

# Laboratory Exercise 6

## Finite State Machines

The purpose of this exercise is to learn how to create and use finite state machines.

### Preparation Before the Lab

You are required to complete Parts I to III of the lab by writing and testing Verilog code and compiling it with Quartus II. Show your schematics, Verilog, and simulations for Parts I to III and state diagrams for Parts II and III to the teaching assistants. You must simulate your circuit with ModelSim using reasonable test vectors.

### In-lab Work

You are required to implement and test all of Parts I to III of the lab. You need to demonstrate all parts to the teaching assistants.

### Part I

We wish to implement a finite state machine (FSM) that recognizes two specific sequences of applied input symbols, namely four consecutive 1s or the sequence 1101. There is an input  $w$  and an output  $z$ . Whenever  $w = 1$  for four consecutive clock pulses, or when the sequence 1101 appears on  $w$  across four consecutive clock pulses, the value of  $z$  has to be 1; otherwise,  $z = 0$ . Overlapping sequences are allowed, so that if  $w = 1$  for five consecutive clock pulses the output  $z$  will be equal to 1 after the fourth and fifth pulses. Figure 1 illustrates the required relationship between  $w$  and  $z$ . A state diagram for this FSM is shown in Figure 2.

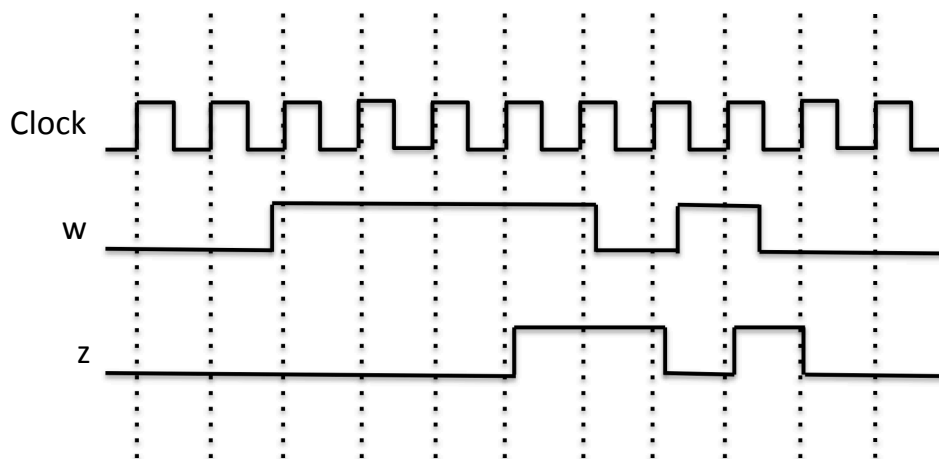


Figure 1: Required timing for the output  $z$ .

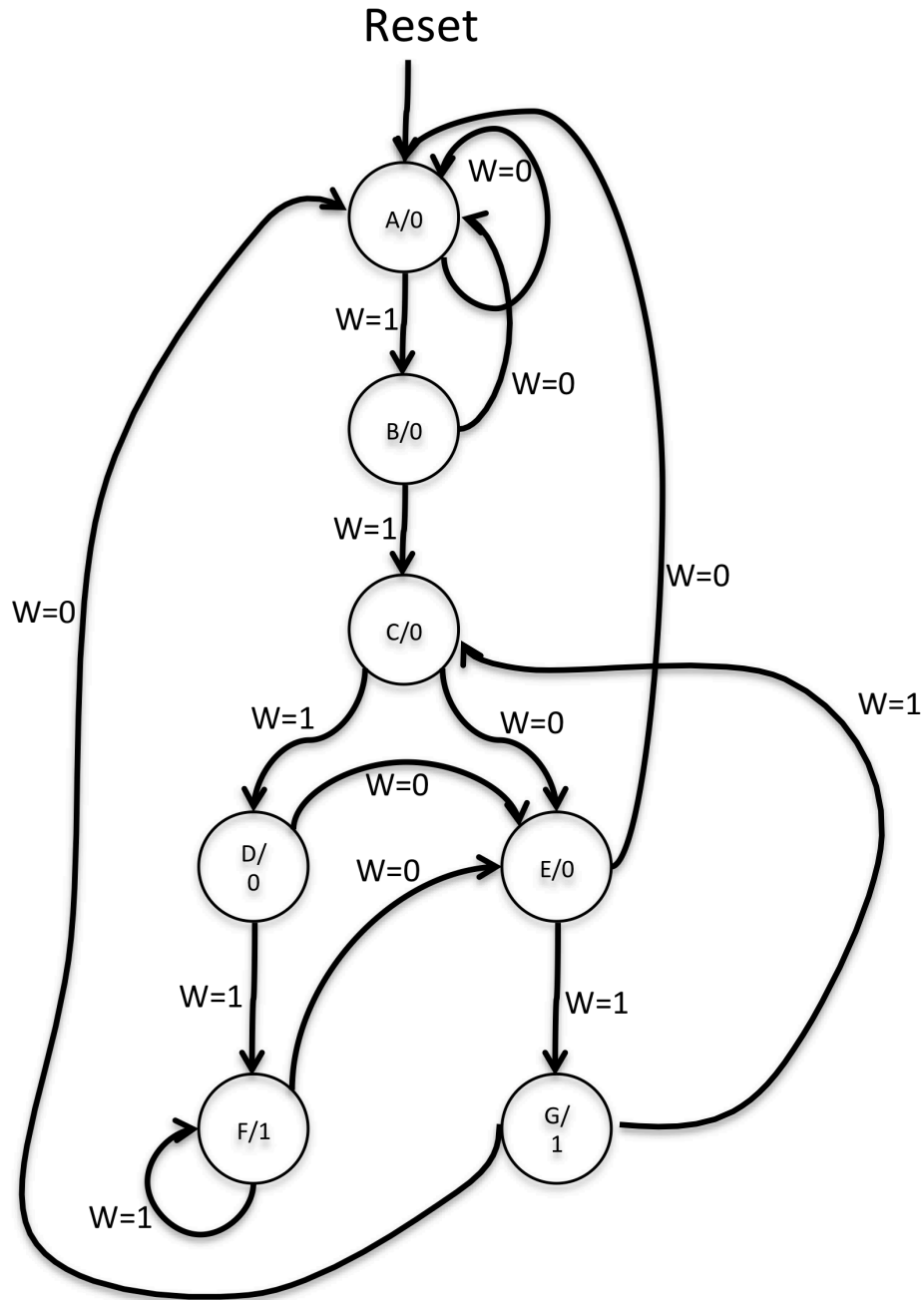


Figure 2: A state diagram for the FSM.

Figure 3 shows a partial Verilog file for the required state machine. Study and understand this code as it provides a model for how to clearly describe a finite state machine that will both simulate and synthesize properly.

The toggle switch  $SW_0$  on the DE1-SoC board is an active-low synchronous reset input for the FSM,  $SW_1$  is the  $w$  input, and the pushbutton  $KEY_0$  is the clock input that is applied manually. The LED  $LEDR_9$  is the output  $z$ , and the state flip-flop outputs are assigned to  $LEDR_{3-0}$ .

```

//SW[0] reset when 0
//SW[1] input signal

//KEY[0] clock signal

//LEDR[3:0] displays current state
//LEDR[9] displays output

module sequence_detector(SW, KEY, LEDR);
    input [9:0] SW;
    input [3:0] KEY;
    output [9:0] LEDR;

    wire w, clock, reset_b;

    reg [3:0] y_Q, Y_D; // y_Q represents current state, Y_D represents next state
    wire out_light;

    parameter A = 4'b0000, B = 4'b0001, C = 4'b0010, D = 4'b0011, E = 4'b0100, F = 4'b0101, G = 4'b0110;

    assign w = SW[1];
    assign clock = ~KEY[0];
    assign reset_b = SW[0];

    // State table
    // The state table should only contain the logic for state transitions
    // Do not mix in any output logic. The output logic should be handled separately.
    // This will make it easier to read, modify and debug the code.

    always @(*)
        begin: state_table
            case (y_Q)
                A: begin
                    if (!w) Y_D = A;
                    else Y_D = B;
                end
                B: begin
                    if (!w) Y_D = A;
                    else Y_D = C;
                end
                C: ???
                D: ???
                E: ???
                F: ???
                G: ???
                default: Y_D = A;
            endcase
        end // state_table

    // State Registers

    always @(posedge clock)
        begin: state_FFS
            if(reset_b == 1'b0)
                y_Q <= 4'b0000;
            else
                y_Q <= Y_D;
        end // state_FFS

    // Output logic
    // Set out_light to 1 to turn on LED when in relevant states

    assign out_light = ((y_Q == ???) | (y_Q == ???));

    // Connect to I/O

    assign LEDR[9] = out_light;
    assign LEDR[3:0] = y_Q;
endmodule

```

Figure 3: Verilog code for the FSM.

Perform the following steps:

1. Copy the code into a file and name it `sequence_detector.v`. It is available online as `sequence_detector-partial.v`.
2. Complete the state table and the output logic.
3. Draw a schematic describing the circuit.
4. Simulate your circuit with ModelSim for a variety of input settings, ensuring the output waveforms are correct.
5. Compile the project.
6. Download the compiled circuit into the FPGA. Test the functionality of the circuit on your board.

## Part II

Most non-trivial digital circuits can be separated into two main functions. One is the *datapath* where the data flows and the other is the *control path* that manipulates the signals in the datapath to control the operations performed and how the data flows through the datapath. In previous labs, you learned how to construct a simple ALU, which is a common datapath component. In Part I of this lab you have already constructed a simple *finite state machine* (FSM), which is the most common component used to implement a control path. Now you will see how to implement an FSM to control a datapath so that a useful operation is performed. This is an important step towards building a microprocessor as well as any other computing circuit.

In this part, you will be given a block diagram of a datapath. You are to implement the datapath and an FSM that performs the quadratic function:

$$Ax^2 + Bx + c$$

using the given datapath. The values of  $x$ ,  $A$ ,  $B$  and  $C$  will be preloaded before the computation begins.

Figure 4 shows the block diagram of the datapath you will build. Resets are not shown, but do not forget them. The datapath will carry 8-bit unsigned values. Assume that the input values are small enough to not cause any overflows at any point in the computation, i.e., no results will exceed  $2^8 - 1 = 255$ . The ALU needs only to perform addition and multiplication, but you could use a variation of the ALU you built previously to have more operations available for solving other equations if you wish to try some things on your own. There are four registers  $R_x$ ,  $R_A$ ,  $R_B$  and  $R_C$  used at the start to store the values of  $x$ ,  $A$ ,  $B$  and  $C$ , respectively. The registers  $R_A$  and  $R_B$  can be overwritten during the computation. There is one output register,  $R_{OUT}$ , that captures the output of the ALU and displays the value in binary on the LEDs and in hex on the Hex displays. Two 4-to-1 multiplexers at the inputs to the ALU,  $MuxA$  and  $MuxB$ , are used to select which register values are input to the ALU.

All registers have enable signals to determine when they are to load new values and an active high synchronous reset.

The circuit operates in the following manner. After an active high synchronous *Reset* on  $KEY_0$ , you will preload registers  $R_x$  and  $R_A$  when  $KEY_1$  is pushed and then registers  $R_B$  and  $R_C$  are loaded when  $KEY_2$  is pushed. After  $KEY_2$  is released, the computation is performed and the circuit returns to wait for new values to be loaded. The final result should be loaded into  $R_{OUT}$  for display.

Use  $SW_{3-0}$  for the initial values of  $x$  and  $B$  and  $SW_{7-4}$  for the initial values of  $A$  and  $C$ . Note that the registers are 8-bits and you only have four switches to specify the value. You should load the upper nybble (upper four bits) with 0's, i.e., you can only initialize with values in the range 0 to 15.

The final result is displayed on  $LEDR_{7-0}$  in binary and  $HEX0$  and  $HEX1$  in hex.

You will use  $CLOCK_{50}$  as your clock.

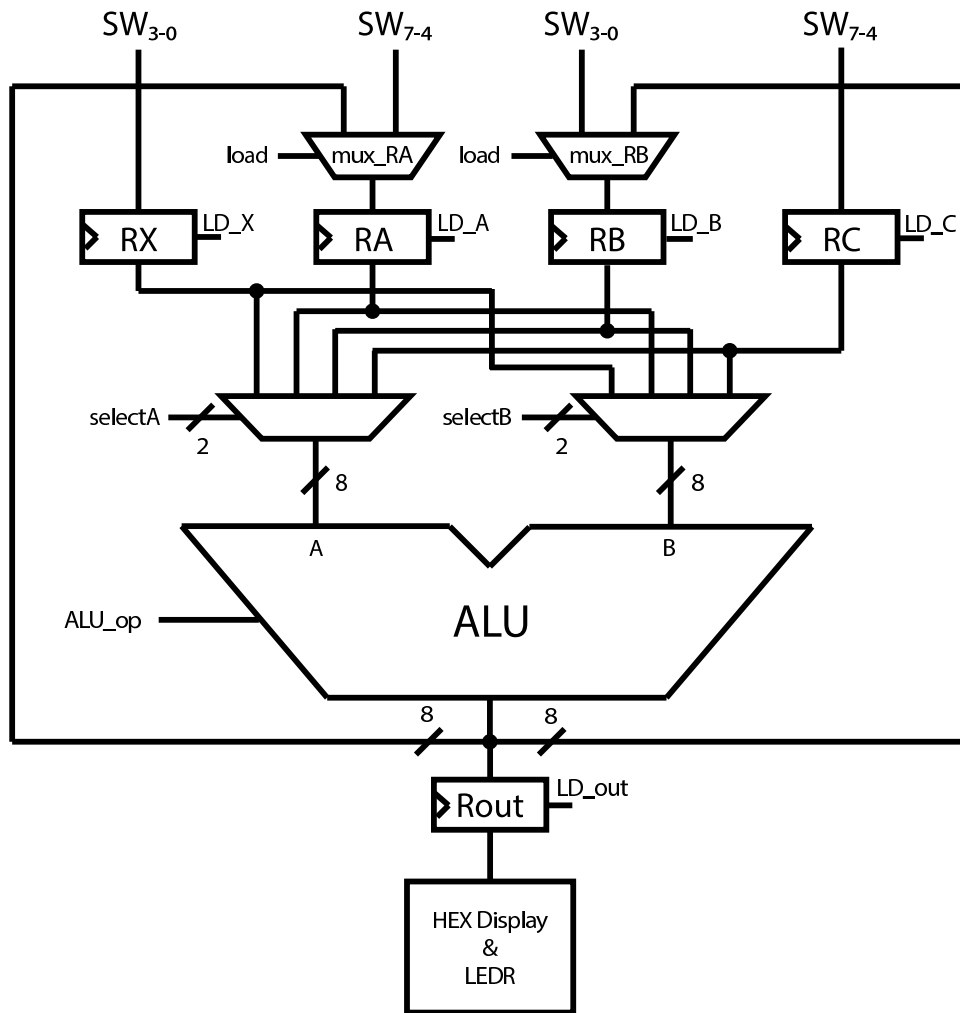


Figure 4: Block diagram of datapath.

## Structuring Your Code

Within your top-level module you will have two separate modules for the datapath and the control path. The main connections between the datapath and control path modules will be the control signals coming from the control path and entering the datapath. You will also instantiate your hex decoders at this level.

From Figure 4 you can see that there are five registers that are similar in structure. Write a generic register module that you can instantiate with the appropriate inputs and outputs for the five registers. Also write modules for the 2-to-1 and 4-to-1 multiplexers since you need more than one of them as well.

Build your datapath module by instantiating and connecting the registers and multiplexers according to Figure 4. You can write your ALU as an always block, or also instantiate a module, such as the one you built in a previous lab. You may use the add and multiply operator symbols, i.e., you do not have to build your own operator logic.

The control path module should use a structure similar to what you were provided in Figure 3 for Part I. The main difference will be for the *Output logic* because there are many more output signals. You can use a style similar to Figure 3 by creating a number of `assign` statements. Figure 5 shows a code fragment for the controller output logic using `assign` statements.

```
// Alternate code for output logic
// It's ugly, so won't do too much here.

assign LD_A = ((PresentState == AX_S) | (PresentState == AXX_S) | ...);
assign selectA = (PresentState == AX_S) ? RA :
                 ((PresentState == AXX_S) ? RA :
                  ...
                  RX);
```

Figure 5: Verilog code fragments for the controller using `assign` statements

A better approach in this design is to use a second case statement that has the same states as the case statement for the *State table*, but assign the outputs instead of the next states. The example code fragment shown in Figure 6 computes  $Ax^2$  in two cycles of the state machine. In this style, it is easy to see what the control signals are set to in each state.

You could be lazy and combine the state table and the output logic case statements. However, this makes the code more difficult to read, and worse, it makes it much more difficult for the synthesis tool to figure out what you want to do. This may lead to unexpected interpretation of your code by the tool and bugs that can be hard to find. The result is that being lazy often results in a lot more work and frustration as you try to debug your code, thus defeating the goal of being lazy in the first place! The choice is yours...

```

// Some of the states:
// RESET_S : reset state
// AX_S : A * X
// AXX_S : (A * X) * X
// BX_S : B * X
//
// Some of the control signals as labeled on Figure 4
// LD_A : load RA
// LD_B : load RB
// load : select for Mux_RA and Mux_RB
// selectA : MuxA select
// selectB : MuxB select
// ALU_OP : select ALU operation

// Part of the State table

always @(*)
begin: state_table
    case (PresentState)
        ...
        AX_S: // Compute A * X
            NextState = AXX_S;
        AXX_S: // Compute (A * X) * X
            NextState = BX_S;
        ...
        default: NextState = RESET_S;
    endcase
end // state_table

// Part of the Output logic

always @(*)
begin: output_logic
    case (PresentState)
        // In each state assign a value to all control signals

        ...

        AX_S: // Compute A * X and store in A
            begin
                LD_A = 1; // Store result in RA
                LD_B = 0;
                load = 0; // Select ALU output
                selectA = RA; // ALU A input gets RA
                selectB = RX; // ALU B input gets RX
                ALU_OP = MULT;
                // Set values for rest of control signals
            end
        AXX_S: // Compute (A * X) * X and store in A
            begin
                LD_A = 1; // Store result in RA
                LD_B = 0;
                load = 0; // Select ALU output
                selectA = RA; // RA now has A * X
                selectB = RX;
                ALU_OP = MULT;
                // Set values for rest of control signals
            end
        ...

        default:
            begin
                LD_A = 0;
                LD_B = 0;
                load = 0;
                selectA = RX;
                selectB = RX;
                ALU_OP = ADD;
                // Set values for rest of control signals
            end
    endcase
end // output_logic

```

Figure 6: Verilog code fragments for the controller

Perform the following steps.

1. Draw a state diagram for your controller.
2. Draw a schematic of your circuit. Here you may find that a hierarchical schematic will make things easier. You can draw separate schematics for the datapath and the control path. Figure 4 is almost all that you need for the datapath, but you should add other labels like the wires and show the HEX displays and LEDs in more detail. You can make another schematic representing the controller. Clearly label the inputs and outputs of these schematics. For the complete design, draw a top-level schematic that incorporates two boxes representing the datapath and controllers and show how their inputs and outputs are connected in your top-level module. Do not forget to show the inputs and outputs to your top-level module.
3. Write a Verilog file that realizes the required circuit and synthesize it.
4. To examine the circuit produced by Quartus II open the RTL Viewer tool (Tools > Netlist Viewers > RTL Viewer). Find (on the left panel) and double-click on the box shown in the circuit that represents the finite state machine, and determine whether the state diagram that it shows properly corresponds to the one you have drawn. To see the state codes used for your FSM, open the Compilation Report, select the **Analysis and Synthesis** section of the report, and click on **State Machines**.

The state codes after synthesis may be different from what you originally specified. This is because the tool may have found a way to optimize the logic better by choosing a different state assignment. If you really need to use your original state assignment, there is a setting to keep it.

5. Simulate your circuit with ModelSim for a variety of input settings, ensuring the output waveforms are correct. It is recommended that you start by simulating the datapath and controller modules separately. Only when you are satisfied that they are working individually should you combine them into the full design. Why is this approach better? (Hint: Consider the case when your design has 20 different modules.)
6. After you are satisfied with your simulations, download and test the functionality of the circuit on the FPGA board.



## Part III

Division in hardware is the most complex of the four basic operations. Add, subtract and multiply are much easier to build in hardware. For this part, you will be designing a 4-bit restoring divider using a finite state machine.

Figure 7 shows an example of how the restoring divider works. This mimics what you do when you do long division by hand. The restoring divider starts with *Register A* set to 0. The *Dividend* is shifted left and the bit shifted out of the left most bit of the *Dividend* (called the most significant bit or MSB) is shifted into the least significant bit (LSB) of *Register A* as shown in Figure 8.

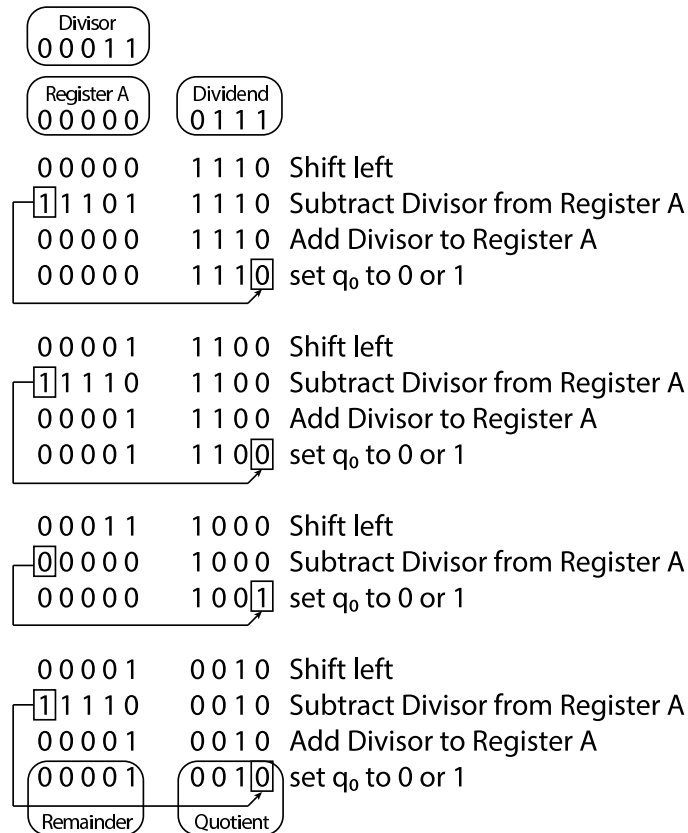


Figure 7: An example showing how the restoring divider works.

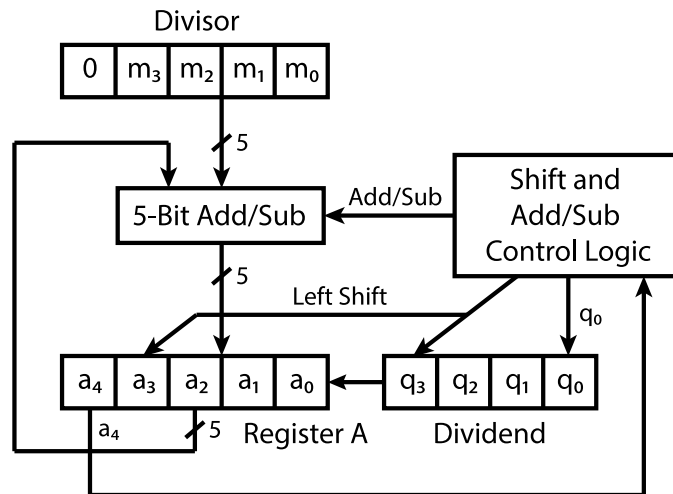


Figure 8: Block diagram of restoring divider.

The *Divisor* is then subtracted from *Register A*. If the MSB of *Register A* is a 1, then we restore *Register A* back to its original value by adding the *Divisor* back to *Register A*, and set the LSB of the *Dividend* to 0. Else, we do not perform the restoring addition and immediately set the LSB of the *Dividend* to 1.

This cycle is performed until all the bits of the *Dividend* have been shifted out. Once the process is complete, the new value of in the *Dividend* register is the *Quotient*, and *Register A* will hold the value of the *Remainder*.

To implement this part, you will use  $SW_{3-0}$  for the divisor value and  $SW_{7-4}$  for the dividend value. Use  $CLOCK_{50}$  to for the clock signal,  $KEY_0$  as a synchronous active high reset, and  $KEY_1$  as the *Go* signal to start computation. The output of the *Divisor* will be displayed on  $HEX0$ , the *Dividend* will be displayed on  $HEX2$ , the *Quotient* on  $HEX4$ , and the *Remainder* on  $HEX5$ . Set the remaining HEX displays to 0. Also display the *Quotient* on  $LEDR$ .

Structure your code in the same way as you were shown in Part II.

Perform the following steps.

1. Draw a schematic for the datapath of your circuit. It will be similar to Figure 8. You should show how you will initialize the registers, where the outputs are taken, and include all the control signals that you require.
2. Draw the state diagram to control your datapath.
3. Draw the schematic for your controller module.
4. Draw the top-level schematic showing how the datapath and controller are connected as well as the inputs and outputs to your top-level circuit.
5. Write the Verilog code that realizes your circuit.
6. Simulate your circuit with ModelSim for a variety of input settings, ensuring the output waveforms are correct.
7. After you are satisfied with your simulations, download and test the functionality of the circuit on the FPGA board.