Modeling and Extraction of SiGe HBT Noise Parameters from Measured Y-Parameters and Accounting for Noise Correlation

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***Abstract*— In this paper, we present a set of new equations for the noise parameters of SiGe HBTs and a new methodology to extract the noise parameters from the y-parameters of the transistors without fitting to measured noise data. This method was verified to provide excellent agreement to both simulated and measured noise parameter data. A NF_{MIN} that is approximately 1 dB lower at 60 GHz when noise correlation is account for is predicted using this method.**

***Index Terms*—Noise modeling, noise correlation, millimetre-wave, SiGe HBTs.**

I. Introduction

Recent publications on 60-GHz SiGe HBT circuits reported that the measured phase noise of VCOs [1] and noise figure of LNAs [2] are systematically lower than simulated values. At the same time, bipolar transistor models (SGP, HICUM) currently available in simulators do not account for the correlation between the base and collector noise current sources. It is well established that correlation, typically captured in the noise transit time [3], can be ignored at lower frequencies with minimal impact on noise model accuracy. However, as the operation frequency of the transistor increases, neglecting it leads to an overestimation of transistor noise figure [4]. Expanding the y-parameter based noise equations in [5] to account for correlation, this paper describes a method to extract all noise parameters of millimetre-wave SiGe HBTs from measured S-parameters, without fitting to measured noise parameters.

II. Theory

Expression of SiGe HBT common-emitter noise parameters are derived as functions of the two-port y-parameters based on the noise equivalent circuit illustrated in Fig. 1. Instead of using an explicit equivalent circuit, the intrinsic transistor is modeled by its y-parameters as a black box. This approach allows the results to be applied to other devices, for instance MOSFETs, whose noise can be adequately modeled by this equivalent circuit.

The thermal noise contributions from the emitter and base series resistances are modeled by lumped elements R_E and R_B respectively. R_B includes the contribution from both the intrinsic and extrinsic base resistances, i.e.

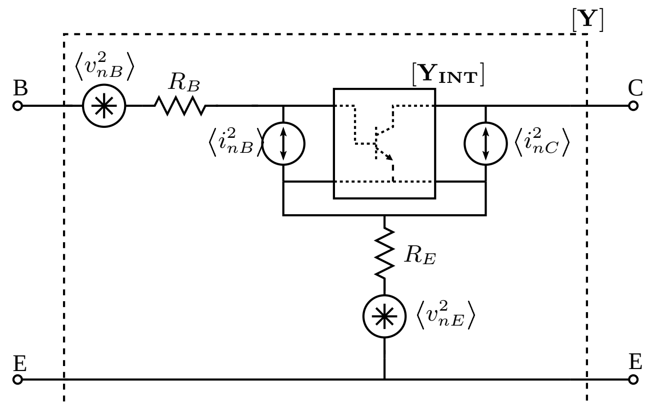

Fig. 1. SiGe HBT noise equivalent circuit defining the intrinsic and extrinsic y-parameters

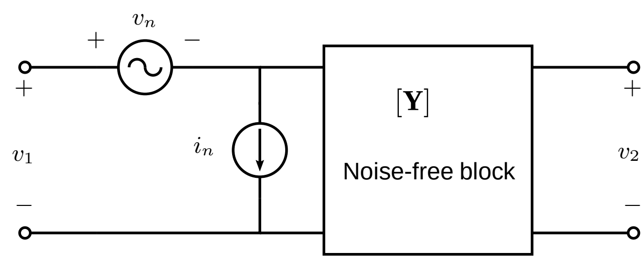

Fig. 2. Chain matrix representation of a linear noisy two-port network.

$R_B = R_{BI} + R_{BX}$. The base and collector shot noises are represented by two noise current sources with power spectral densities given by

$$\langle i_{nB}^2 \rangle = 2qI_B \quad (1)$$

$$\langle i_{nC}^2 \rangle = 2qI_C, \quad (2)$$

where I_B and I_C are the DC base and collector currents, respectively, and q is the positive electron charge. The correlation between the two shot noise sources is given by [3]

$$\langle i_{nB} i_{nC}^* \rangle = 2qI_C \left[\exp\left(j\omega\tau_n\right) - 1\right], \quad (3)$$

where τ_n represents the noise transit time. This modeling parameter describes the delay between the base and collector shot noise currents. Although τ_n is commonly extracted

$$\langle v_n^2\rangle = \langle v_{nB}^2\rangle + \left|1+\frac{y_{22}}{y_{21}}\right|^2\langle v_{nE}^2\rangle + \left|R_B + R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right|^2\langle i_{nB}^2\rangle + \left|y_{21}^{-1} - R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right|^2\langle i_{nC}^2\rangle$$
$$- \frac{2}{|y_{21}|^2}\Re\left\{(y_{21}R_B + R_E[y_{21}+y_{22}])\times(1-R_E[y_{21}+y_{22}])^*\langle i_{nB}i_{nC}^*\rangle\right\} \qquad (4)$$

$$\langle i_n^2\rangle = \left|1+\frac{R_E\det[\mathbf{Y}]}{y_{21}}\right|^2\langle i_{nB}^2\rangle + \left|\frac{y_{11}-R_E\det[\mathbf{Y}]}{y_{21}}\right|^2\langle i_{nC}^2\rangle + \left|\frac{\det[\mathbf{Y}]}{y_{21}}\right|^2\langle v_{nE}^2\rangle$$
$$- \frac{2}{|y_{21}|^2}\Re\left\{(y_{21}+R_E\det[\mathbf{Y}])\times(y_{11}-R_E\det[\mathbf{Y}])^*\langle i_{nB}i_{nC}^*\rangle\right\} \qquad (5)$$

$$\langle v_n^* i_n\rangle = \frac{\det[\mathbf{Y}]}{y_{21}}\left(1+\frac{y_{22}}{y_{21}}\right)^*\langle v_{nE}^2\rangle - \left(\frac{R_E\det[\mathbf{Y}]}{y_{21}}+1\right)\left(y_{21}^{-1} - R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right)^*\langle i_{nB}i_{nC}^*\rangle$$
$$- \left(\frac{y_{11}-R_E\det[\mathbf{Y}]}{y_{21}}\right)\left(R_B + R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right)^*\langle i_{nB}^* i_{nC}\rangle + \left(\frac{R_E\det[\mathbf{Y}]}{y_{21}}+1\right)\left(R_B + R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right)^*\langle i_{nB}^2\rangle$$
$$+ \left(\frac{y_{11}-R_E\det[\mathbf{Y}]}{y_{21}}\right)\left(y_{21}^{-1} - R_E\left[1+\frac{y_{22}}{y_{21}}\right]\right)^*\langle i_{nC}^2\rangle \qquad (6)$$

by fitting to measured noise data, as in [4], [6], it is related to and hence can be extracted from the high frequency transconductance of the device [7].

Following a lengthy derivation to properly transfer all the internal noise sources to the input of the transistor, the expressions of the power spectral densities of the input referred noise voltage and noise current as defined in Fig. 2 are obtained as equations (4)-(6), where y_{ij} describe the y-parameters of the extrinsic transistor, including the effects of R_B and R_E. The noise parameters are then obtained using [8], [9]

$$R_n = \frac{\langle v_n^2\rangle}{4k_BT} \qquad (4)$$

$$Y_{OPT} = \sqrt{\frac{\langle i_n^2\rangle}{\langle v_n^2\rangle} - \left[\Im\left(\frac{\langle v_n i_n^*\rangle}{\langle v_n^2\rangle}\right)\right]^2} + j\Im\left(\frac{\langle v_n i_n^*\rangle}{\langle v_n^2\rangle}\right) \qquad (5)$$

$$F_{MIN} = 1 + \frac{\langle v_n i_n^*\rangle + \langle v_n^2\rangle Y_{OPT}^*}{2k_BT}, \qquad (6)$$

where R_n is the equivalent noise resistance, Y_{OPT} is the optimum source impedance, F_{MIN} is the minimum noise factor, k_B is the Boltzmann constant and T is the absolute temperature in kelvin. Note that in (4)-(6), R_B and R_E appear both explicitly and implicitly through the expressions of y_{ij}. The latter important aspect might be missed, as in [10].

III. Simulation Results

Equations (4)-(6) were first verified by device simulation on a two-dimensional SiGe HBT structure. The equations were applied to simulated y-parameters to obtain the noise parameters of the device. The values calculated from the equations were compared to those calculated directly by the device simulator, which employs the impedance field method [11].

The two-dimensional SiGe HBT structure was constructed using Silvaco's TCAD process simulator, Athena. The emitter length was assumed to be 1μm. Since doping profiles and detailed device geometry are confidential and hence unavailable, the simulated device does not correspond exactly to the fabricated devices that will be discussed in section IV. Instead, both devices have similar f_T and f_{MAX} values of approximately 160 GHz.

The y-parameters and noise parameters of this SiGe HBT device were simulated up to 100 GHz using the device simulator, Atlas. The simulation employed Fermi-Dirac statistics and drift-diffusion equations and accounted for band gap narrowing and self-heating effects.

The emitter and extrinsic base series resistances were extracted from low frequency $\Re\{z_{12}\}$ and high frequency $\Re\{z_{11}-z_{12}\}$ characteristics, respectively, as in [5]. The intrinsic base resistance was extracted using the modified impedance circle method [12]. Finally, the noise transit time τ_n was extracted at the peak f_{MAX} bias in the high frequency domain where the phase of $g_m(\omega)$ is linear as

$$\tau_n = -\frac{\partial}{\partial\omega}\text{phase}[g_m(\omega)] = -\frac{\partial}{\partial\omega}\left(y_{21}^{\text{INT}} - y_{12}^{\text{INT}}\right), \qquad (7)$$

where y_{ij}^{INT} are the intrinsic y-parameters of the transistor (Fig. 3). They are obtained from the extrinsic y-parameters as

$$\mathbf{Y}_{\text{INT}} = \frac{1}{D}\left(\mathbf{Y} - \det[\mathbf{Y}]\begin{bmatrix} R_E + R_C & -R_E \\ -R_E & R_B + R_E\end{bmatrix}\right), \qquad (8)$$

where $D = 1 - y_{11}R_B - y_{22}R_C - R_E\sum_{ij} y_{ij} + (R_BR_C + R_BR_E + R_ER_C)\det[\mathbf{Y}]$.

Figs. 4-5 compare the noise parameters of the SiGe HBT calculated by the device simulator with those obtained from equations (4)-(6). The noise parameters in the absence of shot noise correlation were obtained from the same equations by setting τ_n to zero. Excellent agreement is obtained,

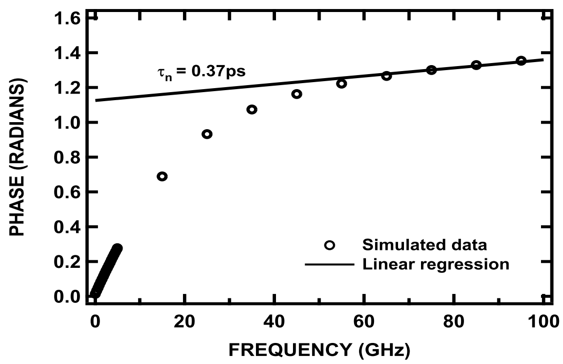

Fig. 3. Extracting noise transit time τ_n from the high frequency transconductance of the simulated SiGe HBT.

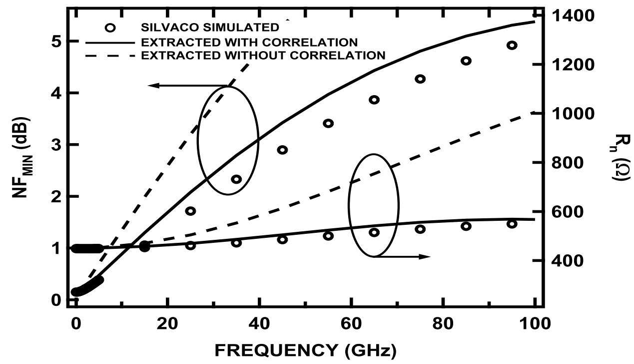

Fig. 4. Simulated and extracted NF_{MIN} and R_n with and without correlation at minimum noise bias. The emitter area is $0.2 \times 1\mu\text{m}^2$.

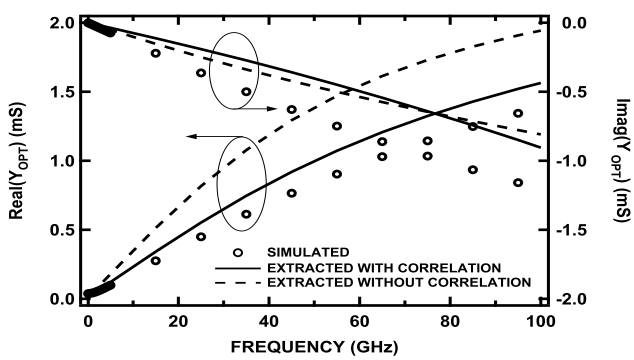

Fig. 5. Simulated and extracted Y_{OPT} with and without correlation at minimum noise bias. The emitter area is $0.2 \times 1\mu\text{m}^2$.

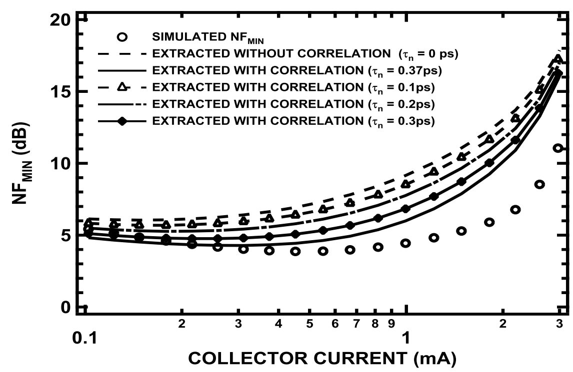

Fig. 6. Simulated and extracted NF_{MIN} at 65 GHz for different values of τ_n. The emitter area is $0.2 \times 1\mu\text{m}^2$.

even at 60 GHz, when shot noise correlation is accounted for using the derived equations with τ_n extracted as in equation (7). NF_{MIN} at 65 GHz was also calculated for different values of τ_n and plotted in Fig. 6. The deviation of the calculated values from those predicted directly by the simulator at high current bias is due to the breakdown of the uncorrelated carrier assumption that is fundamental to the p-n junction shot noise expressions in (1)-(3).

IV. Experimental Results

Experimental validation was carried out by applying the equations to the measured y-parameters of commercial SiGe HBTs featuring f_T and f_{MAX} values of approximately 160 GHz. The noise parameters derived from y-parameters were then compared with those obtained directly from noise parameter measurements.

The S-parameters of the SiGe HBTs were measured up to 65 GHz and de-embedded using the open-short technique [13] before applying the equations (4)-(6) to extract the noise parameters. In addition, directly measured noise parameters were obtained up to 18 GHz using a Focus Microwaves tuner system and a HP8970B/HP8971C noise figure test set.

Fig. 7 shows the high frequency transconductance of the measured SiGe HBT. A value of 0.28 ps was extracted for the noise transit time using equation (7). Figs. 8-11 compare the measured noise parameters of the transistor (with pad contribution) to those calculated solely from measured y-parameters using (4)–(6) with and without accounting for correlation. These results confirm that, for 160-GHz SiGe HBTs, the impact of noise current correlation is insignificant up to 18 GHz and is typically drowned in the noise measurement scatter. In contrast to noise measurements, high frequency S-parameter measurements are less scattered. Considering also the complexity of a noise measurement setup and the difficulty of de-embedding millimetre-wave noise measurements, the extraction of noise parameters from measured S-parameters is preferable and likely more accurate. Finally, using the measured y-parameters, the technique predicts a noise figure that is about 1 dB lower at 60 GHz when correlation is considered (Fig. 12). The later is in good agreement with the measured

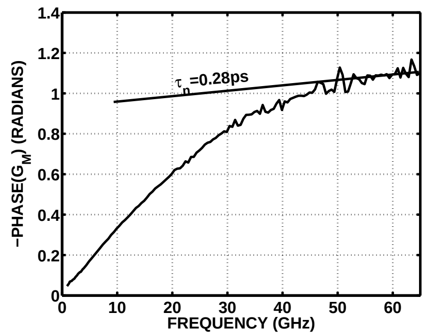

Fig. 7. Measured phase of $g_m(\omega)$ at $V_{CE} = 1.5\,V$.

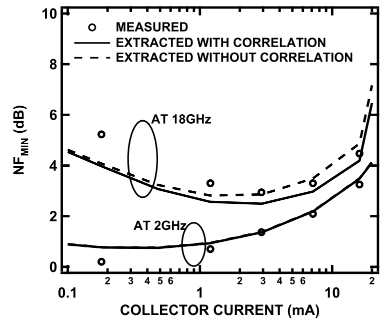

Fig. 8. Measured and modeled NF_{MIN} at 2 and 18 GHz vs. I_C (with pad parasitics).

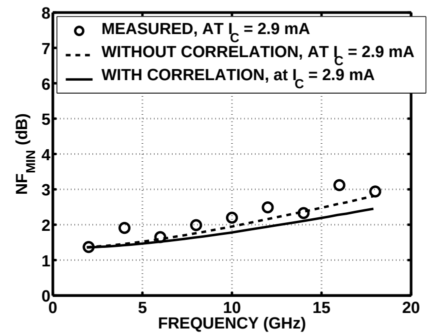

Fig. 9. Measured and modeled NF_{MIN} at minimum noise bias (with pad parasitics).

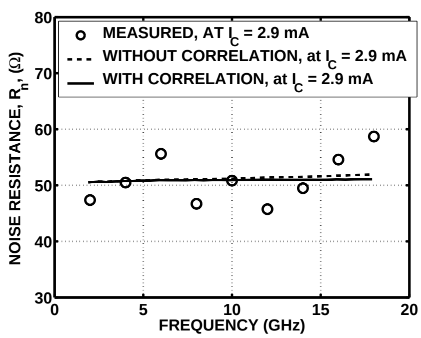

Fig. 10. Measured and modeled R_n at minimum noise bias (with pad parasitics).

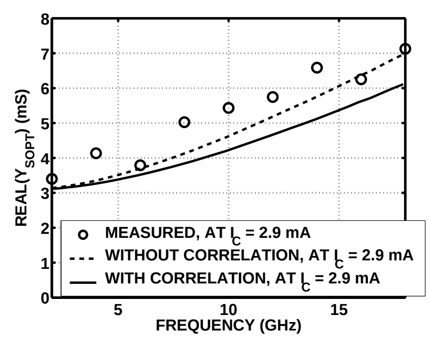

Fig. 11. Measured and modeled $\Re(Y_{SOPT})$ at minimum noise bias (with pad parasitics).

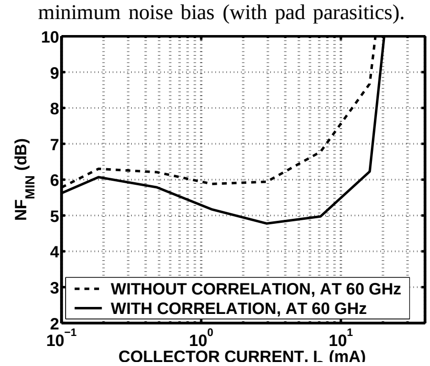

Fig. 12. NF_{MIN} vs. I_C at 60 GHz as extracted from y-parameters after de-embedding pad parasitics.

phase noise of 60-GHz SiGe HBT VCOs [1].

V. Conclusion

A set of new equations for the noise parameters of millimetre-wave SiGe HBTs was presented and a new method was developed to extract the noise transit time solely from HBT y-parameters, without fitting to noise parameter data. The equations and the extraction methodology were verified through device simulations and noise measurements. By applying the new method to the measured y-parameters of 160-GHz SiGe HBTs, the NF_{MIN} at 60 GHz is predicted to be 1 dB lower when correlation is accounted for, in agreement with published VCO and LNA results.

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