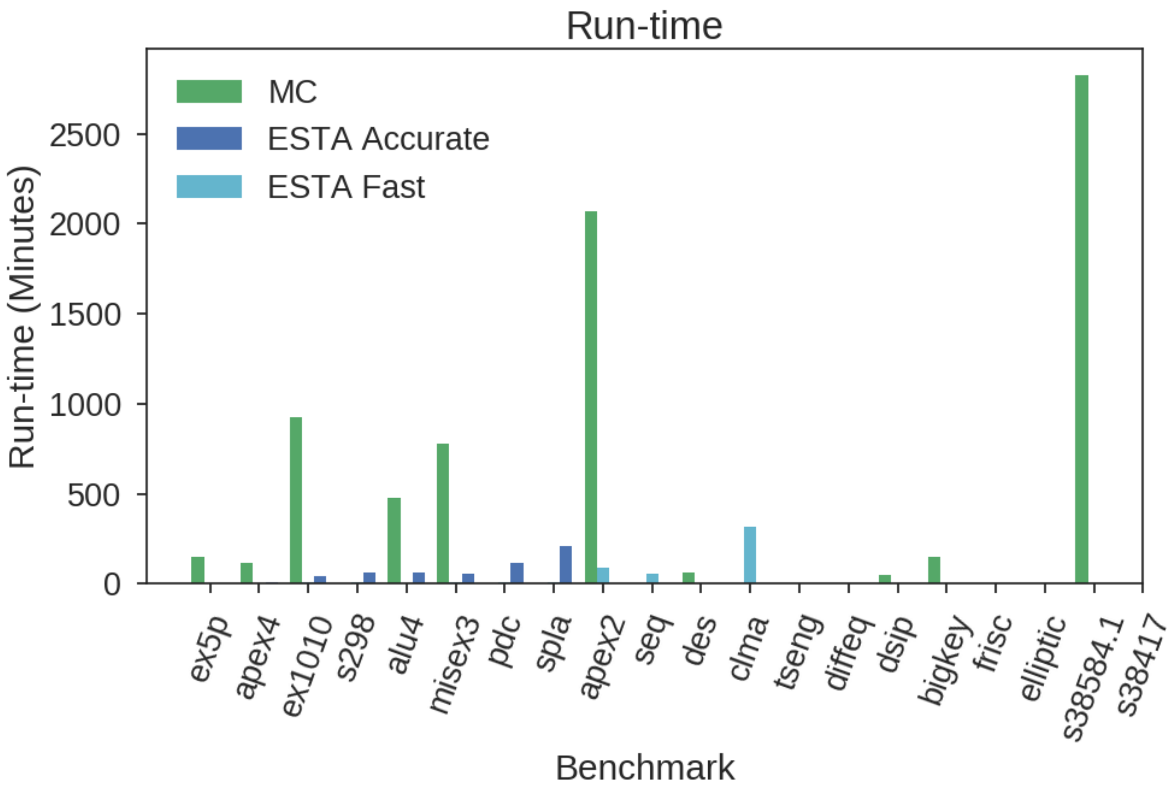


Experimental Results: Quality



- **ESTA:**
  - **46% to 32% less pessimistic than STA**

Experimental Results: Run-time



- **48 hour completion:**
  - **MC: 10**
  - **ESTA: 11**
- **ESTA Speed-up:**
  - **14.6x to 44.0x**

Technique Comparison

	MC	ESTA	STA
Scalable			
Safe Analysis			
Overly Pessimistic			
False Paths			
Statistical Delays		*	

* Future work

Conclusion & Future Work

- **Introduced ESTA method:**
  - **Performs timing analysis with non-worst-case switching**
  - **Calculates timing path activation probabilities using #SAT**

- **Described an ESTA implementation:**
  - **46% to 32% less pessimistic than STA**
  - **14.6x to 44.0x faster than MC Timing Simulation**

- **Future work:**
  - **Improve scalability**
  - **More general input distributions**
  - **Feedback analysis results to optimization tools (Place & Route)**
  - **Application studies**

Thanks!

Questions?

Backup

Complexity

- **Exhaustive Simulation:**

$$\boldsymbol{O(n \cdot 4^{I})}$$

- **ESTA:**

$$\boldsymbol{O(n \cdot L^{K} + 4^{I})}$$

Complexity

- **Exhaustive Simulation:**

$$\boldsymbol{O}(\underbrace{\boldsymbol{n}}_{\text{Circuit Size}} \cdot \underbrace{\boldsymbol{4}^{\overbrace{\boldsymbol{I}}^{\text{\# of Primary Inputs}}}}_{\text{\# of cases}})$$

- **ESTA:**

$$\boldsymbol{O}(\boldsymbol{n} \cdot \boldsymbol{L}^{\boldsymbol{K}} + \boldsymbol{4}^{\boldsymbol{I}})$$

Complexity

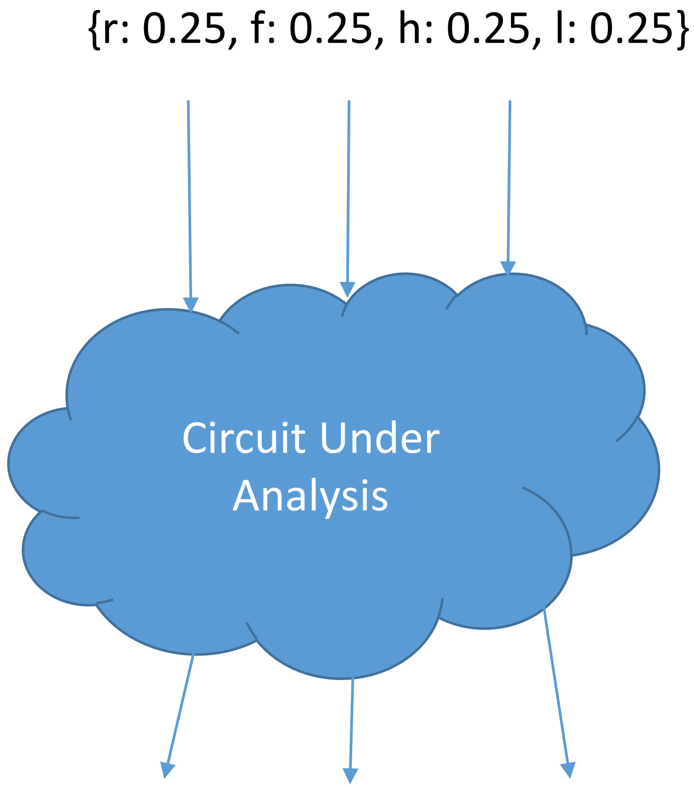
- **Exhaustive Simulation:**

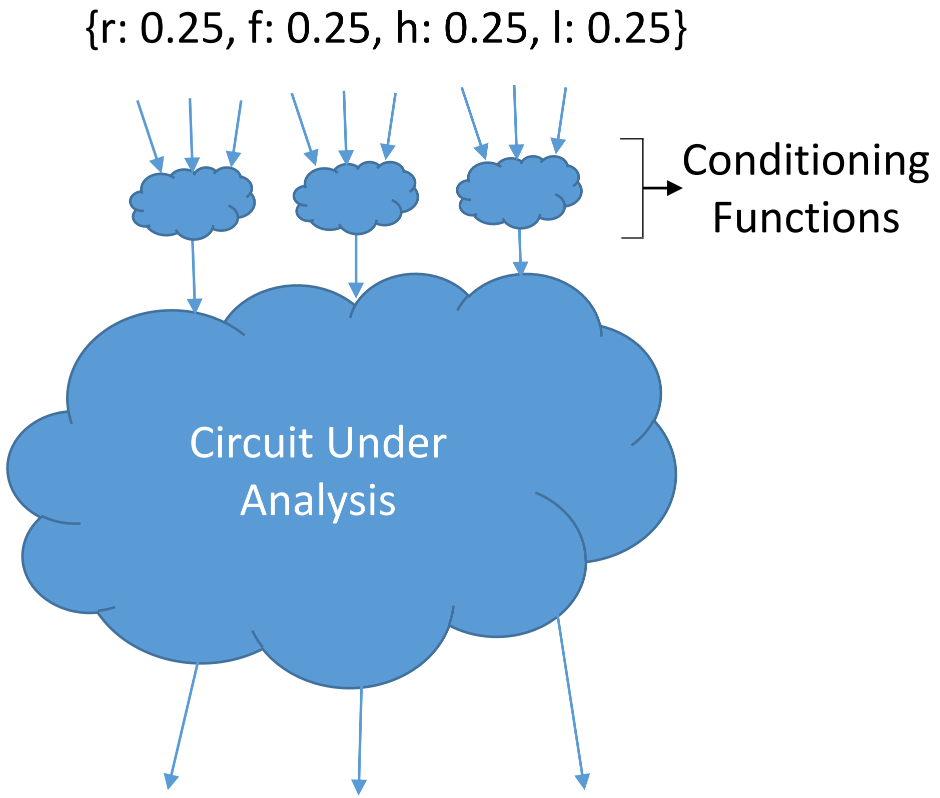
$$\boldsymbol{O}(\underbrace{\boldsymbol{n}}_{\text{Circuit Size}} \cdot \underbrace{\boldsymbol{4}^{\overbrace{\boldsymbol{I}}^{\text{\# of Primary Inputs}}}}_{\text{\# of cases}})$$

- **ESTA:**

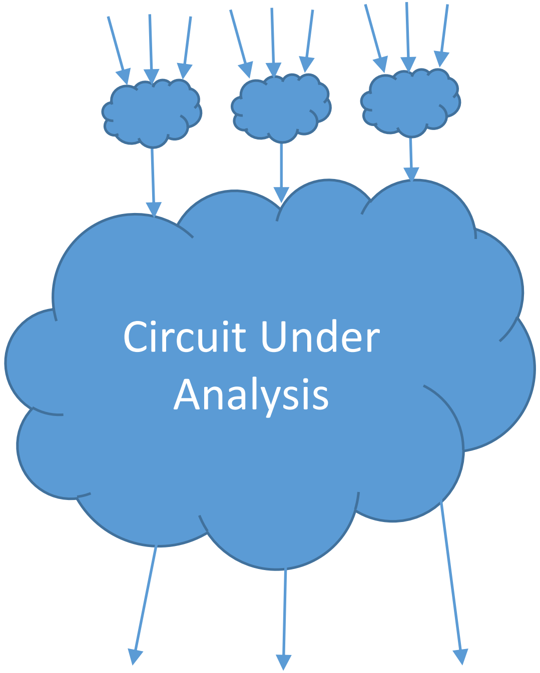
$$\boldsymbol{O}(\underbrace{\boldsymbol{n} \cdot \boldsymbol{L}^{\boldsymbol{K}}}_{\text{Graph Traversal}} + \underbrace{\boldsymbol{4}^{\boldsymbol{I}}}_{\text{\#SAT}})$$

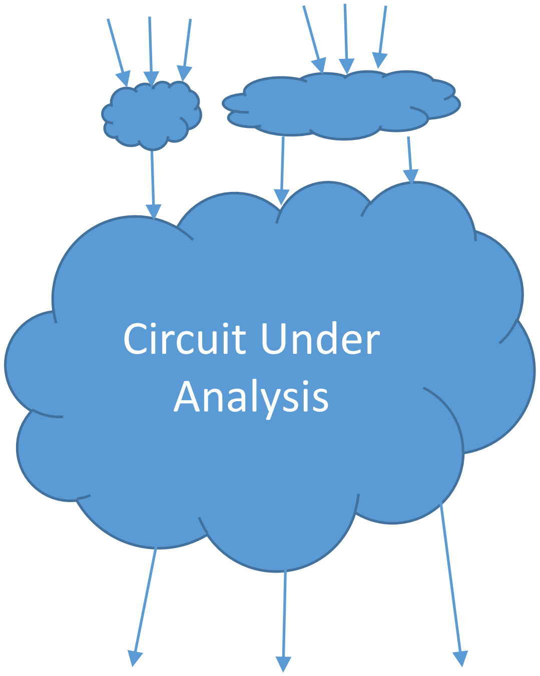
Non-uniform Input Probabilities





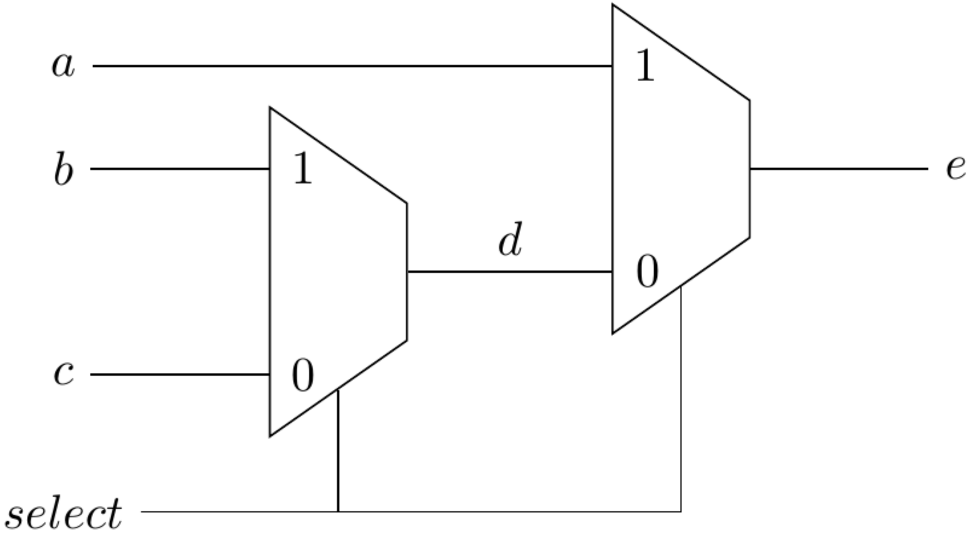
Correlated Input Probabilities





- **Correlation determined by:**
  - **Number of shared variables**
  - **Activation function cover overlaps**

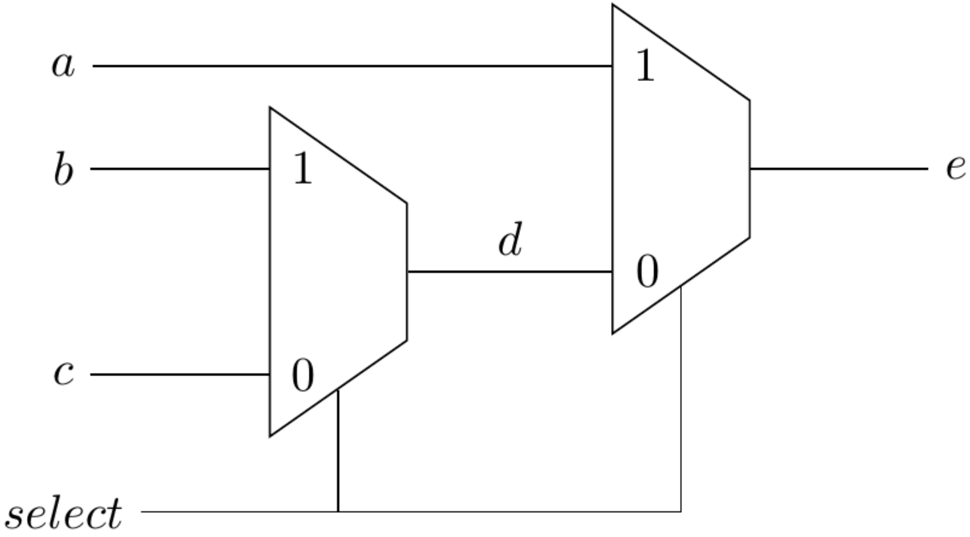
False Paths



	STA	MC	ESTA	
s298	1.000	0.981	0.981	
clma	1.000	0.959	0.984	
frisc	1.000	0.965	0.999	*
elliptic	1.000	0.974		

- ***b-d-e*** **is a false path**

False Paths



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- **MC underestimates clma worst-case delay**