Analog Filter Adaptation Using a Dithered Linear Search Algorithm

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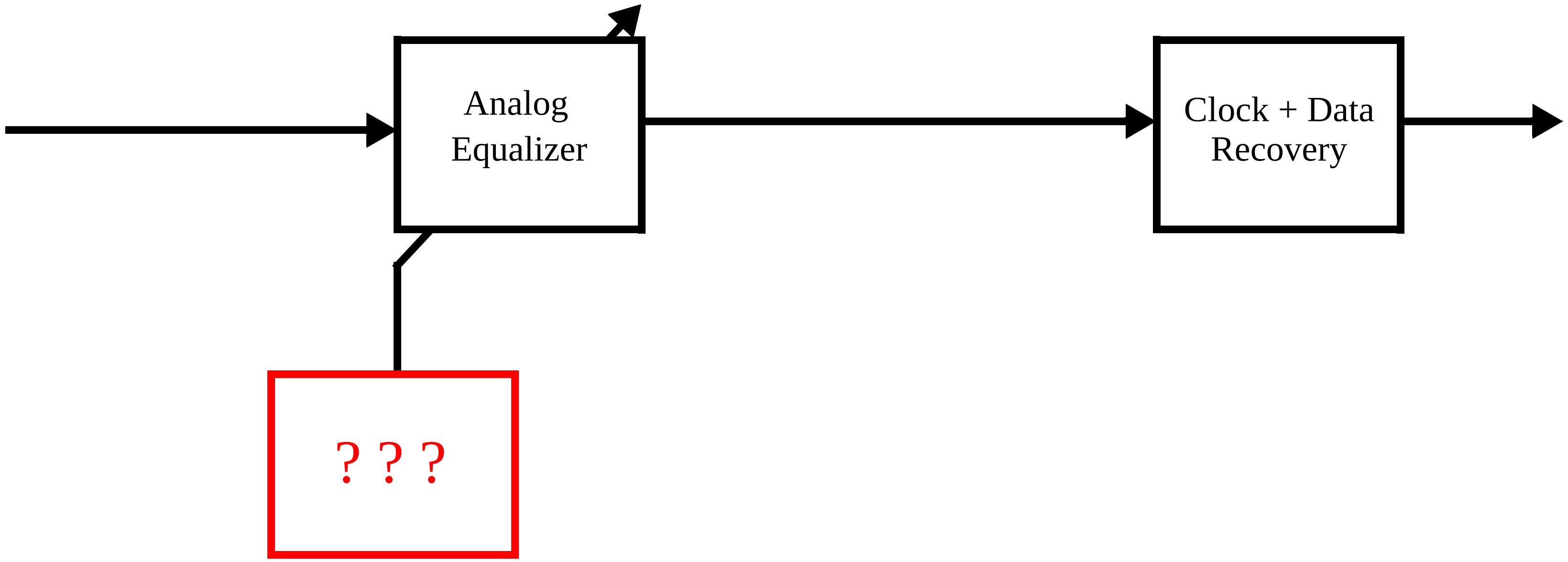
Outline

- Motivation - analog adaptive filters
- Background - LMS algorithm
- Dithered Linear Search
- Results
- Conclusions

Motivation

- Analog adaptive filters offer several important advantages in high speed mixed signal systems:
  - reduced specifications on the A/D converter
  - reduced specifications on the analog line driver (echo cancellation application)
  - moves the equalizer outside of the timing recovery loop
  - potential for power and area savings over digital filters (at high speeds and long impulse responses)



- **Problem**: How are the filter parameters adapted/optimized???

Background - LMS Algorithm

- LMS adaptation is popular for digital filters due to its simple and robust hardware implementation
- filter parameters are updated according to:

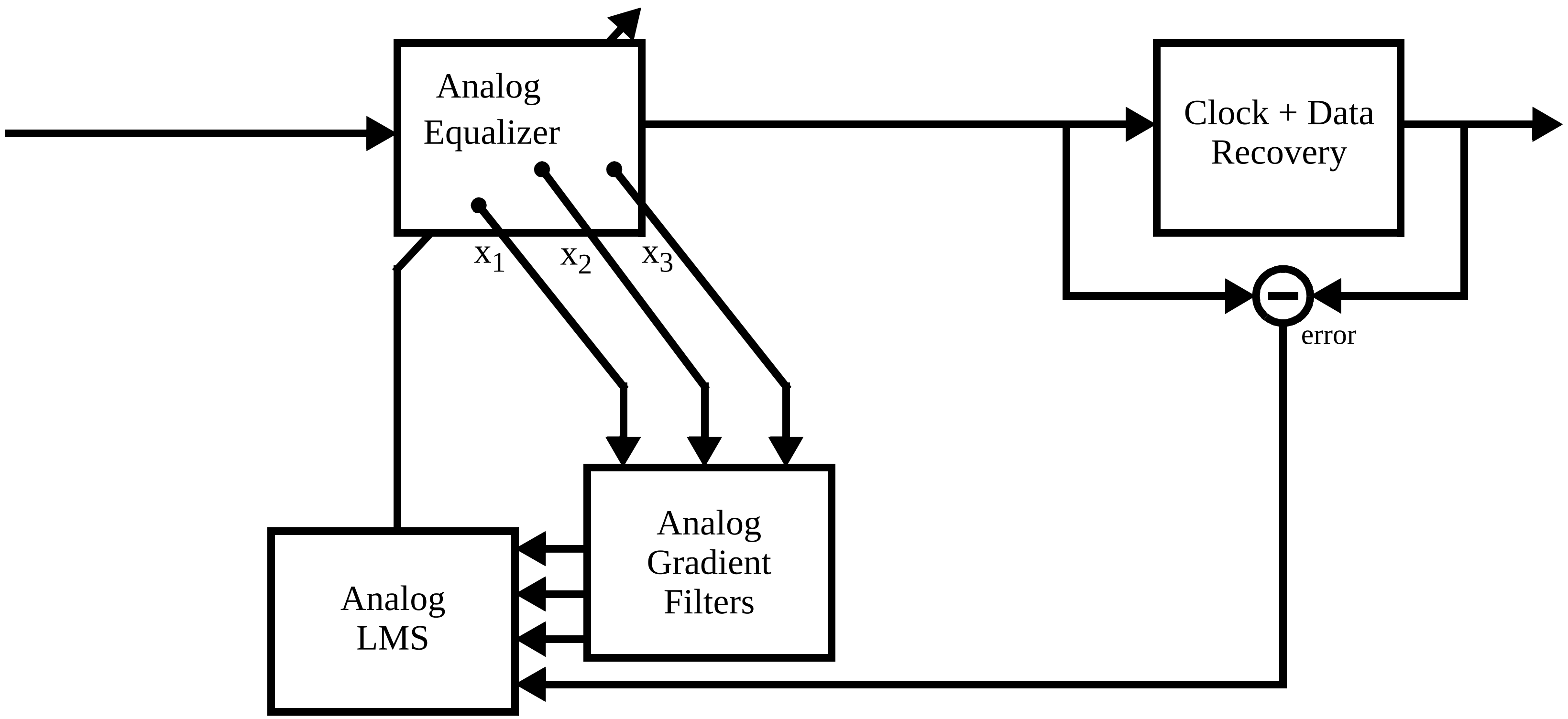
$$p(k+1) = p(k) + 2\mu \cdot \phi(k) \cdot e(k)$$

$$\phi = \frac{\partial y}{\partial p}$$

- **Problem**: For an analog filter, the LMS algorithm is neither simple nor robust!!!
- it can be very difficult to obtain the gradient signals, ϕ
- dc offsets on the gradient and error signals lead to inaccurate convergence

Example: Equalization in Digital Communications

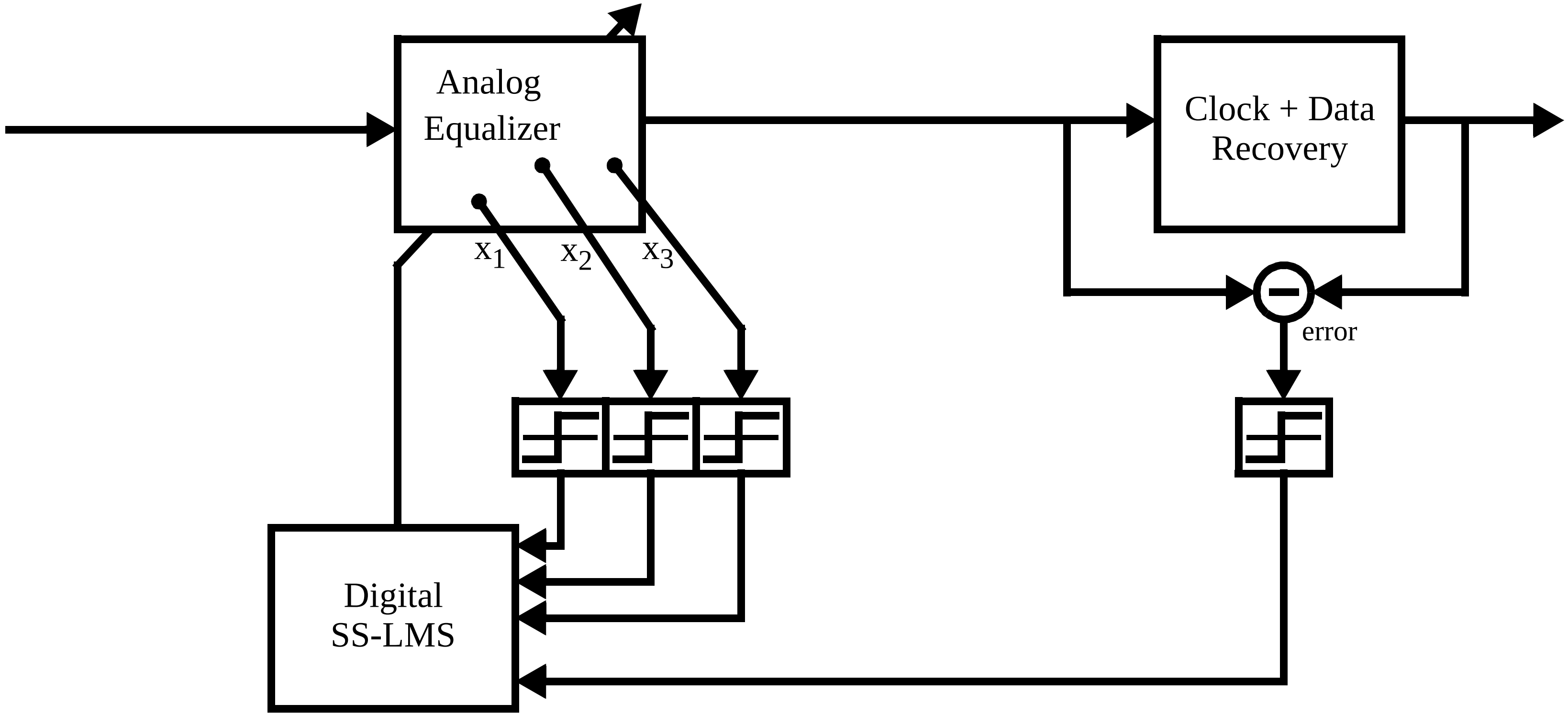
- LMS algorithm implemented with analog circuits



- lots of high-speed analog design required
- dc offsets will hinder adaptation

Example: Equalization in Digital Communications

- Sign-Sign LMS algorithm implemented digitally



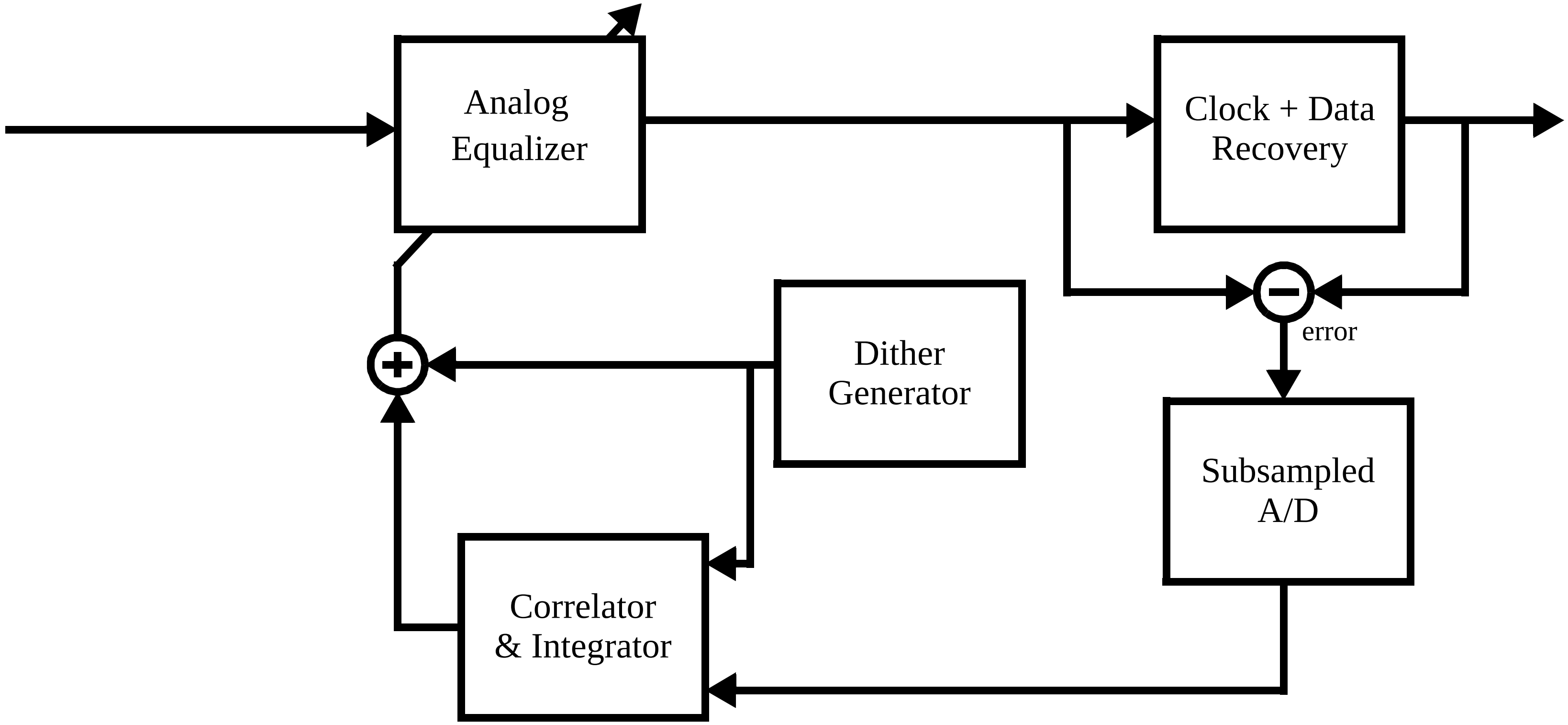
+ comparators may be subsampled and the digital circuitry run at a slow rate

+ dc offset effects can be eliminated

- many additional comparators may be required, each loading the filter's internal nodes

- does not work for some analog equalizers (depends on the filter topology)

Dithered Linear Search



+ applicable to general filter structures

+ dc offset effects can be eliminated

+ the A/D converter and all the digital circuitry can be subsampled & hence, operated at a relatively slow rate

- adaptation can be slow due to averaging required in the correlator

Dithered Linear Search

- general gradient descent algorithm:

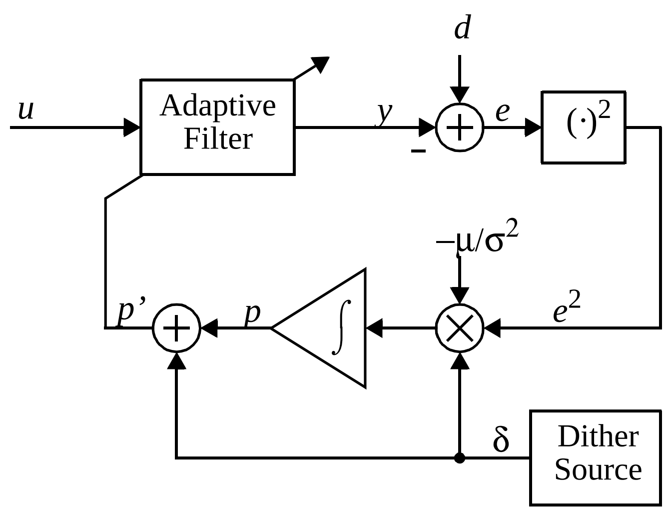
$$p(k+1) = p(k) - \mu \cdot \frac{\partial E[e^2(k)]}{\partial p(k)}$$

- can be shown that,

$$E\left[\frac{1}{\sigma^2} \cdot \delta(k) \cdot e^2(k)\right] = \frac{\partial E[e^2(k)]}{\partial p(k)}$$

- therefore, the DLS algorithm updates the filter parameters according to:

$$p(k+1) = p(k) - \frac{\mu}{\sigma^2} \cdot \delta(k) \cdot e^2(k)$$



Choice of Dither Signal

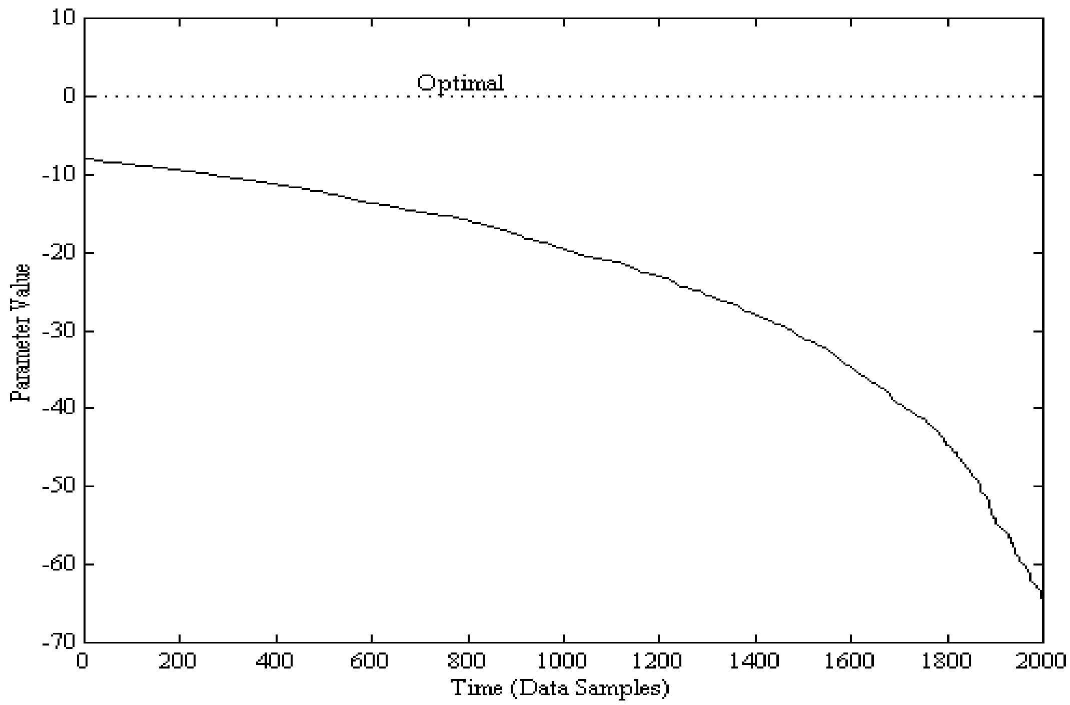
- Binary dither provides straightforward implementation

1. pseudorandom binary sequences

+ simple hardware implementation

+ error introduced by the dither is random

- long sequences of ones or zeros are possible, which can cause the algorithm to diverge



Choice of Dither Signal

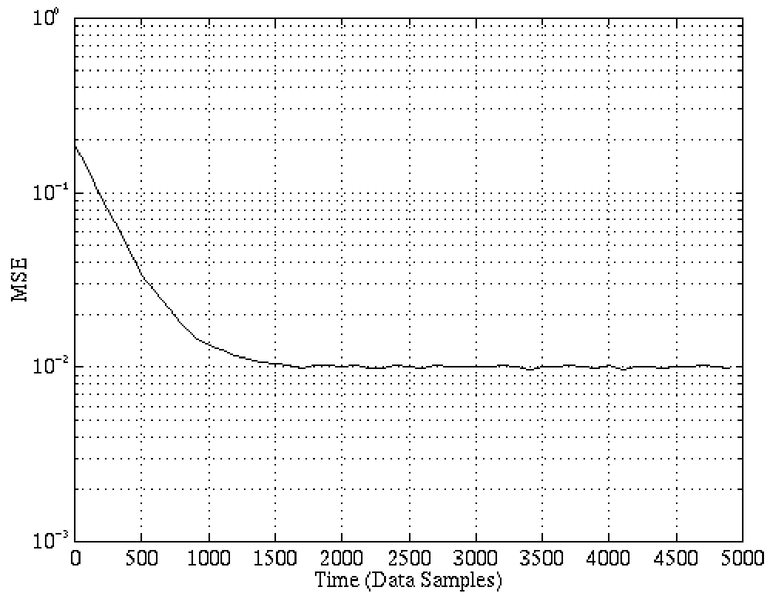
2. orthogonal periodic binary sequences (e.g. Hadamard sequences)

+ simple hardware implementation

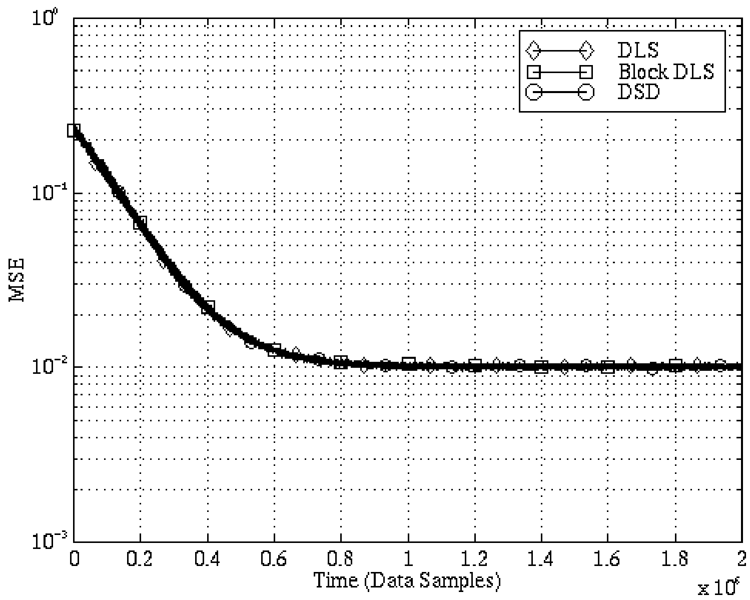
+ no long sequences of ones or zeros

- periodic nature of the dither can cause spurs to appear in the filter output

Not a problem when dither is applied slowly, but DLS is already slow:



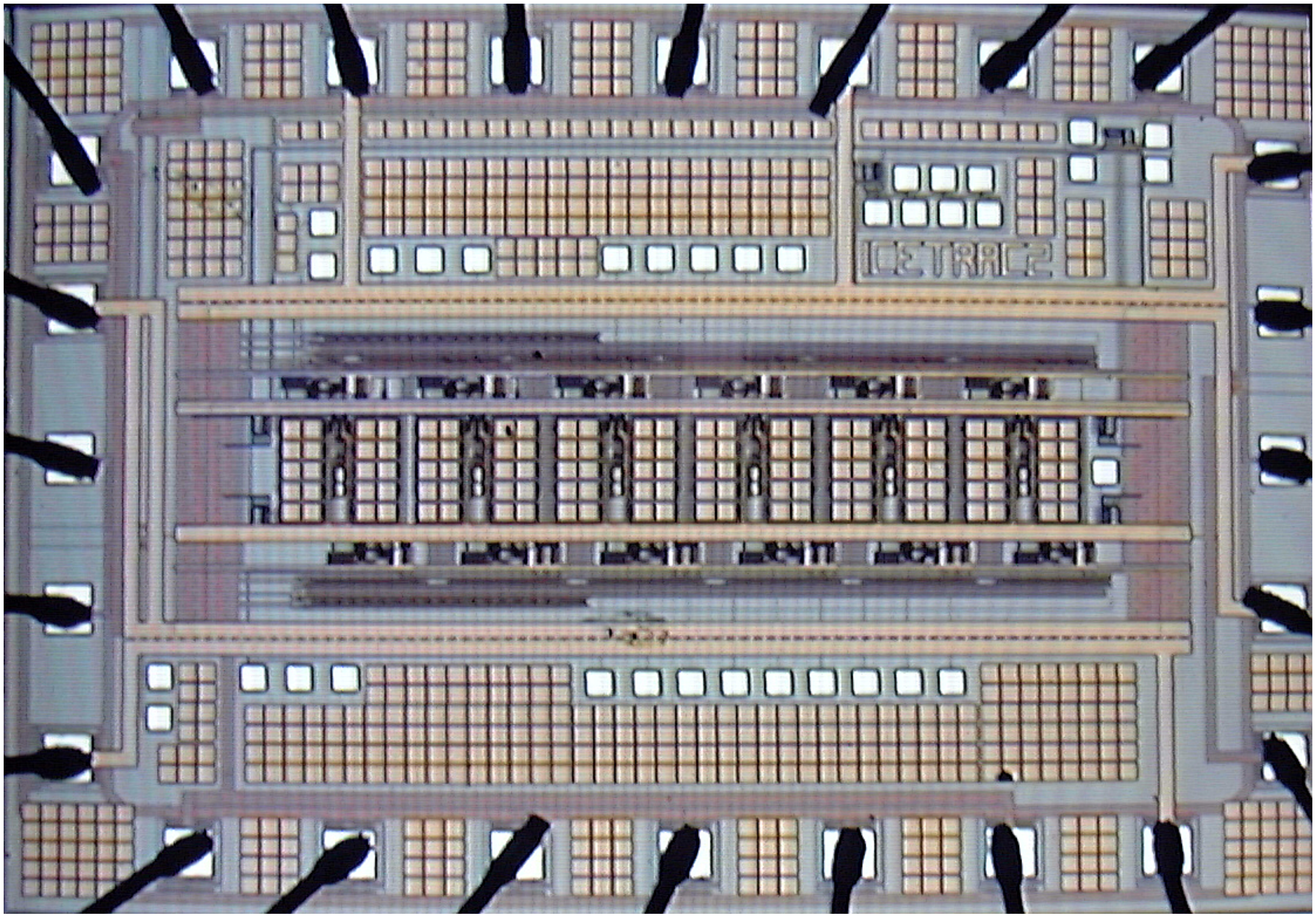
LMS



DLS

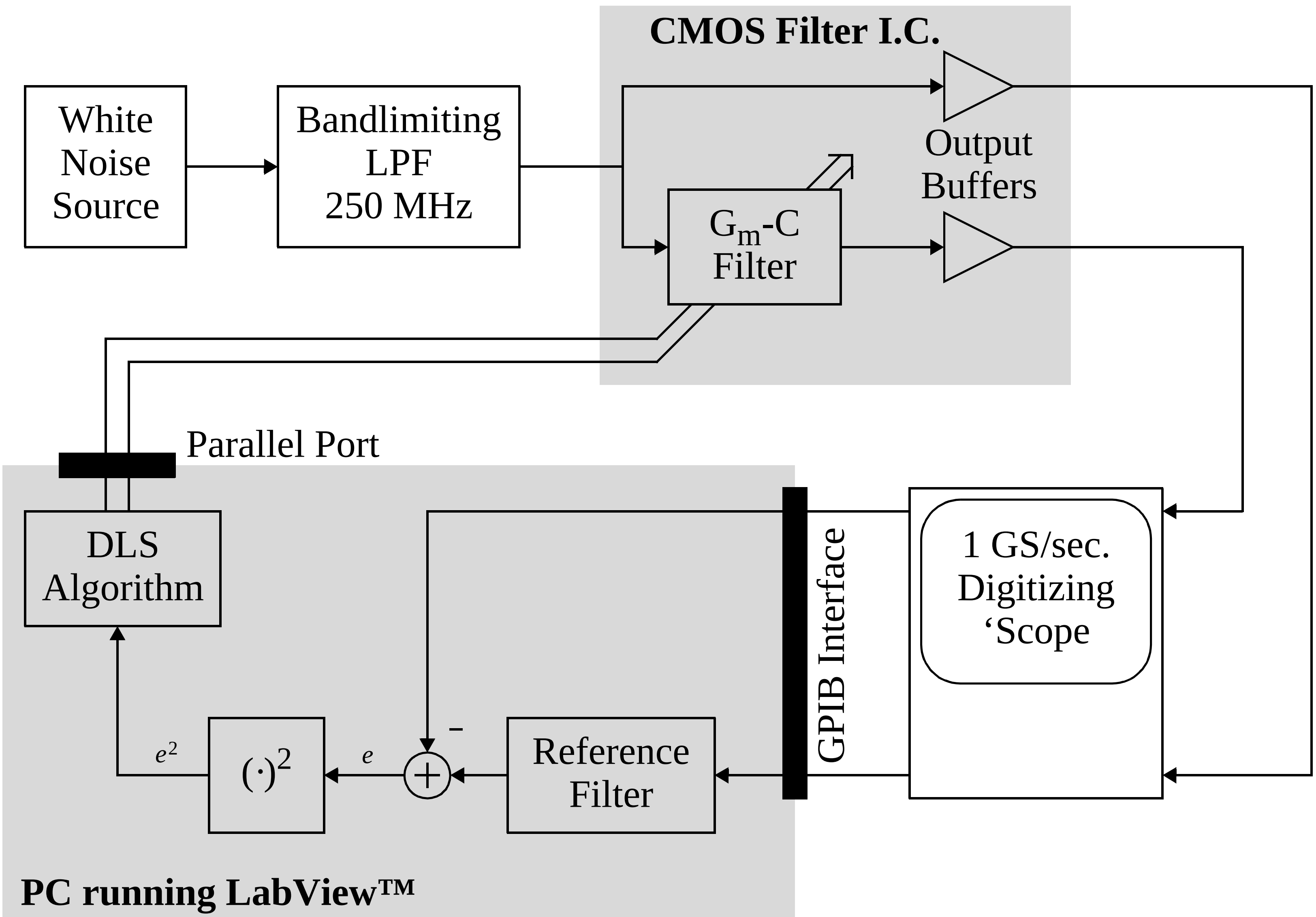
- 1000x slower for a 5-tap FIR filter

Die Photo

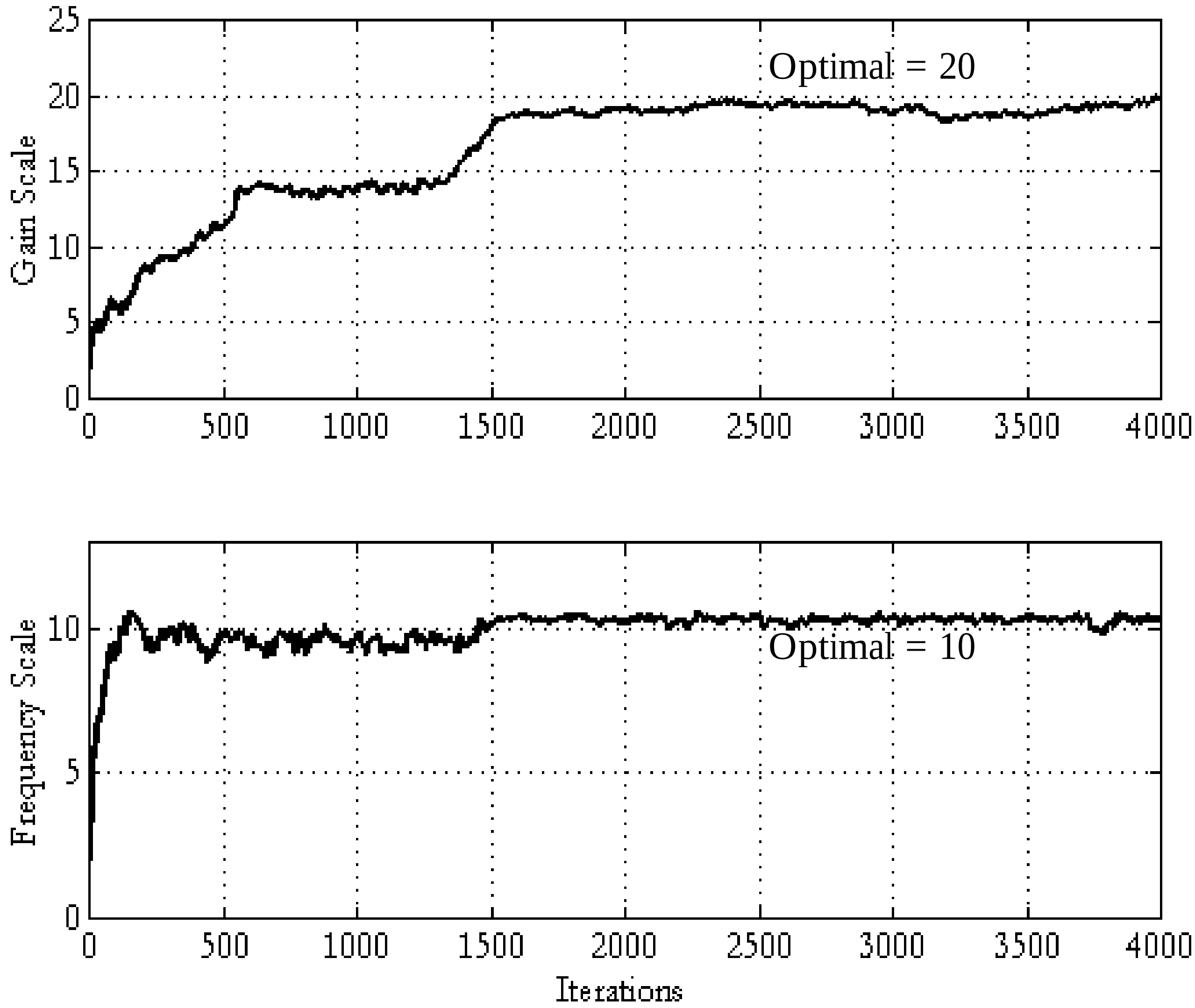


"A 5th order Gm-C filter in 0.25 μm CMOS with digitally programmable poles & zeros," Chan Carusone & Johns, ISCAS '02, Ballroom G

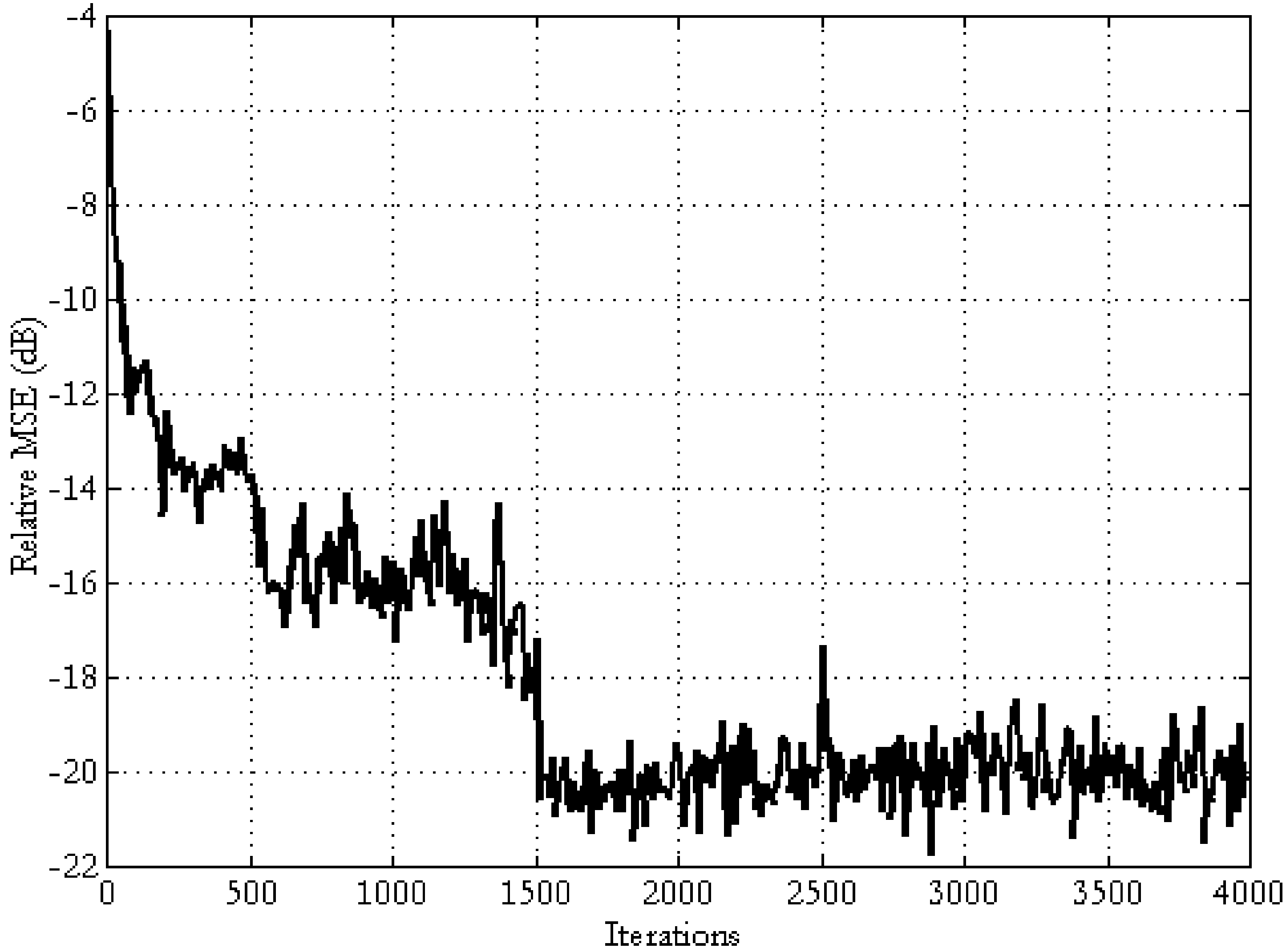
Experimental Setup



Results


Optimal = 20
Gain Scale
25
20
15
10
5
0
0
500
1000
1500
2000
2500
3000
3500
4000
Optimal = 10
Frequency Scale
10
5
0
Iterations

Results


-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
0
500
1000
1500
2000
2500
3000
3500
4000
Relative MSE (dB)
Iterations

Conclusions

- The DLS algorithm is suitable for integrated analog adaptive filters, especially under the following conditions:

  1. the gradient signals required for LMS adaptation are difficult to obtain

  2. slow convergence can be tolerated

- analyses and experimental verification of the algorithm was performed on a 5th order continuous-time integrated filter