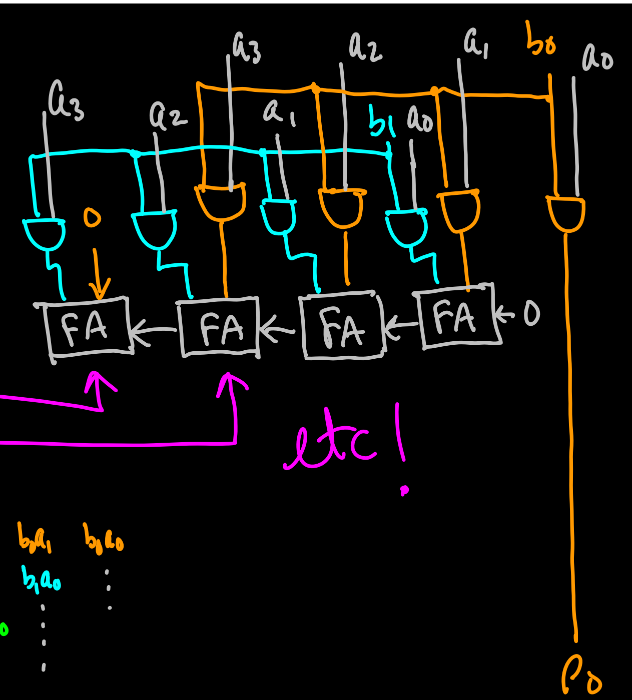
Multiplier Circuit

$$\begin{array}{r} 12 \\ \times\ 11 \\ \hline 12 \\ +\ 12\leftarrow \\ \hline 132 \end{array} \qquad \begin{array}{r} 1100 \\ \times\ 1011 \\ \hline 1100 \\ 1100\leftarrow \\ 0000\leftarrow \\ +\ 1100\leftarrow \\ \hline 10000100 \end{array} \qquad \begin{array}{r} A \\ \times\ B \\ \hline P \end{array}$$

$$\begin{array}{cccccccc} & & & & a_3 & a_2 & a_1 & a_0 \\ & & & \times & b_3 & b_2 & b_1 & b_0 \\ \hline & & & & b_0a_3 & b_0a_2 & b_0a_1 & b_0a_0 \\ & & & b_1a_3 & b_1a_2 & b_1a_1 & b_1a_0 & \vdots \\ & & b_2a_3 & b_2a_2 & b_2a_1 & b_2a_0 & \vdots & \vdots \\ + & b_3a_3 & b_3a_2 & b_3a_1 & b_3a_0 & \vdots & \vdots & \vdots \\ \hline p_7 & p_6 & p_5 & p_4 & p_3 & p_2 & p_1 & p_0 \end{array}$$

– We can implement a circuit using just AND gates & Adders.

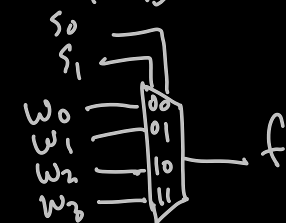


$$\begin{array}{ccccc} & 0 & b_0a_3 & b_0a_2 & b_0a_1 & b_0a_0 \\ & b_1a_3 & b_1a_2 & b_1a_1 & b_1a_0 & \vdots \\ b_2a_3 & b_2a_2 & b_2a_1 & b_2a_0 & \vdots & \\ + b_3a_3 & b_3a_2 & b_3a_1 & b_3a_0 & \vdots & \end{array}$$

Implementing Logic Functions Using Only Muxes

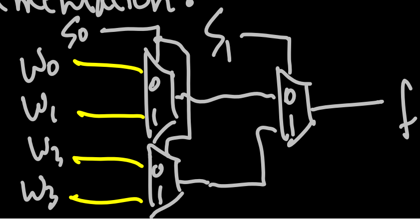
Recall: 2-to-1 mux

4-to-1 mux:

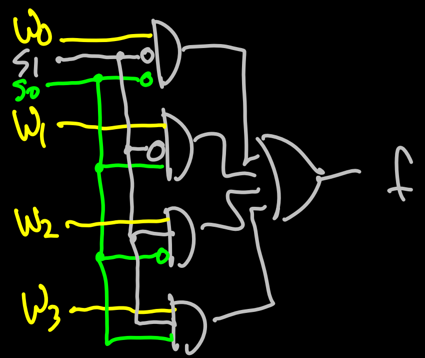


$s_1\ s_0$	f
0 0	w_0
0 1	w_1
1 0	w_2
1 1	w_3

Implementation:

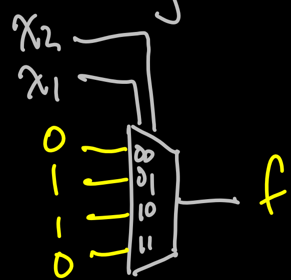


Alternatively:



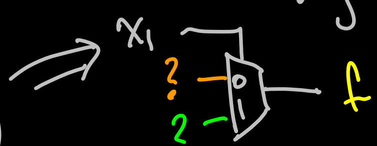
Example: implement f using a 4-to-1 mux

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0



- re-implement f using 2-to-1 muxes.

randomly choose x_1 as the select.

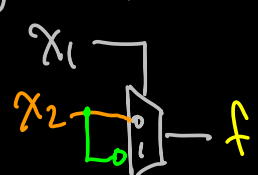


Then, redraw tt:

x_1	f
0	x_2
1	$\overline{x_2}$

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

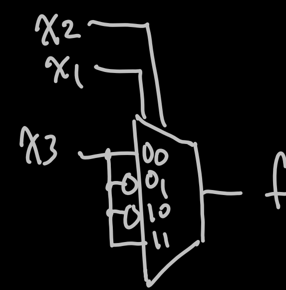
$\Rightarrow$



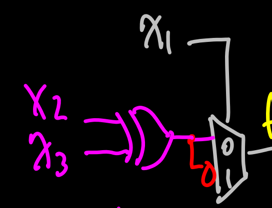
3-Variables

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

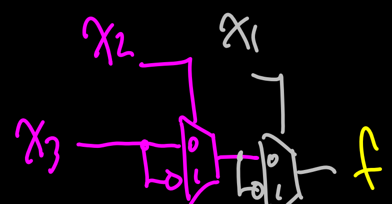
Using 4-to-1



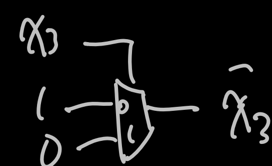
Using 2-to-1



if we need to use only 2-to-1 muxes:



- if we want to eliminate NOT gates:



Summary: any function can be realized using only multiplexers.

Shannon's Theorem

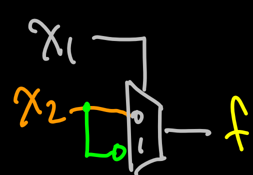
Given a function $f(x_1, x_2, \ldots, x_n)$, the function can be expressed as:

$$f(x_1, x_2, \ldots, x_n) = \overline{x_1} \cdot f(0, x_2, \ldots, x_n) + x_1 \cdot f(1, x_2, \ldots, x_n)$$

Example

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

$\Rightarrow$



$$f(x_1, x_2) = \overline{x_1} \cdot \underbrace{f(0, x_2)}_{x_2} + x_1 \cdot \underbrace{f(1, x_2)}_{\overline{x_2}}$$