OBTAINING DIGITAL GRADIENT SIGNALS FOR ANALOG ADAPTIVE FILTERS

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ABSTRACT

Analog adaptive filters with digitally programmable coefficients can provide speed, power, and area advantages over digital adaptive filters while overcoming the dc offset problems associated with fully analog implementations. However, digital estimates of the filter states and gradient signals must be generated from the filter output in order to perform LMS adaptation. State observers studied in the control literature either require access to the system input or require the system to be minimum phase. Here, approximate time-delayed state estimates are obtained from the filter output by truncating a Taylor series expansion of the inverted non-minimum phase zeros. Simulation results are presented for a 5-tap FIR filter. No steady-state error is introduced by dc and gain offsets.

1. INTRODUCTION

Adaptive filters can be implemented using either digital or analog circuitry. Digital filters are common in low- to moderate- speed applications. They are robust, flexible and easily ported to new process technologies. They do not suffer from the nonlinearities introduced by component mismatches in analog designs. However, at high speeds the silicon area and power consumption of a digital implementation may be prohibitively high. In these cases, analog adaptive filters are required.

Unfortunately, the design of analog adaptive filters is not straightforward due to the challenge of performing the adaptation to minimize some error criterion. When using the LMS algorithm, a significant residual error remains due to the effects of dc offsets which are present on state and error signals [1], [2]. Thus there is a need for finding an adaptation approach which does not suffer in the presence of dc offsets.

In many applications, the signal bandwidths and data rates are increasing but the required adaptation rates are remaining constant in time. So it is natural to consider using analog circuitry for the high-speed signal path while implementing the slower adaptive algorithm using digital circuitry. If a discrete version of the LMS adaptive algorithm is employed, filter coefficients are updated according to (1) where p_i are the filter coefficients, e is the error in the filter output with respect to some reference signal and μ is a variable controlling the adaptation rate.

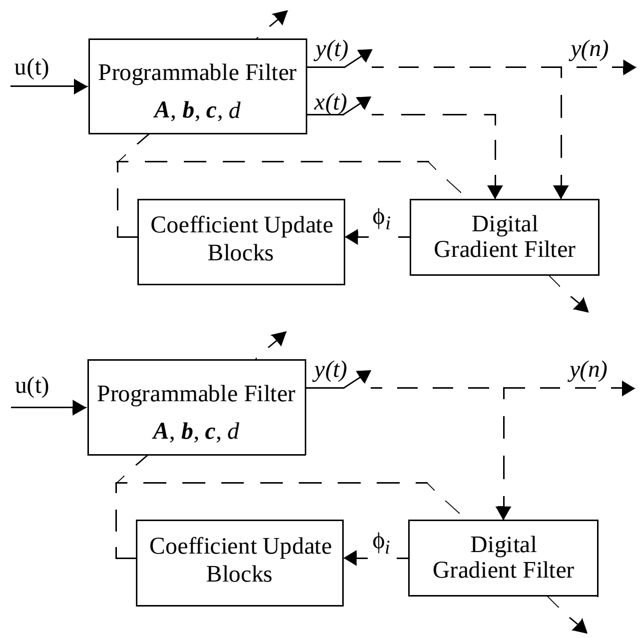

FIGURE 1. Digital Adaptation of an Analog Filter With (1a) and Without (1b) Sampling the Internal State Signals (dashed lines correspond to digitized signals)

$$p_i(n+1) = p_i(n) + 2\mu e(n)\phi_i(n) \tag{1}$$

A potential problem with this approach is that a gradient signal, ϕ_i, is required for each adapted coefficient. If a state-space model is used for the programmable filter, gradient signals can be generated from a knowledge of the filter’s internal states as in [1]. This suggests the architecture depicted in Figure 1a. However, if A/D converters were used to sample each of the filter’s internal states circuit size and complexity may become prohibitive. This work examines a method for estimating the gradient signals of arbitrary state-space adaptive filters using only the digitized filter output, allowing for the modified system architecture depicted in Figure 1b.

Section 2 looks at established techniques for estimating the internal states of a system. Section 3 explores a method

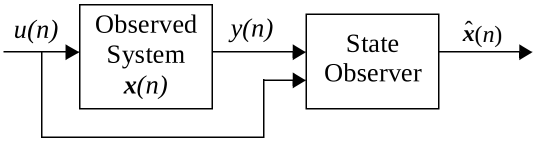

FIGURE 2. Standard Linear State Observation

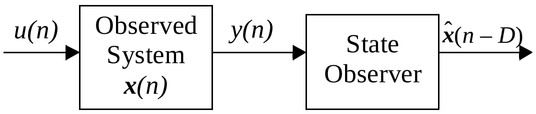

FIGURE 3. Time-Delayed and Unknown Input State Observation

for generating approximate time-delayed state estimates of non-minimum phase systems with unknown inputs. Section 4 provides simulation results in the presence of analog dc offsets.

2. UNKNOWN INPUT OBSERVATION

The estimation of a system's internal states, called state observation, is well documented in control theory [3], [4], [5]. A linear state observer can be designed of order equal to that of the system under observation. Furthermore, the dynamics of a state observer can be designed to meet some desired tracking performance specification. Unfortunately, in general access to both the system inputs and outputs is required to generate these state estimates (Figure 2).

The situation in Figure 1b is somewhat different. The "system under observation" is the programmable filter ($\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, d) and only its output is available. Also, since the programmable filter coefficients can be adapted based upon gradient information from some time in the past (as long as the overall system's adaptation rate is satisfactory) time-delayed state estimates are sufficient. A directly analogous problem appears in the control literature and is called "time-delayed and unknown input observation" (Figure 3). It was shown in [6] that this problem is equivalent to inversion of the observed system. This is intuitively satisfying because if the observed system can be inverted then its input can be reproduced from its output and the problem of state observation becomes trivial.

Unfortunately, it is difficult to ensure that an adaptive filter will always be invertible, particularly if the zeros are being adapted. In this case the filter may become non-minimum phase and the resulting inverted system would have unstable poles.

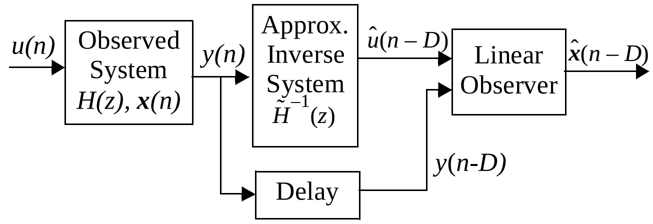

FIGURE 4. Generation of an Approximate Time-Delayed State Estimate

3. APPROXIMATE TIME-DELAYED STATE OBSERVATION

This section describes a method for providing time-delayed state estimates of an arbitrary state-space system. The approach taken is to approximate the inverse transfer function of the system under observation by introducing delay. Once a delayed estimate of the system input is obtained, standard linear state observers can be used to produce time-delayed state estimates, as in Figure 4.

Consider a stable, real, rational, discrete time transfer function $H(z)$ with zeros z_i and poles p_k.

$$H(z) = A \cdot \frac{\prod_k (1 - z_k z^{-1})}{\prod_l (1 - p_l z^{-1})} \tag{2}$$

This could be the transfer function of any real stable discrete state space system, or the equivalent transfer function of a continuous time state space system sampled at or above the Nyquist rate. The (possibly unstable) inverse transfer function is:

$$H^{-1}(z) = \frac{1}{A} \cdot \frac{\prod_l (1 - p_l z^{-1})}{\prod_k (1 - z_k z^{-1})} \tag{3}$$

This can be factored into two parts: a stable part, $H_S(z)$, including all zeroes and stable poles, and an unstable part, $H_U(z)$, including only unstable terms:

$$H_U(z) = \frac{1}{\prod_i (1 - z_i z^{-1})} \tag{4}$$

If we define $a_i = 1/z_i$, each binomial factor of $H_U(z)$ can be approximated by the following truncated Taylor Series expansion about $z = 0$:

$$\begin{aligned} 1/(1 - z_i z^{-1}) &= (-a_i z)/(1 - a_i z) \\ &= (-a_i z)(1 + a_i z + a_i^2 z^2 + \dots) \\ &= z^{d+1}(-a_i z^{-d} - a_i^2 z^{-d+1} - \dots - a_i^{d+1}) \\ &= H_i(z) \end{aligned} \quad (5)$$

Note that in (5) the series was truncated after d terms. This allows the unstable pole to be approximated by an FIR filter with d taps. The z^{d+1} term indicates that a delay of $(d + 1)$ is introduced by the approximation. The choice of d depends upon several factors: the required adaptation rate, the number of coefficients which can be reasonably computed and stored, and the amount of approximation error which can be tolerated.

Expanding each binomial term in $H_U(z)$ this way provides the following stable FIR approximation for $H^{-1}(z)$:

$$H^{-1}(z) = H_S(z) \cdot H_U(z) \approx H_S(z) \cdot \prod_i H_i(z) = \tilde{H}^{-1}(z) \quad (6)$$

Since any complex zeros of $H(z)$ will appear in conjugate pairs, their corresponding series expansions will have imaginary terms which cancel when combined leaving purely real coefficients on the right side of (6). This transfer function can then be used in conjunction with the system depicted in Figure 4 to generate time-delayed state estimates of a non-minimum phase system.

4. SIMULATION RESULTS

The ideas described above were simulated in MATLAB [7]. An adaptive 5-tap FIR filter was chosen since analog realizations of digitally programmable FIR filters have proven to be practical in real-world applications (for instance, [8]). The gradient signals for FIR filters are simply time-shifted versions of the filter input, so the linear observer and gradient filter can be replaced by a tapped delay line. The simulated system is shown in Figure .

The input $u(n)$ was driven by a zero-mean Gaussian white noise source with unit power. Finite dc offsets, $\boldsymbol{m_x}$, and gain errors, $\boldsymbol{g}$, were introduced at each tap of the programmable filter to simulate the effects of component mismatches. A constant tap element was used for dc offset-cancellation. No quantization noise was included in order to isolate the effect of dc offsets. The impulse response of stable IIR terms in (6) were truncated; this allowed $H^{-1}(z)$ to be completely approximated by a simple FIR filter of length $N = 50$. Simulations were performed with several sets of fixed

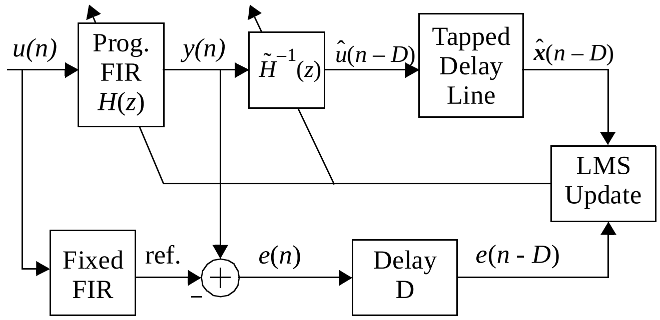

FIGURE 5. Model-Matching Configuration for Adaptive FIR Filter Simulation

filter tap weights, $\boldsymbol{c}$, to ensure accurate adaptation with transfer function zeros both inside and outside of the unit circle.

The filter coefficients and error signal, $e(n)$, are plotted over time in Figure 6 and Figure 7 for one simulation run. The dc offsets, gain errors, and fixed filter coefficients used are:

$$\boldsymbol{m}_x = \begin{bmatrix} 0.01 & 0.03 & -0.08 & -0.02 & 0.04 \end{bmatrix}^T \quad (7)$$

$$\boldsymbol{g} = \begin{bmatrix} 1.04 & 0.98 & 0.92 & 1.03 & 0.95 \end{bmatrix}^T \quad (8)$$

$$\boldsymbol{c} = \begin{bmatrix} 0.9 & -0.9 & -0.4 & -0.2 & 0.1 \end{bmatrix}^T \quad (9)$$

Similar experiments modeled for a fully-analog LMS adaptive system in [2] indicated finite steady-state errors approximately 10 to 20 dB below the input signal level. Zero steady-state error is present using the techniques described in this paper. Presumably, this is because the state signals used for adaptation are being generated from the dc offset-free output, $y(n)$, rather than using the values $\boldsymbol{x}(n)$ directly with their associated offsets.

5. CONCLUSIONS

Techniques for state observation which have been studied in the control literature either require access to both the system inputs and outputs, or require the system under observation to be minimum phase. An approximate time-delayed state estimator was proposed which can be applied to any state space system with unknown inputs. The strategy was applied in simulations to the adaptation of a 5-tap FIR filter with success. Simulations indicated that the technique is immune to the dc offset effects.

6. REFERENCES

[1] D.A. Johns, W.M. Snelgrove and A.S. Sedra, "Continuous-Time LMS Adaptive Recursive Filters," *IEEE*

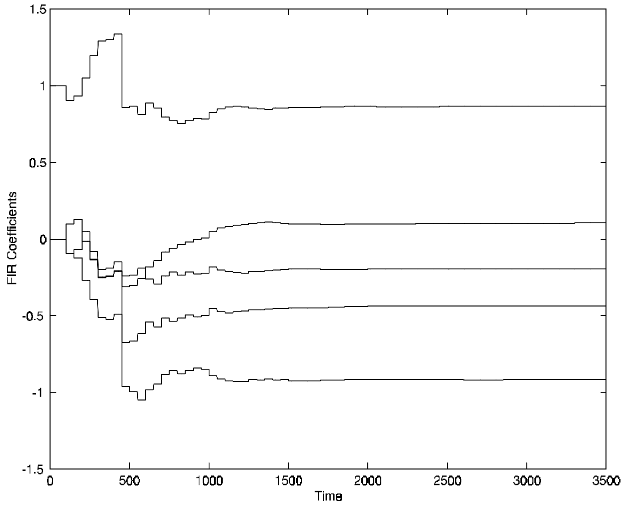

FIGURE 6. 5-Tap Adaptive FIR Filter Coefficients over time

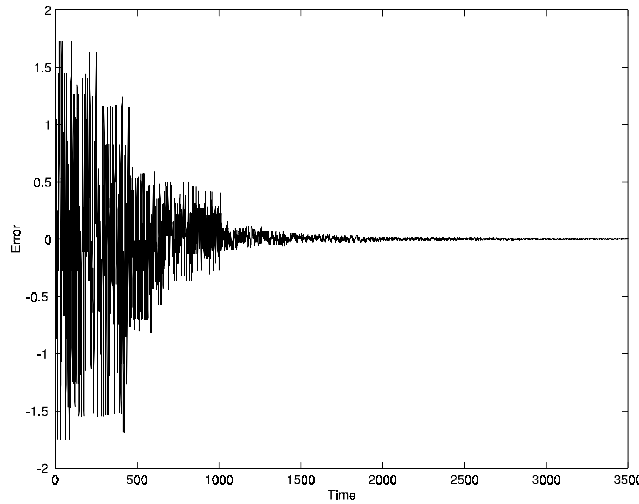

FIGURE 7. 5-Tap Adaptive FIR Filter Error Signal over time

Trans. Circuits Syst., vol. CAS-38, pp. 769-778, July 1991.

[2] A. Shoval, D.A. Johns and W.M. Snelgrove, “Comparison of DC Offset Effects in Four LMS Adaptive Algorithms,” *IEEE Trans. Circuits Syst.*, vol. CAS-42, pp. 176-185, March 1995.

[3] R.E. Kalman, “A New Approach to Linear Filtering and Prediction Problems,” *ASME J. of Basic Eng.*, vol. 820, pp. 35-45, 1960.

[4] R.E. Kalman and R.S. Bucy, “New Results in Linear Filtering and Prediction Problems,” *ASME J. of Basic Eng.*, vol. 830, pp. 95-108, 1961.

[5] D.G. Luenberger, “Observing the State of a Linear System,” *IEEE Trans. on Military Electronics*, vol. MIL-8, pp. 74-80, 1964.

[6] J. Jin, M. Tahk and C. Park, “Time-delayed state and unknown input observation,” *Int. J. Control*, vol. 66, pp. 733-745, 1997.

[7] MATLAB, *Matrix Laboratory*, The MathWorks, Inc., Natwick MA, USA, 1996.

[8] S. Kiriaki et al, “A 160-MHz Analog Equalizer for Magnetic Disk Read Channels,” *IEEE J. Solid-State Circuits*, vol. 32, pp. 1839-1850, Nov. 1997.